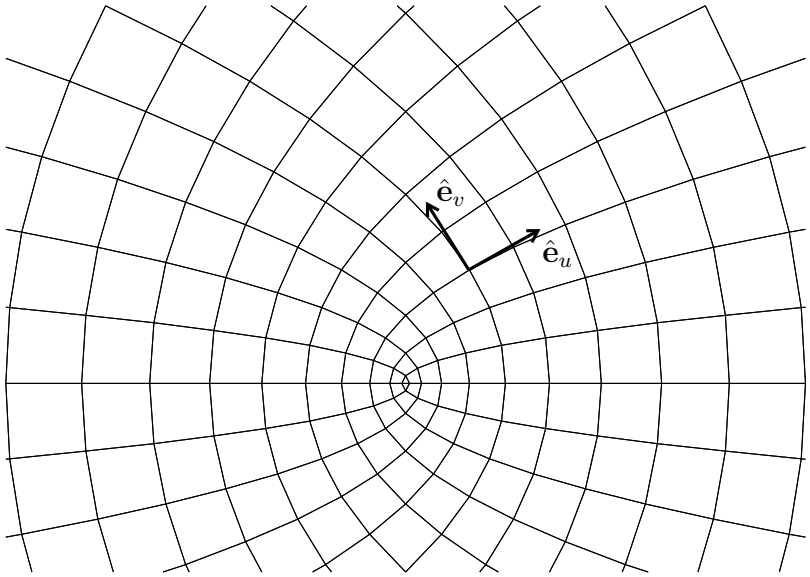
Physics 310 - Assignment #1 - Due September 12^{th}

1. A cylindrical parabolic coordinate system is one in which coordinates (u, v, z) correspond to the Cartesian coordinates given by

$$\begin{aligned} x &= \frac{1}{2}(u^2 - v^2) \\ y &= uv \\ z &= z \end{aligned}$$

as shown:



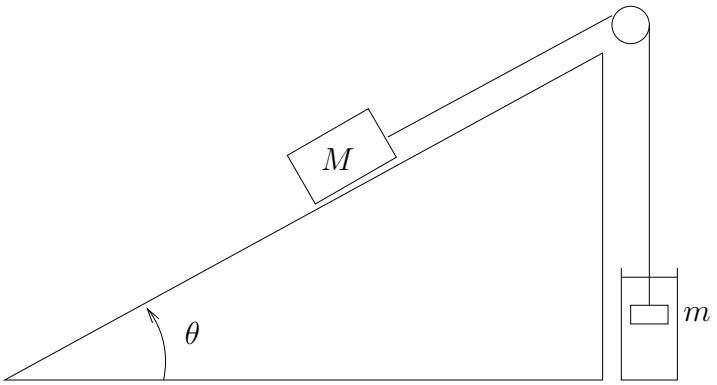
(a) Express the unit vectors $\hat{\mathbf{e}}_u$ and $\hat{\mathbf{e}}_v$ in terms of the Cartesian unit vectors, $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$.
(b) For any vector, $\mathbf{r}$ with coordinates (u, v, z) in the parabolic coordinate system, express it in terms of the $\hat{\mathbf{e}}_u, \hat{\mathbf{e}}_v$ and $\hat{\boldsymbol{k}}$ unit vectors.
(c) Express the derivatives $d\hat{\mathbf{e}}_u/dt$ and $d\hat{\mathbf{e}}_v/dt$ in terms of the unit vectors in the parabolic coordinate system.

2. Find a solution for the motion of an object of mass m with initial velocity v_0, moving through a fluid that produces both linear and quadratic fiscous drag, that is, find $x(t)$ when the only forces acting on the object are

$$F_{\text{drag}} = -c_1 v - c_2 v|v|.$$

Consider separately the two cases when the initial velocity is $v_0 > 0$ and $v_0 < 0$.

3. A box of mass M sits on an frictionless incline plane attached to a thin string that passes over a pulley and is then attached to another mass m which sits in a vat of olive oil as shown.



The oil exerts a resistive force on the mass that is proportional to its velocity. If the box is released from rest, calculate the distance that it moved down the incline plane as a function of time. Describe the solution for the two cases, $M \sin\theta > m$ and $M \sin\theta < m$.

4. For a mass m, to which a force $F(x) = -kx$ is applied, use the formal solution

$$\int_{x_0}^{x} \frac{dx}{\pm\sqrt{\frac{2}{m}[E - V(x)]}} \quad = \quad t - t_0$$

to construct a solution for $x(t)$. Write the solution for the case when the mass is released from rest at $t = 0$, with initial displacement x_0.