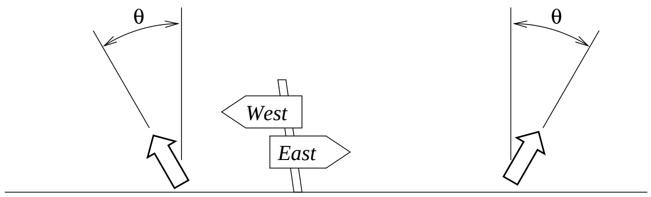
Physics 310 - Second Midterm - November 21, 2006

Instructions

- This exam has 4 pages including this cover sheet.
- There are six questions on this exam - answer any four of them.
- The questions are to be answered in the blue exam booklets.
- Clearly indicate which of the six questions you want marked.
- Write your name on every exam booklet you use.
- No notes, text books, crib sheets, *etc.* are to be used when writing this exam.
- You will not need a calculator for this exam.
- The exam begins at 9:30 am and must be turned in by 11:30 am.
- No guinea-pigs were harmed in the making of this exam.

1. Two rockets are launched from a point on the equator which is a distance r_0 from the center of the earth. Both are launched at an angle θ with respect to the zenith angle, but one is facing east and the other is facing west, as shown below.



(When viewed looking North)

(a) What is the magnitude of the initial velocity of each rocket relative to the center of the earth when its initial velocity relative to the earth's surface is v_0? Clearly state all assumptions and describe all variables used.

(b) What is the minimum velocity, relative to the surface of the earth, that is needed for each rocket to escape the gravitational field of the earth? Assume that the "burn" is short, so that the initial velocity is v_0 when $r = r_0$.

You may recall that

$$\Phi(r) = -\frac{GM}{r}. \tag{1}$$

Also, remember that the sun rises in the east and sets in the west once every 24 hours...

2. A satallite is in an elliptical orbit with eccentricity ϵ at a distance r_0 from the center of the earth at its *apogee*. Show that the change in kinetic energy needed at this point in its orbit that will place the satallite in a circular orbit of radius r_0 is

$$\Delta T = \frac{k\epsilon}{2r_0}. \tag{2}$$

You may wish to use the relation

$$r(\theta) = \frac{m\ell^2/k}{1 + \epsilon\cos\theta} \tag{3}$$

where $\ell = |\boldsymbol{r} \times \boldsymbol{v}|$ =constant.

3. Three particles of equal mass m have initial velocities

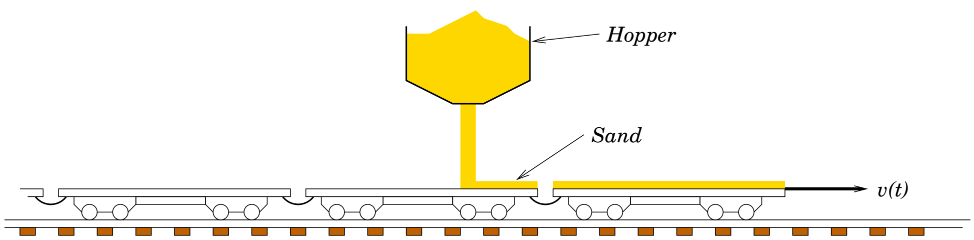
$$\begin{aligned} \boldsymbol{v}_1 &= v\hat{\boldsymbol{\imath}} \\ \boldsymbol{v}_2 &= -v\hat{\boldsymbol{\imath}} \\ \boldsymbol{v}_3 &= v\hat{\boldsymbol{\jmath}} \end{aligned}$$

and are set in motion such that all three particles collide simultaneously.

(a) Calculate the momentum of the center-of-mass of the system after the collision.

(b) If the collision is completely inelastic, calculate the amount of kinetic energy lost in the collision.

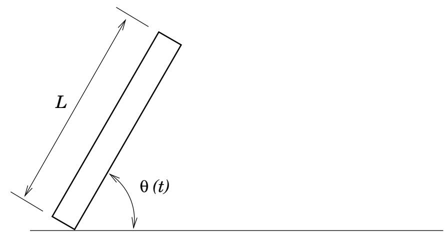
4. A long train is being loaded with sand via a hopper that drops sand into the train cars at a constant rate as shown:



If the train has velocity v_0 and mass m_0 when it first passes under the hopper at $t = 0$, calculate the velocity of the train as a function of time.

Assume that the falling sand exerts no force in the horizontal direction. Also, ignore the tiny cracks between the train cars and assume that no sand spills off the train cars.

5. A thin stick of uniform cross section with mass M and length L is tipping over as shown:

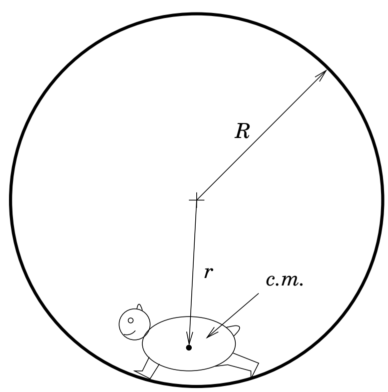


(a) Calculate the moment of inertia of the stick about the point resting on the ground.

(b) Use energy conservation principles to calculate $\omega = \dot{\theta}$ as a function of the angle θ, assuming the stick was initially at rest with $\theta = \pi/2$.

The point $\theta = \pi/2$ is a point of unstable equilibrium, but assume that the stick does indeed tip over.

6. Chipper, the Miller Elementary School grade 4 class guinea pig, runs in his exercise wheel. The wheel can be modelled as a uniform cylinder with mass M and radius r as shown:



(a) When Chipper is stationary in his cage, but runs with velocity v relative to the wheel, calculate the angular momentum of the exercise wheel.

(b) If Chipper then stops running and hangs onto the wheel, calculate the resulting average angular velocity of Chipper and the wheel given that Chipper has a mass m and a moment of inertia I_0 about his center of mass, which is located at a radius r. Ignore any torque produced by gravity since this would average to zero over one revolution.