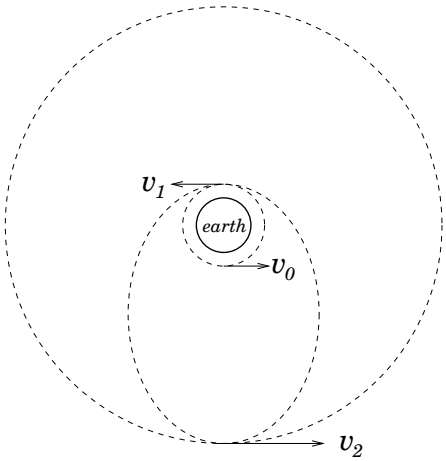
Physics 310 - Assignment #4 - Due November 7^{th}

1. The diagram below shows the orbital maneuvers of a spacecraft that travels with initial velocity v_0 in a circular low earth orbit with an altitude of 200 nautical miles (1 NM = 1.852 km). It changes its velocity to v_1 so that its apogee is at the mean radius of the moon's orbit, $r_m = 384,400$ km. It then changes its velocity to v_2 so that it is in a circular orbit of radius r_m.



(a) Calculate the initial velocity, v_0, of the spacecraft.
(b) Calculate the velocity v_1 that will place it in the second elliptical orbit.
(c) Calculate the eccentricity of the second orbit.
(d) Calculate the velocity v_2 that will place it in the final circular orbit.
(e) Use Kepler's 3rd law to calculate the time it takes to travel from the initial circular orbit to the final circular orbit.

2. Calculate the altitude of a circular geosynchronous orbit, that is, one that has a period of 24 hours. A satallite in a geosynchronous orbit appears to remain in a fixed position above the earth.

3. If two bodies undergo a direct collision, show that the loss of kinetic energy is equal to

$$\frac{1}{2}\mu v^2(1-\epsilon^2)$$

where μ is the reduced mass, v is the relative speed before impact and ϵ is the coefficient of restitution.

4. Show that the angular momentum of a two-particle system is

$$\boldsymbol{r}_{cm} \times m\boldsymbol{v}_{cm} + \boldsymbol{R} \times \mu\boldsymbol{v}$$

where $m = m_1 + m_2$, μ is the reduced mass, $\boldsymbol{R}$ is the relative position vector and $\boldsymbol{v}$ is the relative velocity of the two particles.

5. A uniform heavy chain of length a hangs initially with a part of length b hanging over the edge of a table. The remaining part, of length $a - b$, is coiled up at the edge of the table. If the chain is released from rest, show that the speed of the chain when the last link leaves the end of the table is $[2g(a^3 - b^3)/3a^2]^{1/2}$.