Relativistic turbulence in the pulsar wind

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Purdue Workshop May. 10, 2016

With Krzysztof Nalewajko, William East, Yajie Yuan, Roger Blandford

Overview

- What have we learned from ABC magnetic fields?
- Crab Flares

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What have we learned from ABC fields?

This conference —

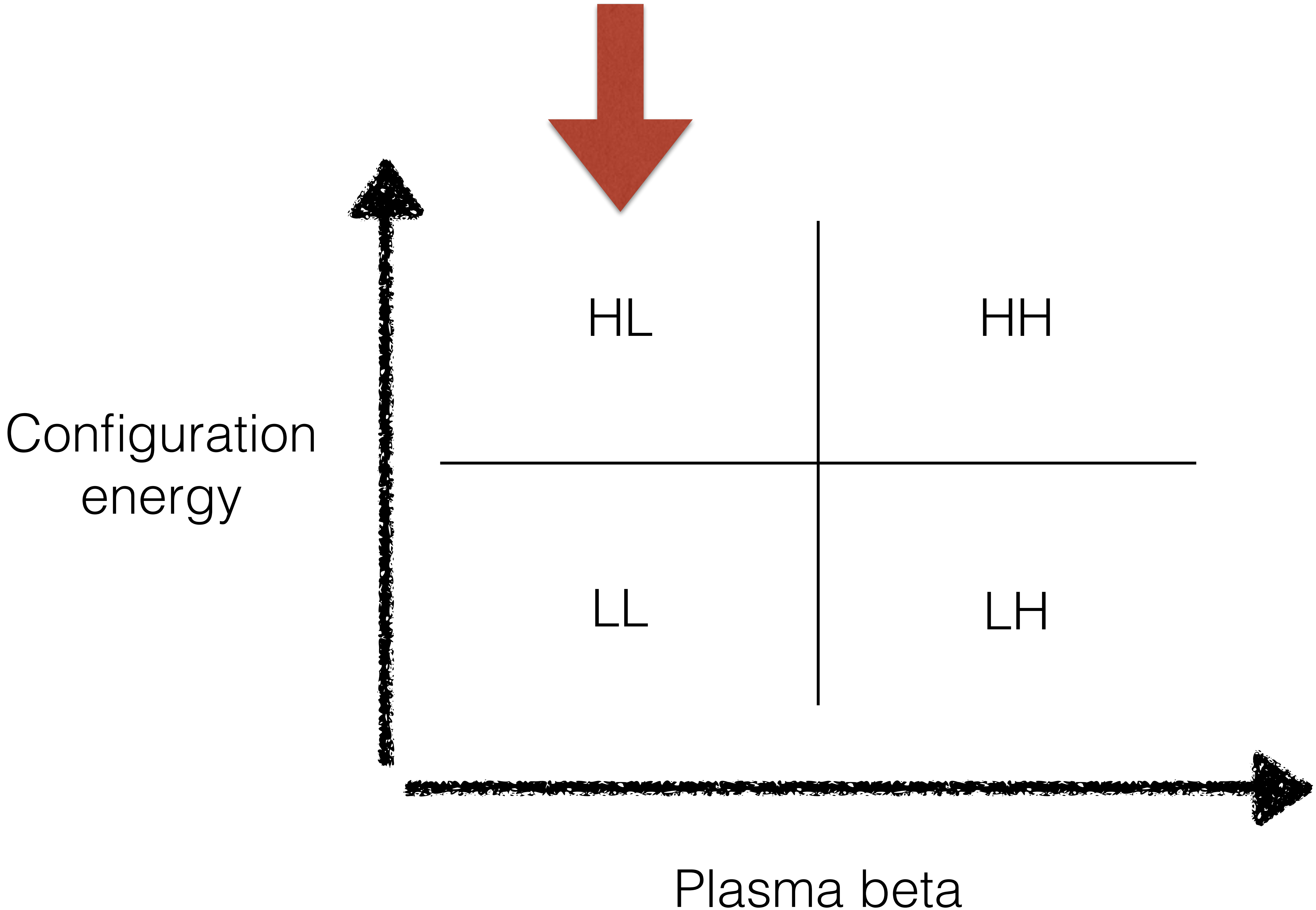
- Strive to study reconnection *in context.*
- *Onset problem* is the million dollar question.

Free energy

$$H = \int \mathbf{A} \cdot \mathbf{B} d^3x$$

"Energy that can be dissipated at constant magnetic helicity"

$$\partial_t H_{\mathcal{V}} = -2 \int_{\mathcal{V}} \mathbf{E} \cdot \mathbf{B} d^3x$$



(from Hui Li's talk)

Conjecture —

"When a force-free equilibrium has free energy, it is linearly unstable."

$$\mathbf{B}^{\mathrm{ABC}}(\mathbf{x}) = \begin{pmatrix} B_3 \cos \alpha_0 z - B_2 \sin \alpha_0 y \\ B_1 \cos \alpha_0 x - B_3 \sin \alpha_0 z \\ B_2 \cos \alpha_0 y - B_1 \sin \alpha_0 x \end{pmatrix}$$

- Force-free equilibrium, J || B
- $\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$
- Have **free energy** when alpha > 1

Spontaneous decay of periodic magnetostatic equilibria

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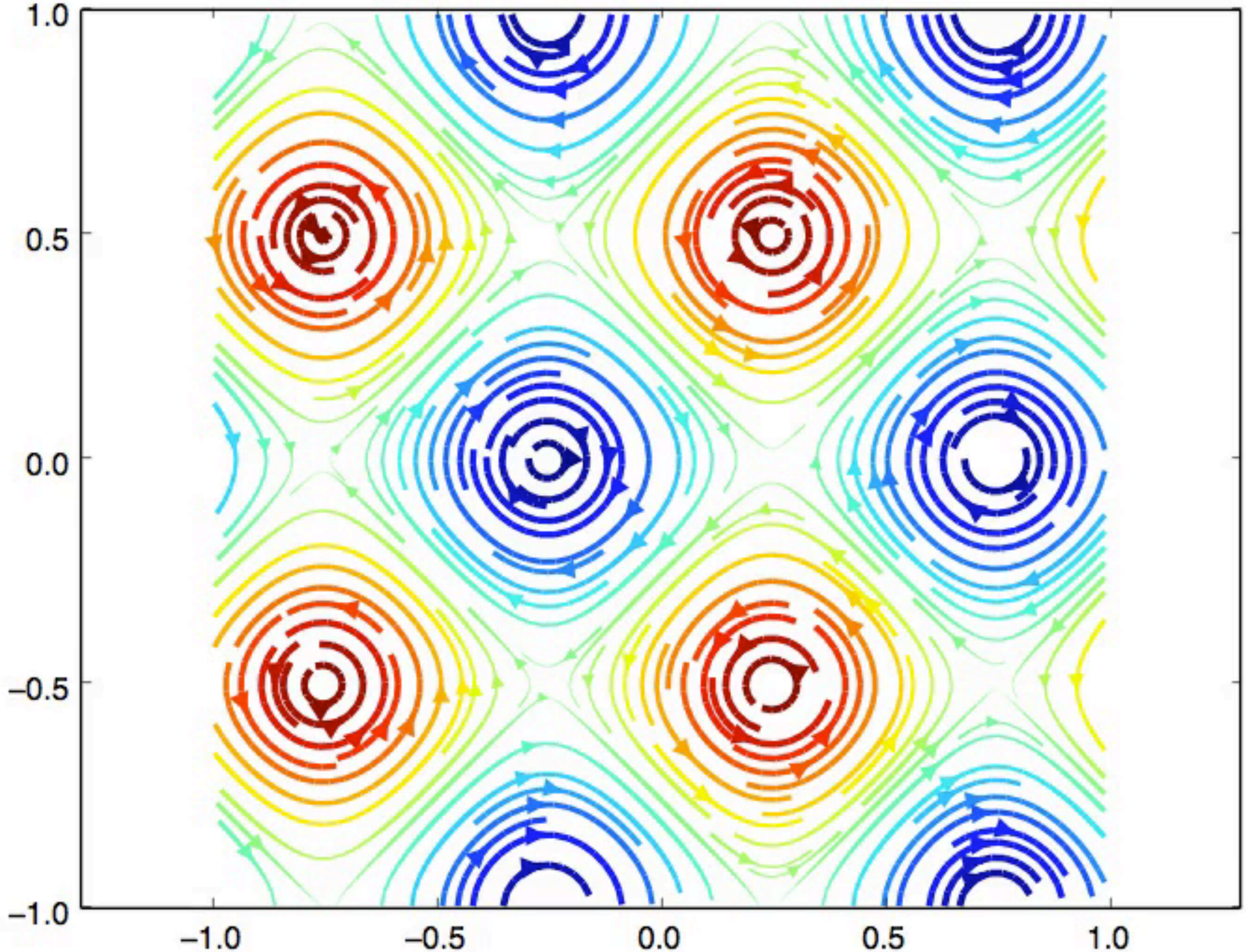

- Instability is linear, ideal… in 2D and in 3D.

- Same results in force-free, RMHD, PIC.

FREELY DECAYING TURBULENCE IN FORCE-FREE ELECTRODYNAMICS

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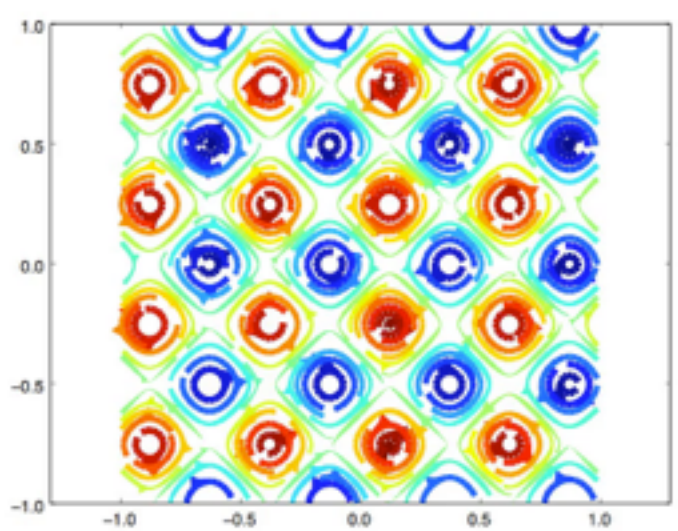
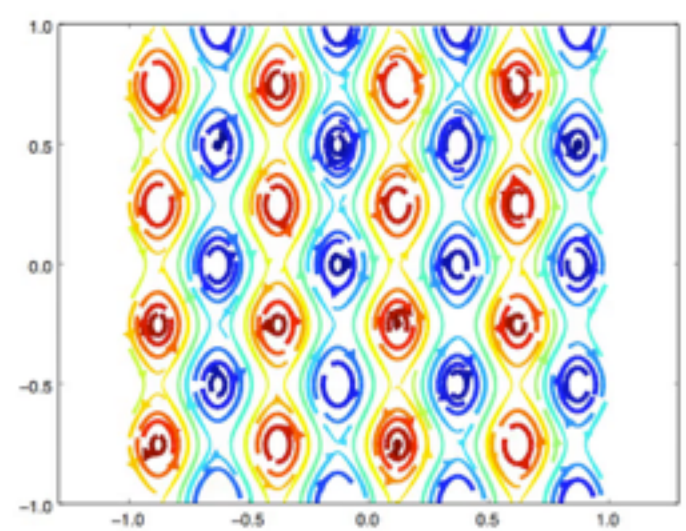
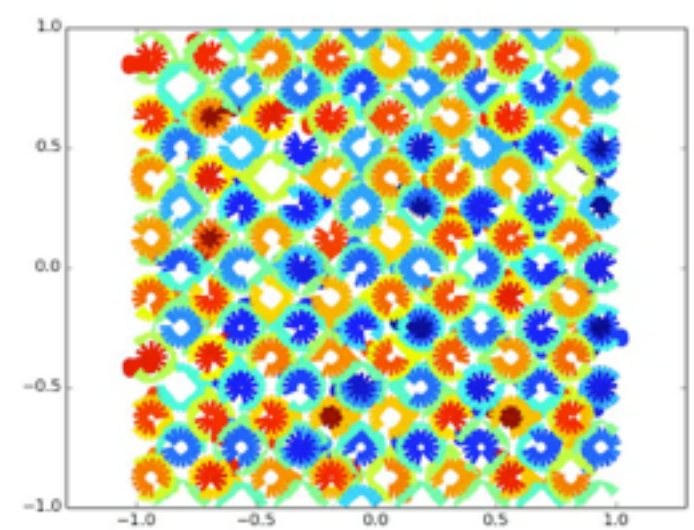
A/B=1, alpha=4



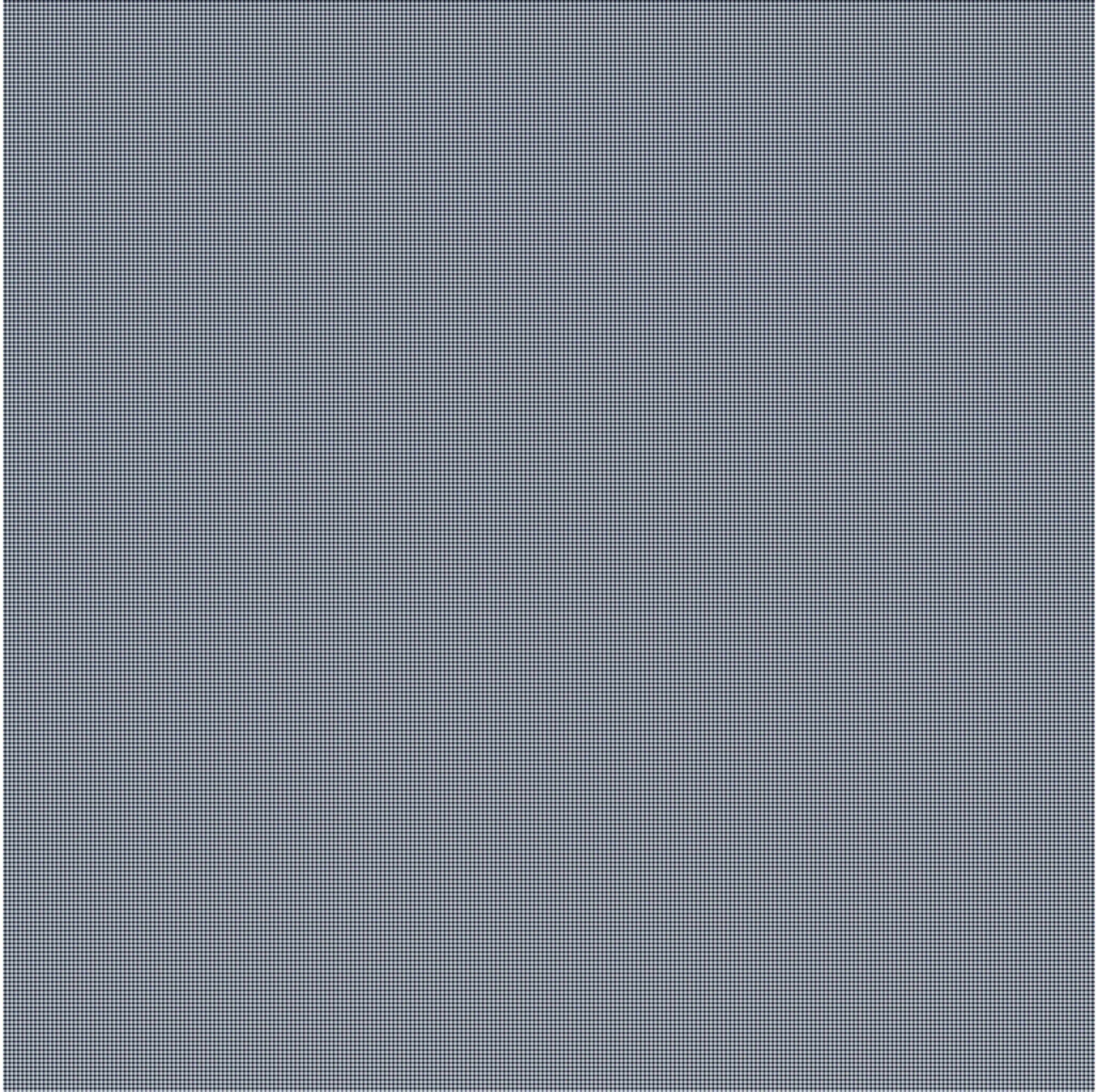
A/B=2, alpha=4



A/B=1, alpha=8



A/B=1, alpha=256 (FFE)



Be careful in 2D

"In 2D, final state is *not* Taylor — infinitely many robust invariants."

$$\mathcal{H}(\psi) = \int \Theta(A_z(\mathbf{x}) - \psi)\mathbf{A} \cdot \mathbf{B} d^3x$$

How about hydrodynamics?

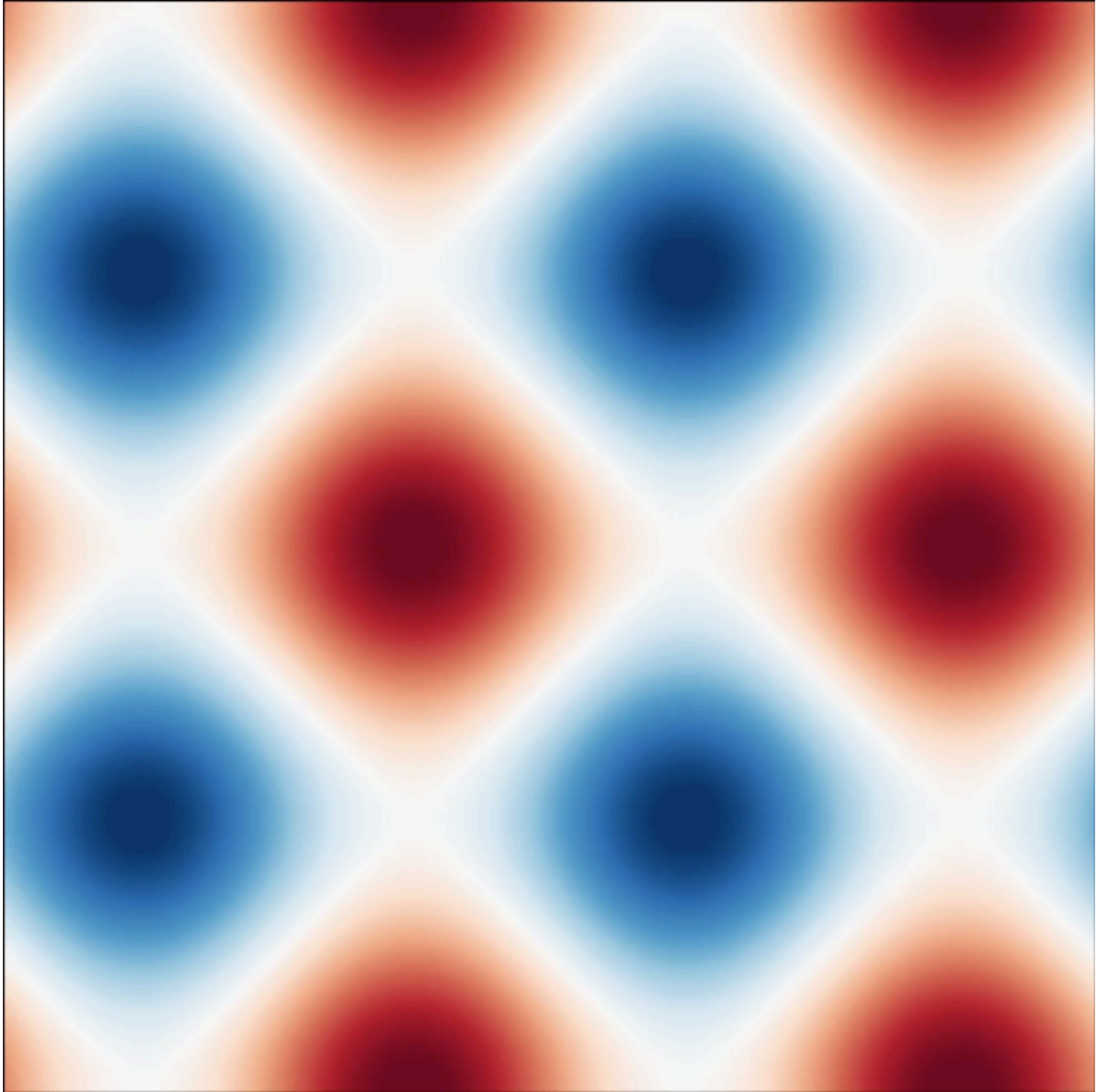
- Repeat the same experiment,
- Replace **B** with **u**.

$$\partial_t \boldsymbol{\omega} = \nabla \times (\boldsymbol{v} \times \boldsymbol{\omega}) \qquad \text{Navier-Stokes}$$

$$\partial_t \boldsymbol{b} = \nabla \times (\boldsymbol{v} \times \boldsymbol{b}) \qquad \text{Faraday's law}$$

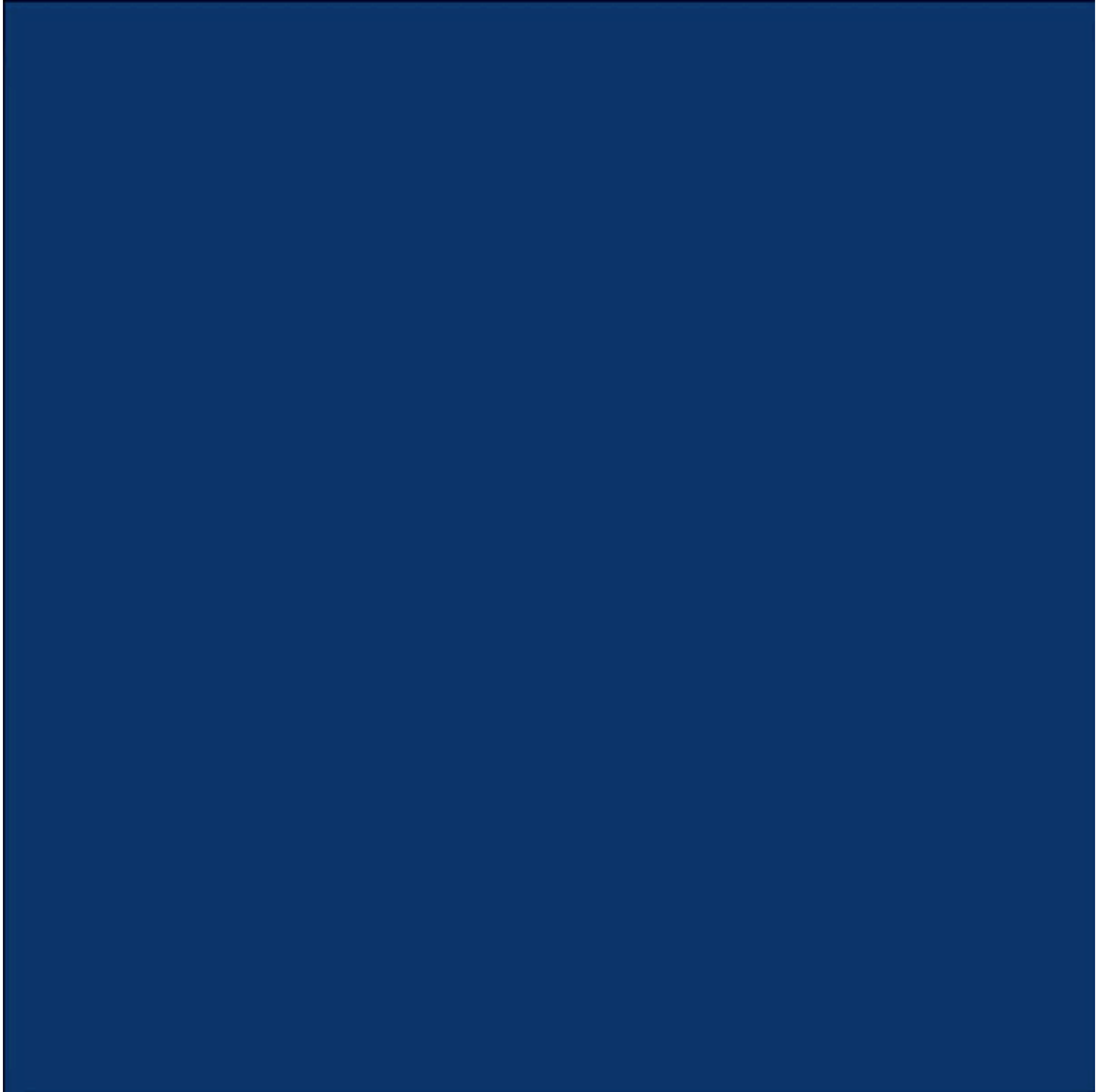
- This is stationary flow. *Stable or unstable?*

2D
$\mathbf{u}_z$

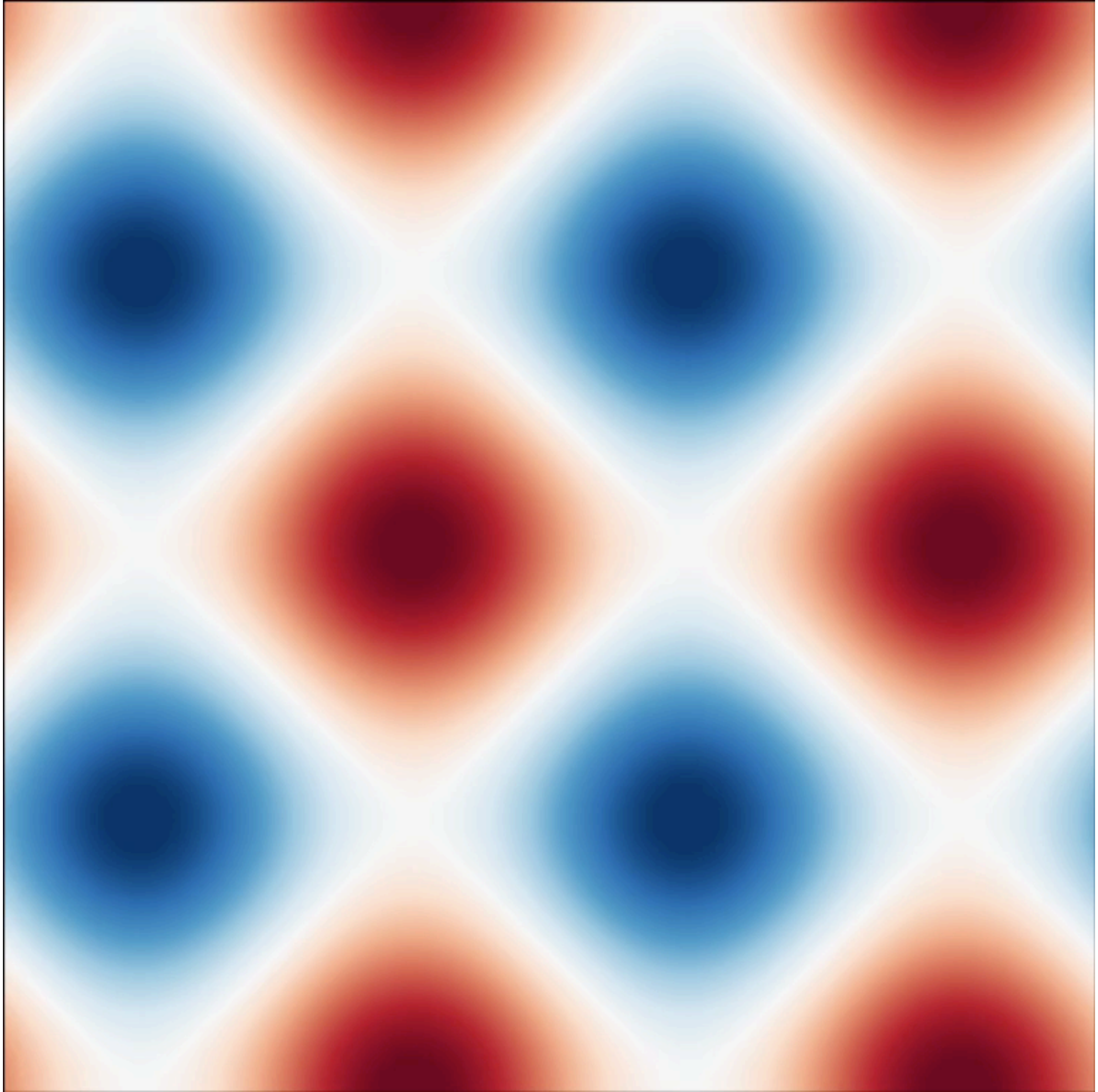


2D

$\Delta \mathbf{u}_z$



3D
$\mathbf{u}_z$



“At fixed *magnetic helicity*, energy **decreases** with wavelength.”

MHD

Coalescence

$$\mathcal{H}_M = \langle \boldsymbol{a} \cdot \boldsymbol{b} \rangle \lesssim k_0^{-1} \mathcal{E}_M$$

$$\mathcal{H}_K = \langle \boldsymbol{v} \cdot \boldsymbol{\omega} \rangle \lesssim k_0 \mathcal{E}_K$$

Hydro

Kelvin-Helmholtz

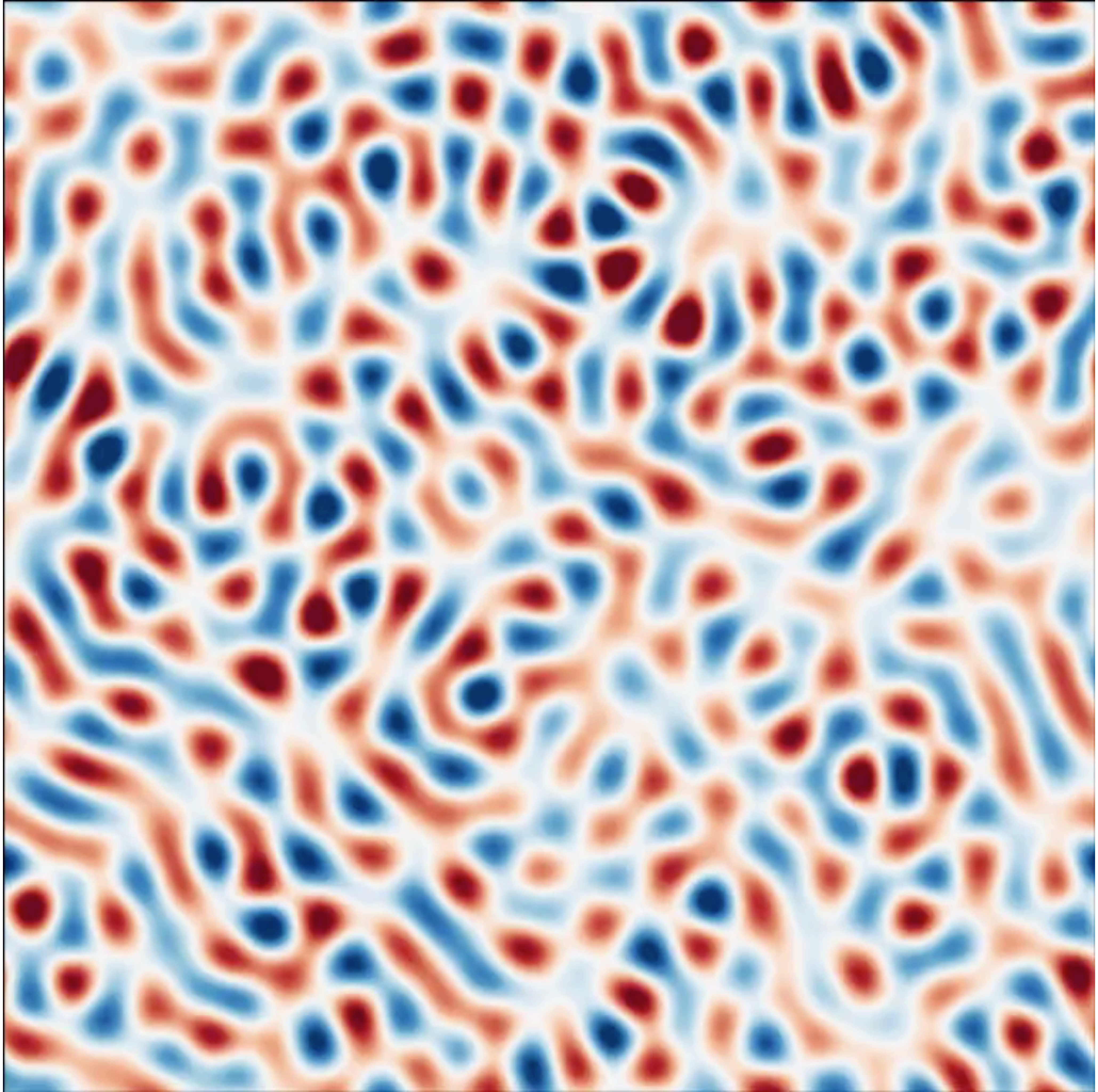
“At fixed *kinetic helicity*, energy **increases** with wavelength…”

On the role of helicity in the inverse energy cascade

Jonathan Zrake (in prep)

"Conjecture: inverse transfer occurs exactly when the primary instability is coalescence."

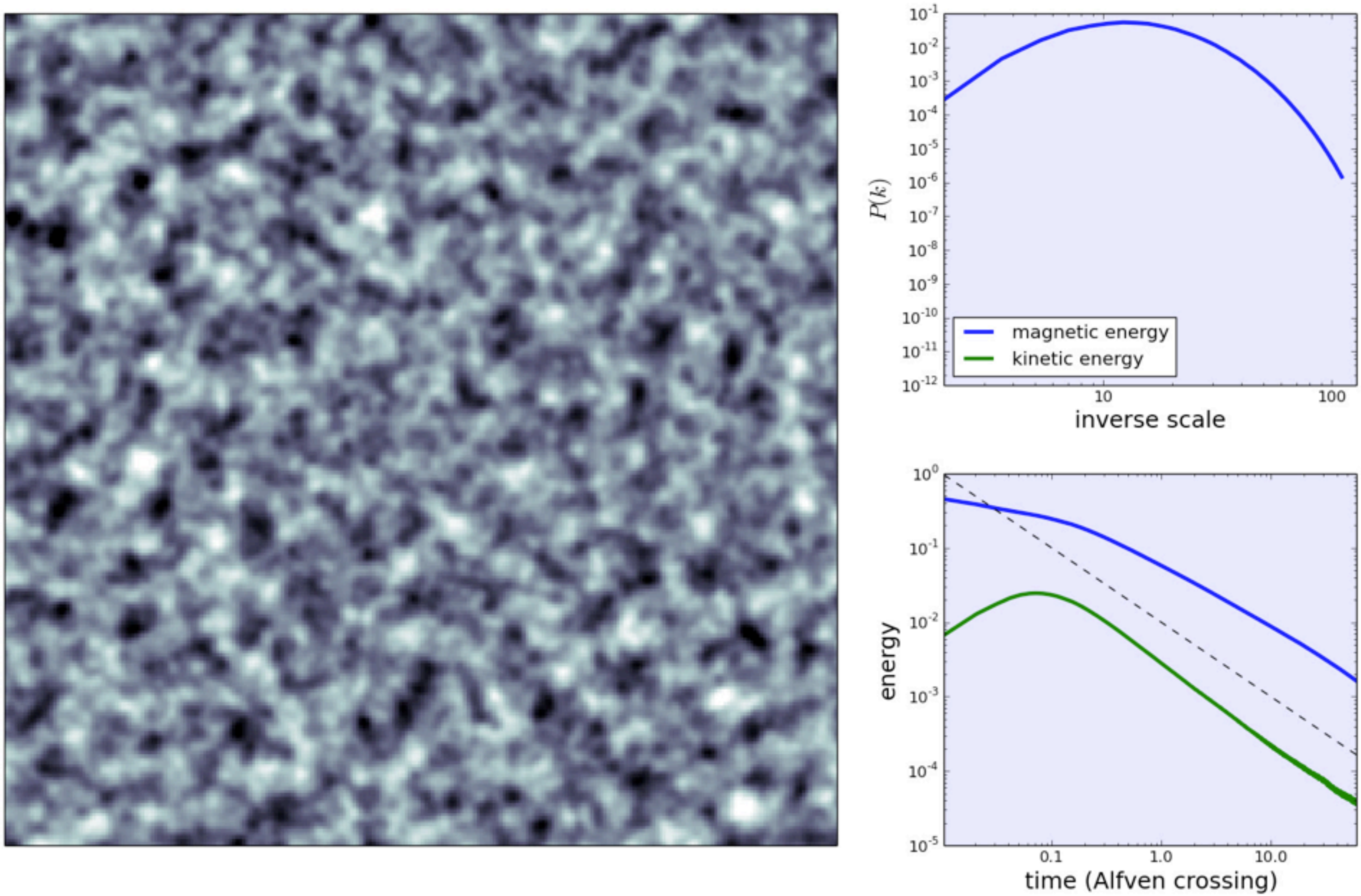
This predicts no self-assembly in 3D hydrodynamics (no surprise here)



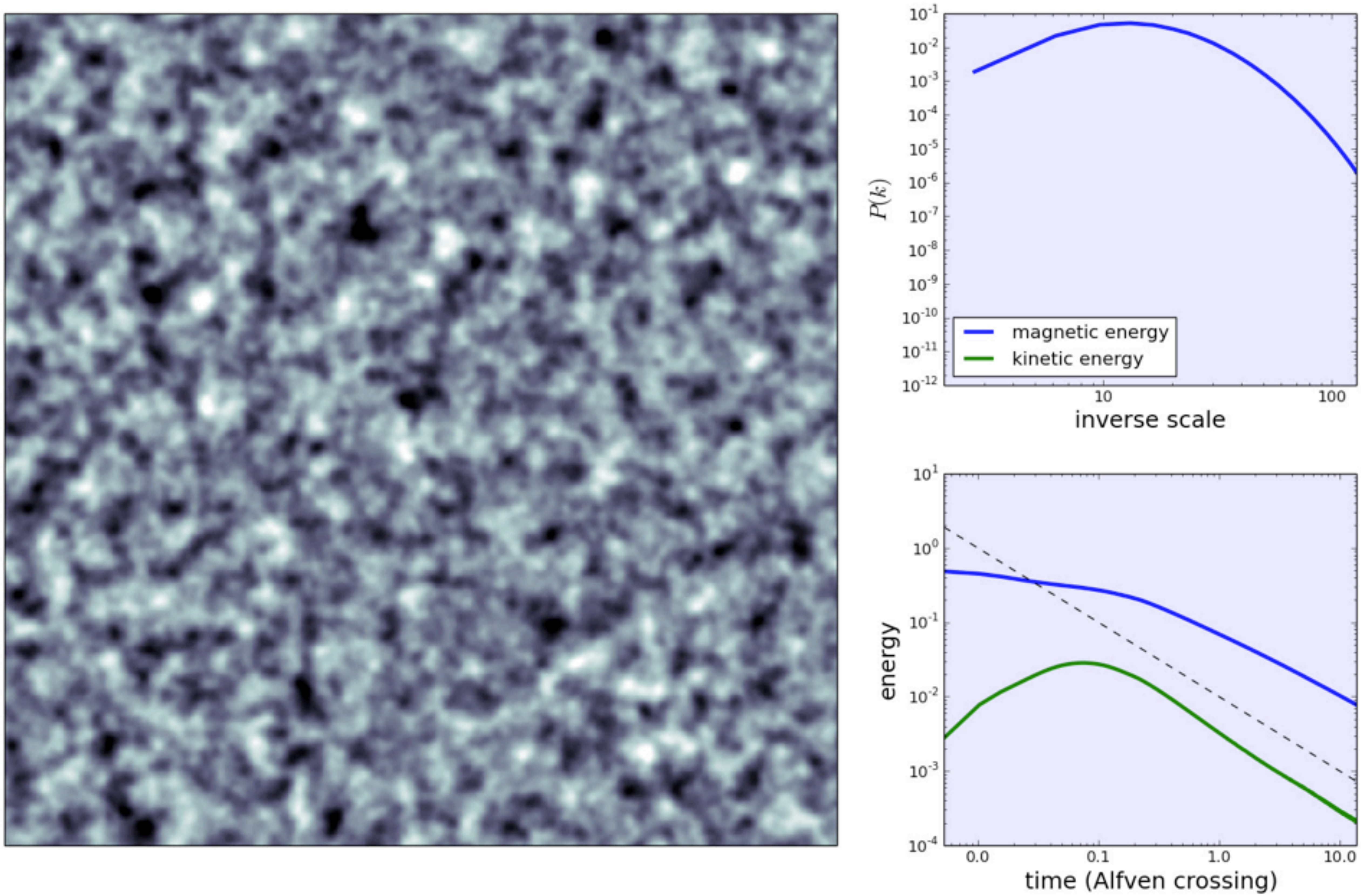
On the role of helicity in the inverse energy cascade

Jonathan Zrake (in prep)

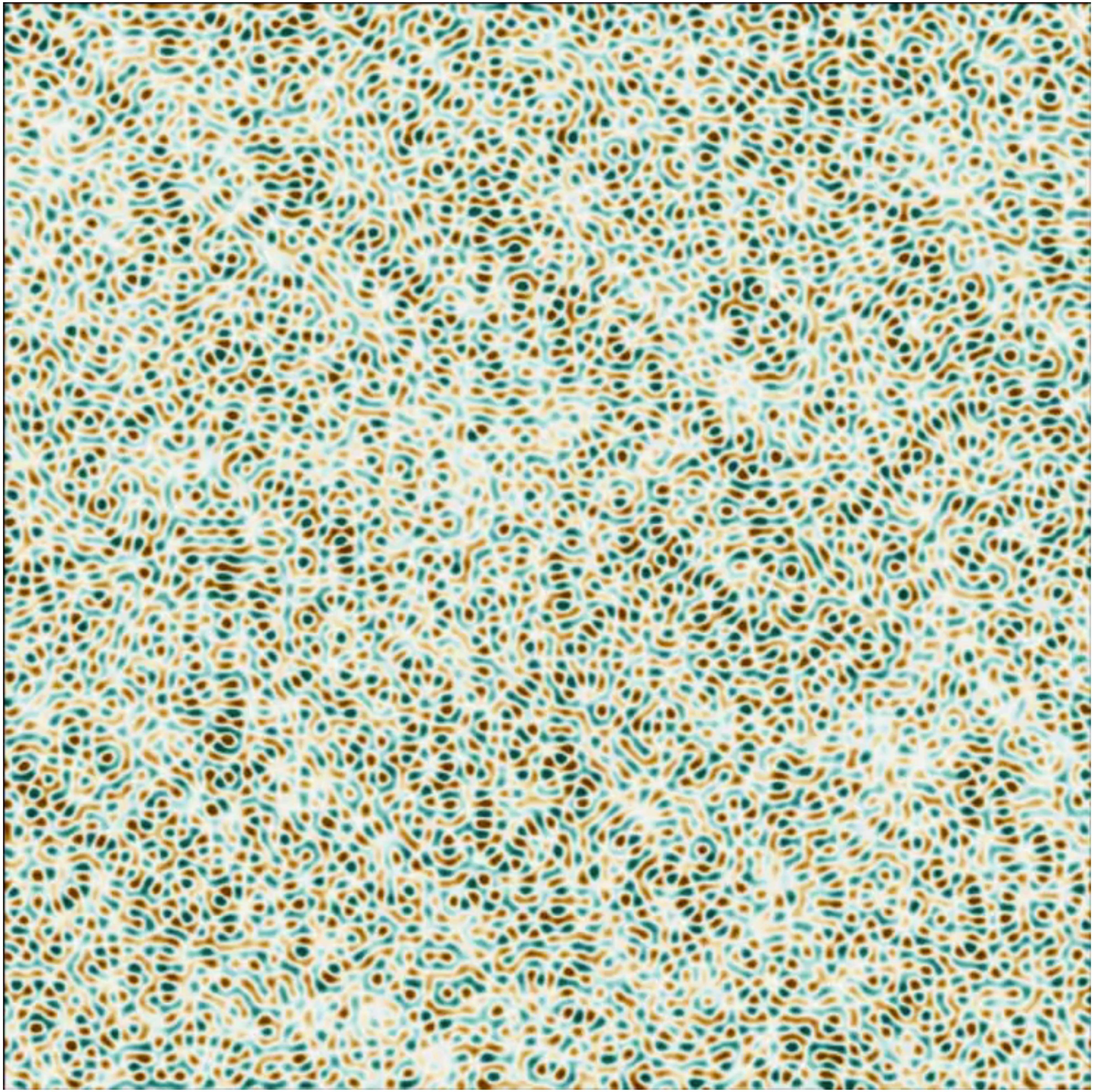
In 3D MHD there is inverse transfer for any value of the net helicity.

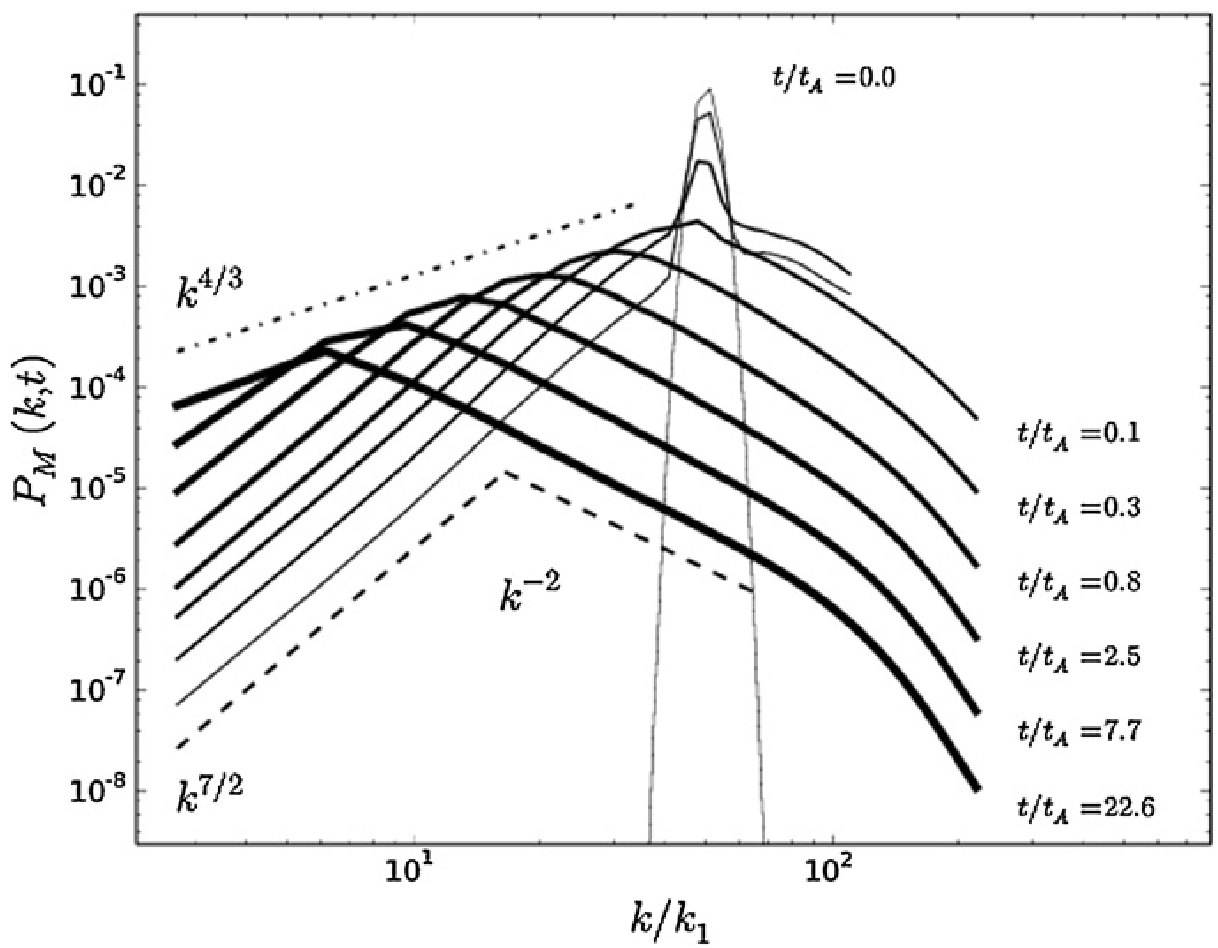


10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
$P(k)$
magnetic energy
kinetic energy
10
100
inverse scale
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
energy
0.1
1.0
10.0
time (Alfven crossing)

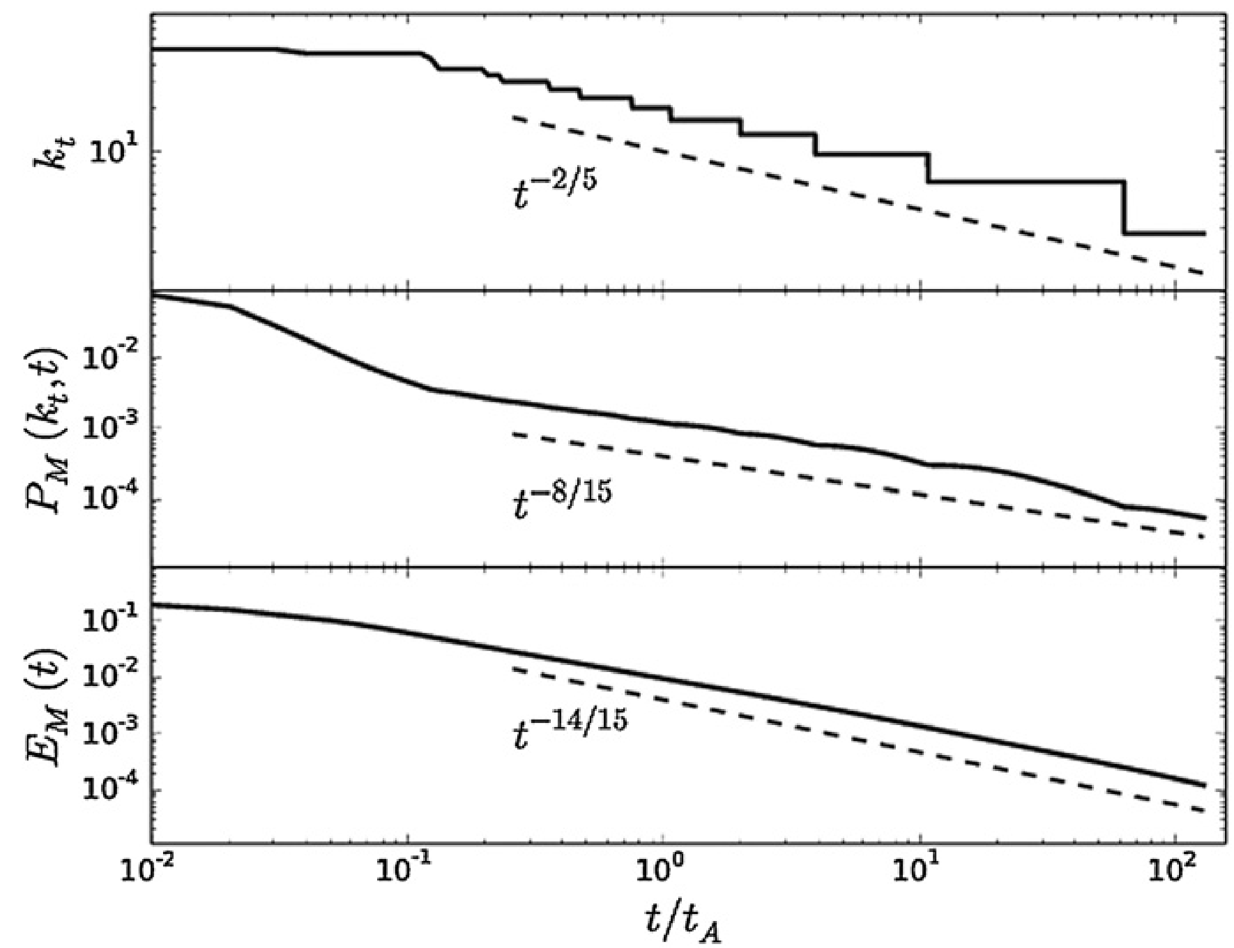


magnetic energy
kinetic energy
P(k)
inverse scale
10
100
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
10^-11
10^-12
energy
time (Alfven crossing)
0.0
0.1
1.0
10.0
10^1
10^0
10^-1
10^-2
10^-3
10^-4

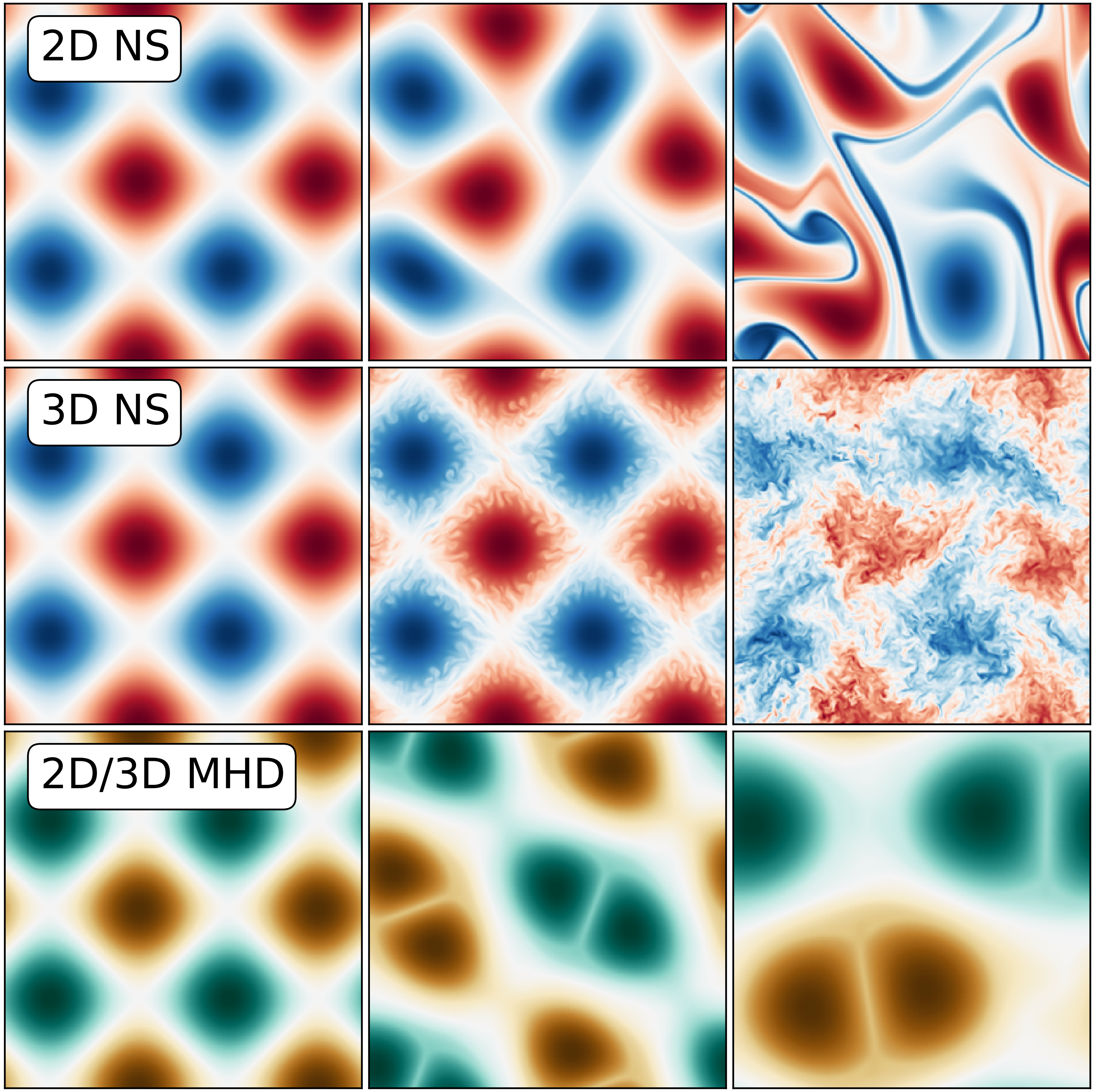




$t/t_A = 0.0$
$k^{4/3}$
k^{-2}
$k^{7/2}$
$t/t_A = 0.1$
$t/t_A = 0.3$
$t/t_A = 0.8$
$t/t_A = 2.5$
$t/t_A = 7.7$
$t/t_A = 22.6$
$P_M\,(k,t)$
k/k_1



k_t
$t^{-2/5}$
$P_M\,(k_t,t)$
$t^{-8/15}$
$E_M\,(t)$
$t^{-14/15}$
t/t_A



On the role of helicity in the inverse energy cascade

Jonathan Zrake (in prep)

On the role of helicity in the inverse energy cascade

Jonathan Zrake (in prep)

$$\mathcal{H}_M = \langle \boldsymbol{a} \cdot \boldsymbol{b} \rangle$$ Coalescence

$$\mathcal{H}_K = \langle \boldsymbol{v} \cdot \boldsymbol{\omega} \rangle$$ Kelvin-Helmholtz

2

Some ideas about the Crab flares.

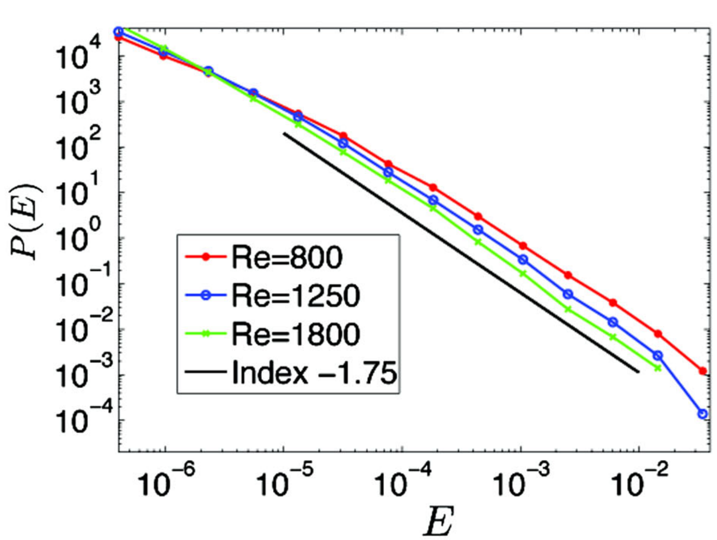
$$\bar{\epsilon} = \frac{L_\star}{\ell_{\rm neb}^3} \sim \frac{10^{38}\,{\rm erg/s}}{(10^{18}\,{\rm cm})^3}$$

$$\epsilon_{\text{flare}} \sim \frac{10^{37}\,\text{erg/s}}{(10^{15}\,\text{cm})^3} = 10^8 \bar{\epsilon}$$

*assuming isotropic emission

Require such excursions at a rate of once per year.

Is this compatible with intermittency of dissipation in MHD turbulence?



Temporal Intermittency of Energy Dissipation in Magnetohydrodynamic Turbulence

Vladimir Zhdankin,[1,*] Dmitri A. Uzdensky,[2] and Stanislav Boldyrev[1]

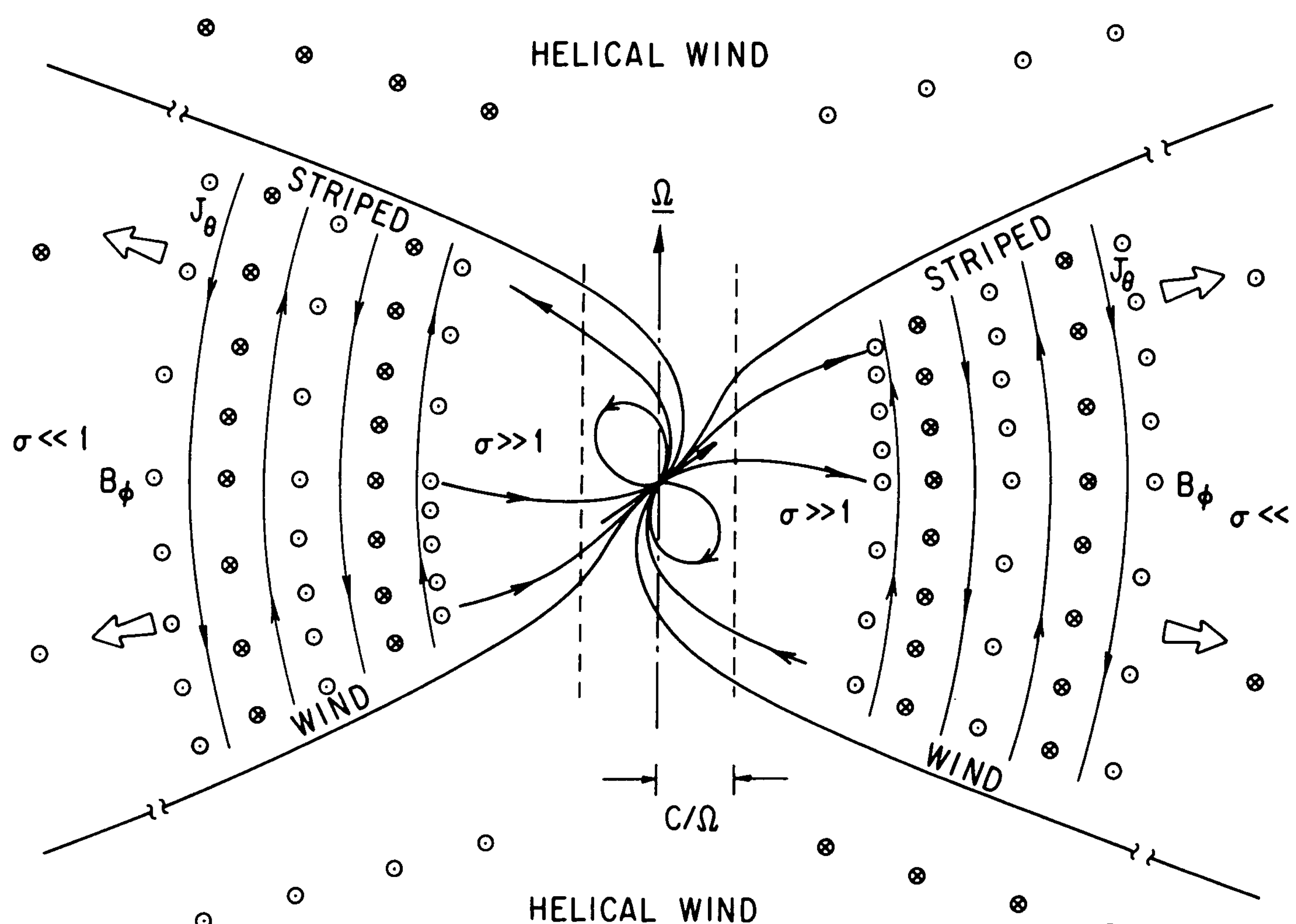
A two-zone model

"A Crab flare is a blob of particles entering a stronger magnetic field, along the line of sight."

- Particles must be accelerated on gyration time
- Flares must come from a high-sigma region

3

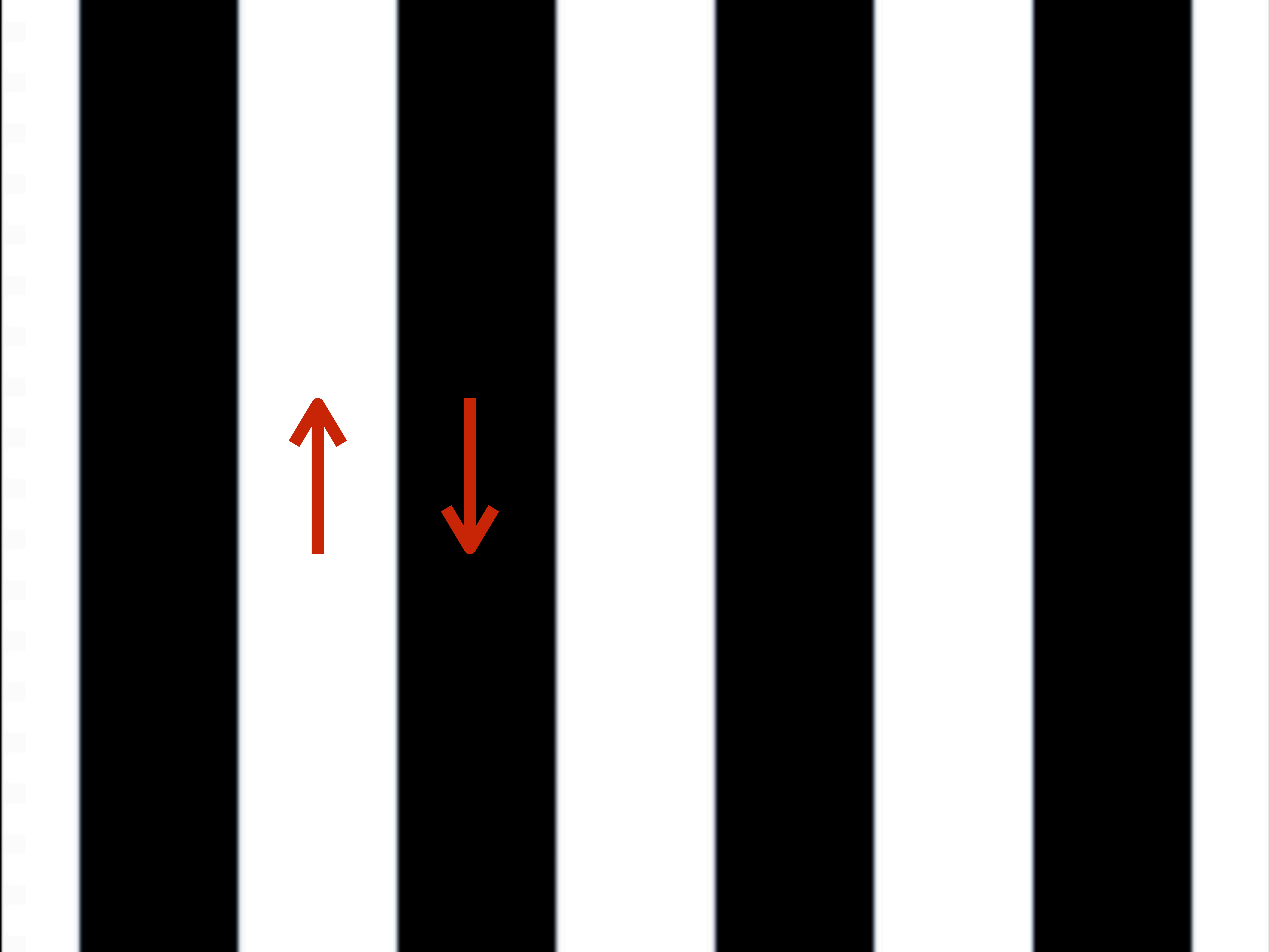
Pulsar striped wind converts to turbulence.

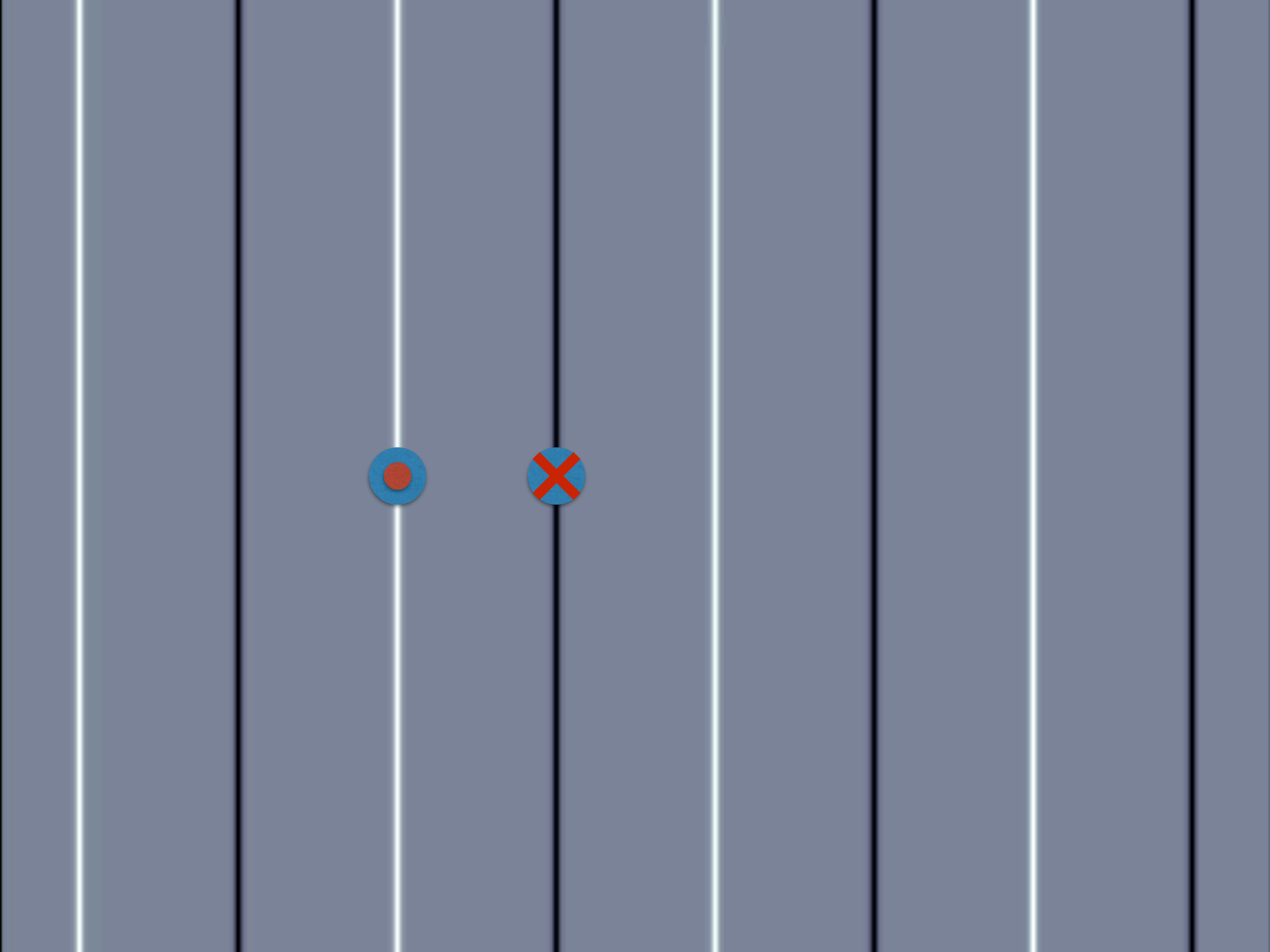


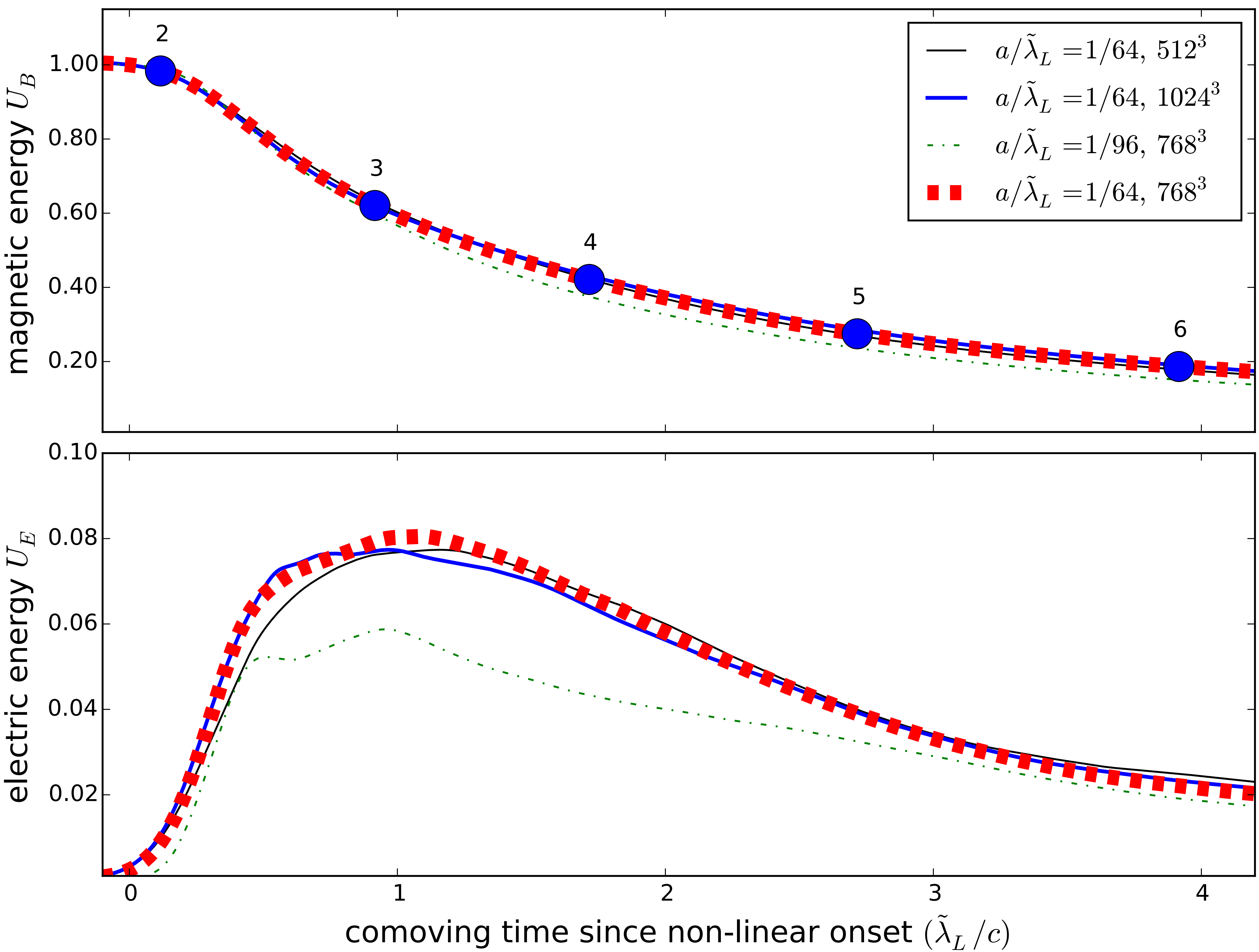
Tearing is fast

- First plasmoids forming at the wind base (thin current layers)

- Corrugation of the current layers, interaction between them, leads to reconnection rates ~0.5 c





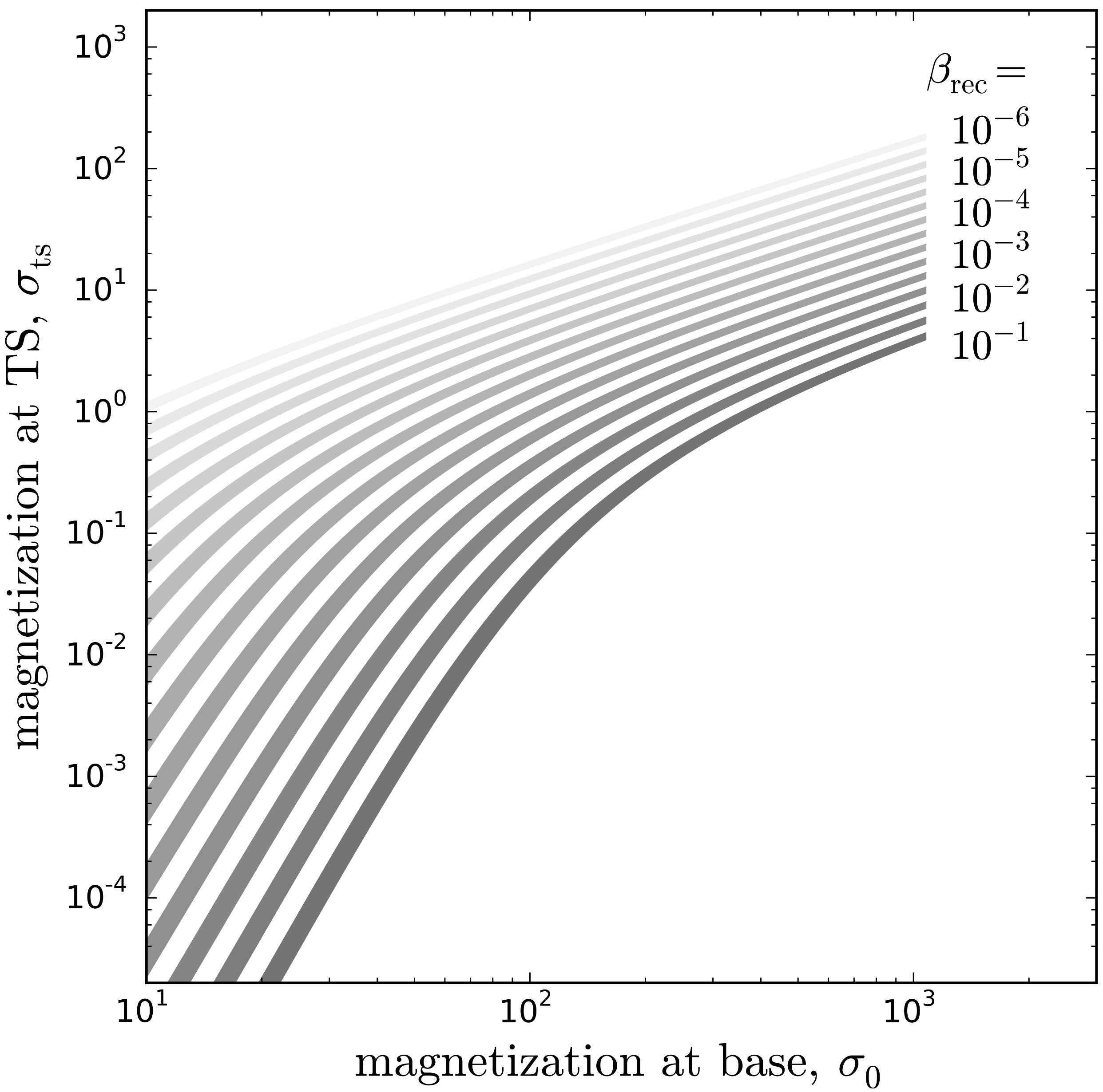


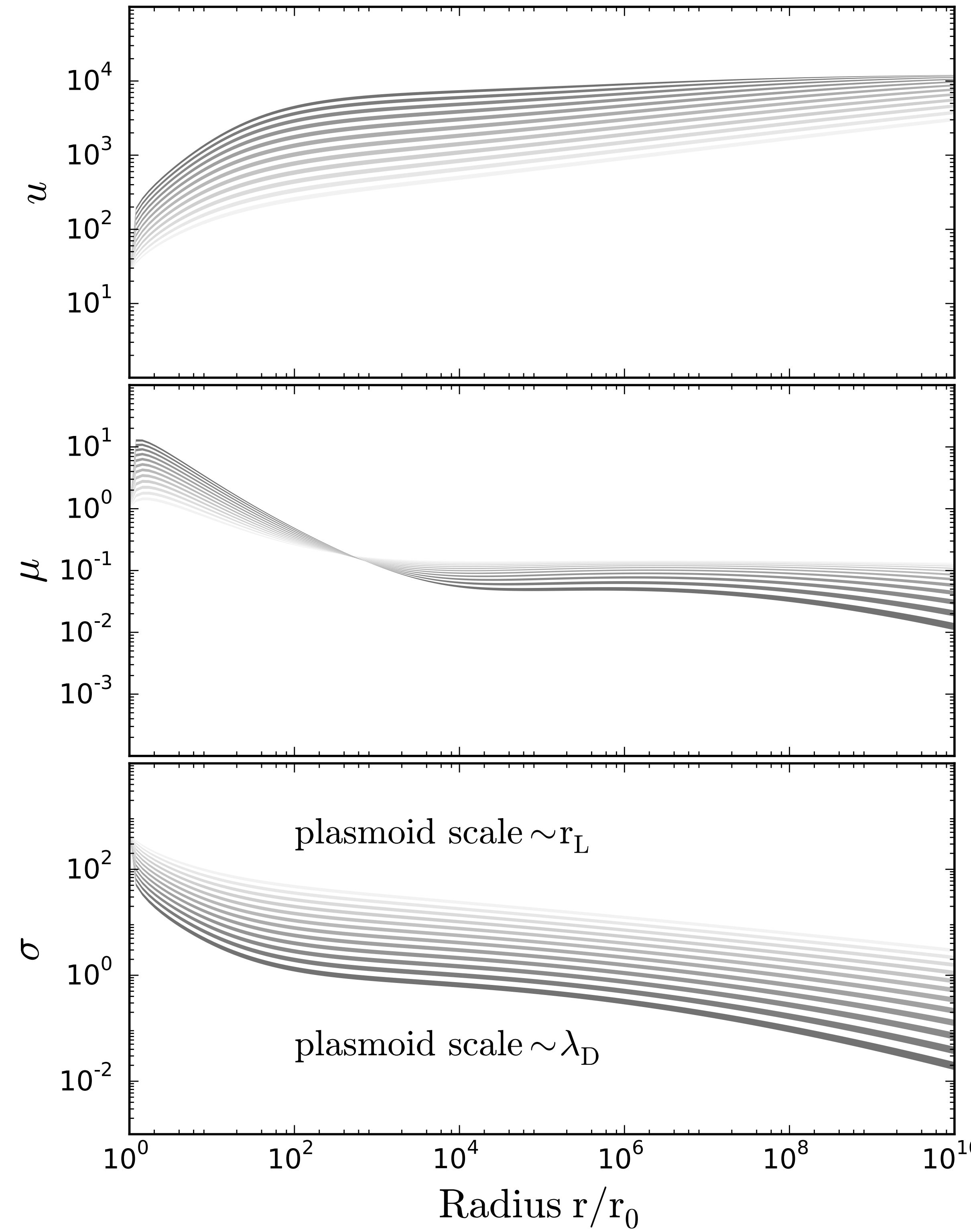
magnetic energy U_B
1.00
0.80
0.60
0.40
0.20
2
3
4
5
6
$a/\tilde{\lambda}_L = 1/64,\ 512^3$
$a/\tilde{\lambda}_L = 1/64,\ 1024^3$
$a/\tilde{\lambda}_L = 1/96,\ 768^3$
$a/\tilde{\lambda}_L = 1/64,\ 768^3$
electric energy U_E
0.10
0.08
0.06
0.04
0.02
0
1
2
3
4
comoving time since non-linear onset $(\tilde{\lambda}_L/c)$

Turbulent striped wind

- Stripes transition to turbulence after 2 comoving magnetosonic wave crossing times $\approx \Gamma^2 \lambda_L$

- Relativistic time dilation -> turbulence transition at closer distance with slower wind





Zrake & Arons (in prep)

Multi-dimensional effects modify Lyubarski & Kirk model.

Turbulent striped wind

- High mass loading (multiplicity ~ 10^5) still required for total dissipation of the AC wind

- Growth of magnetic coherency raises interesting possibility for Crab flares…

CRAB FLARES DUE TO TURBULENT DISSIPATION OF THE PULSAR STRIPED WIND

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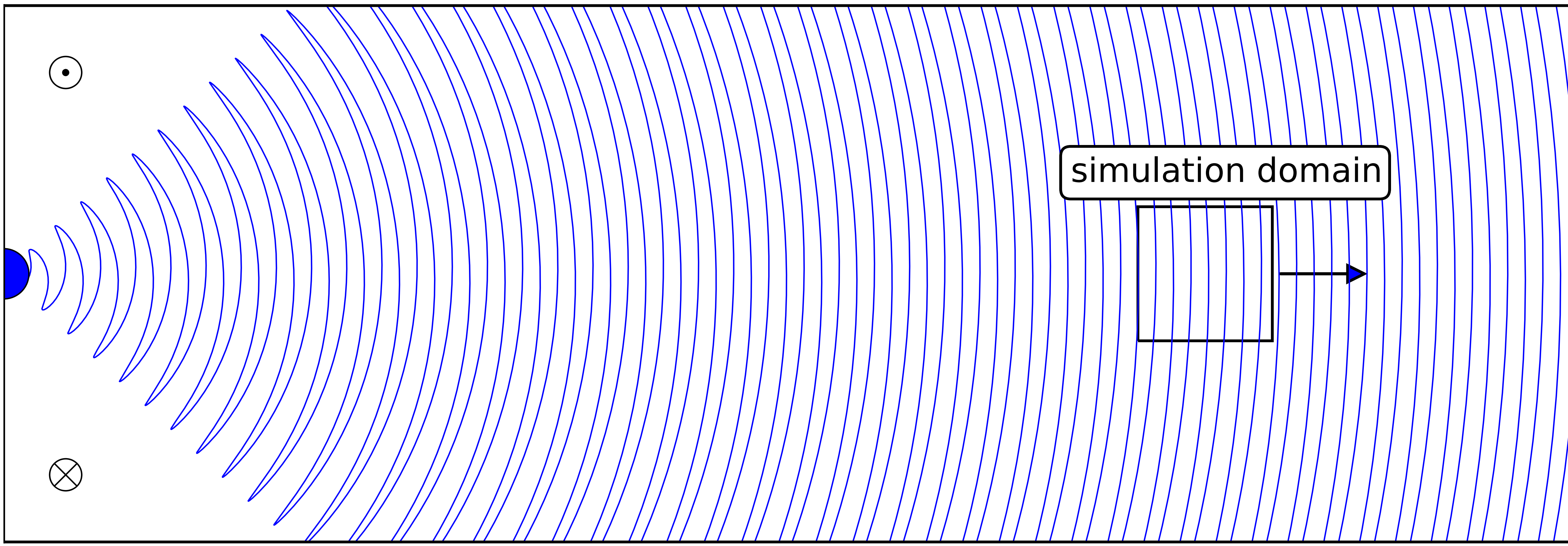
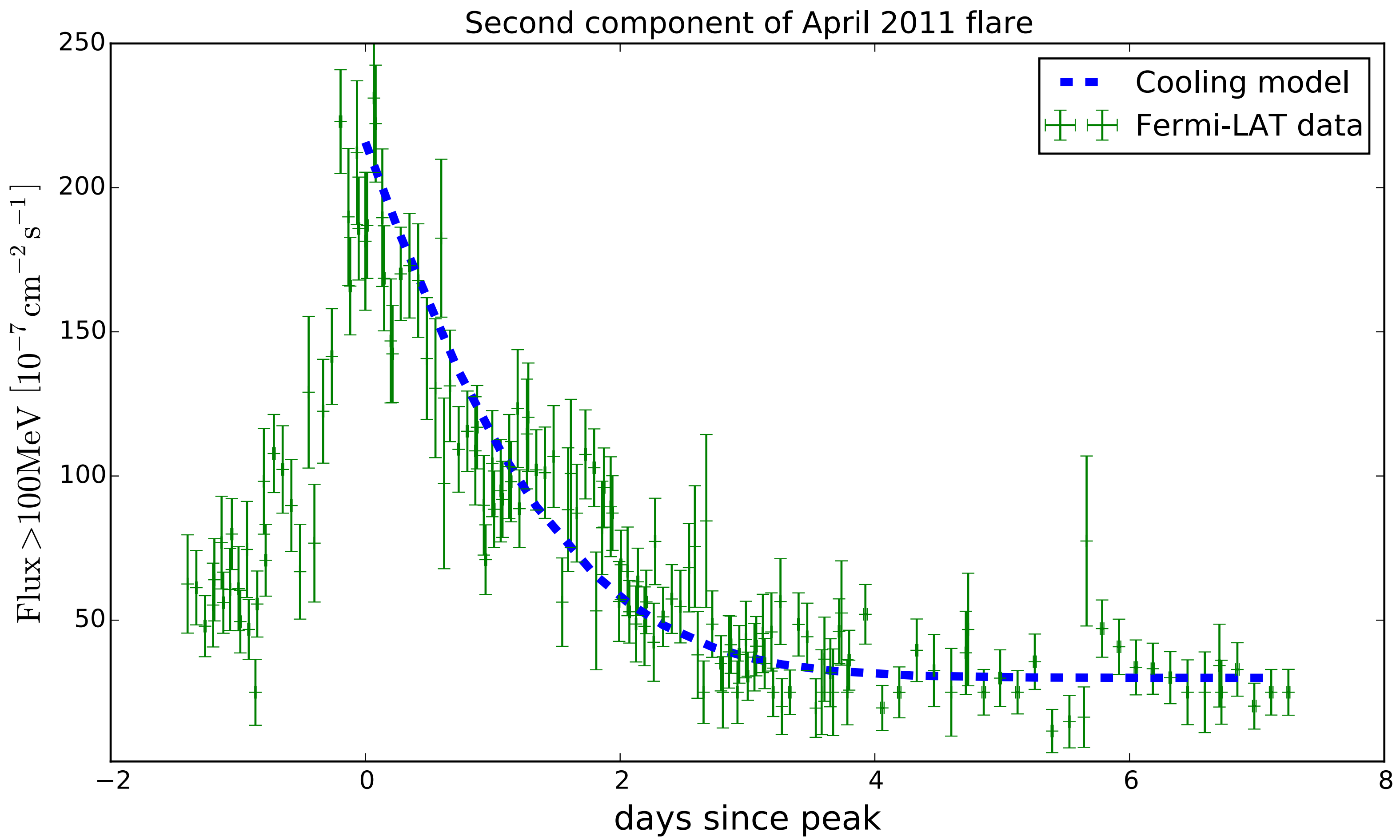


8 light-hour coherency marginally possible if wind Lorentz factor ~ 10^3 - 10^4 (well mass-loaded wind).

Decline phase ~ electron cooling time

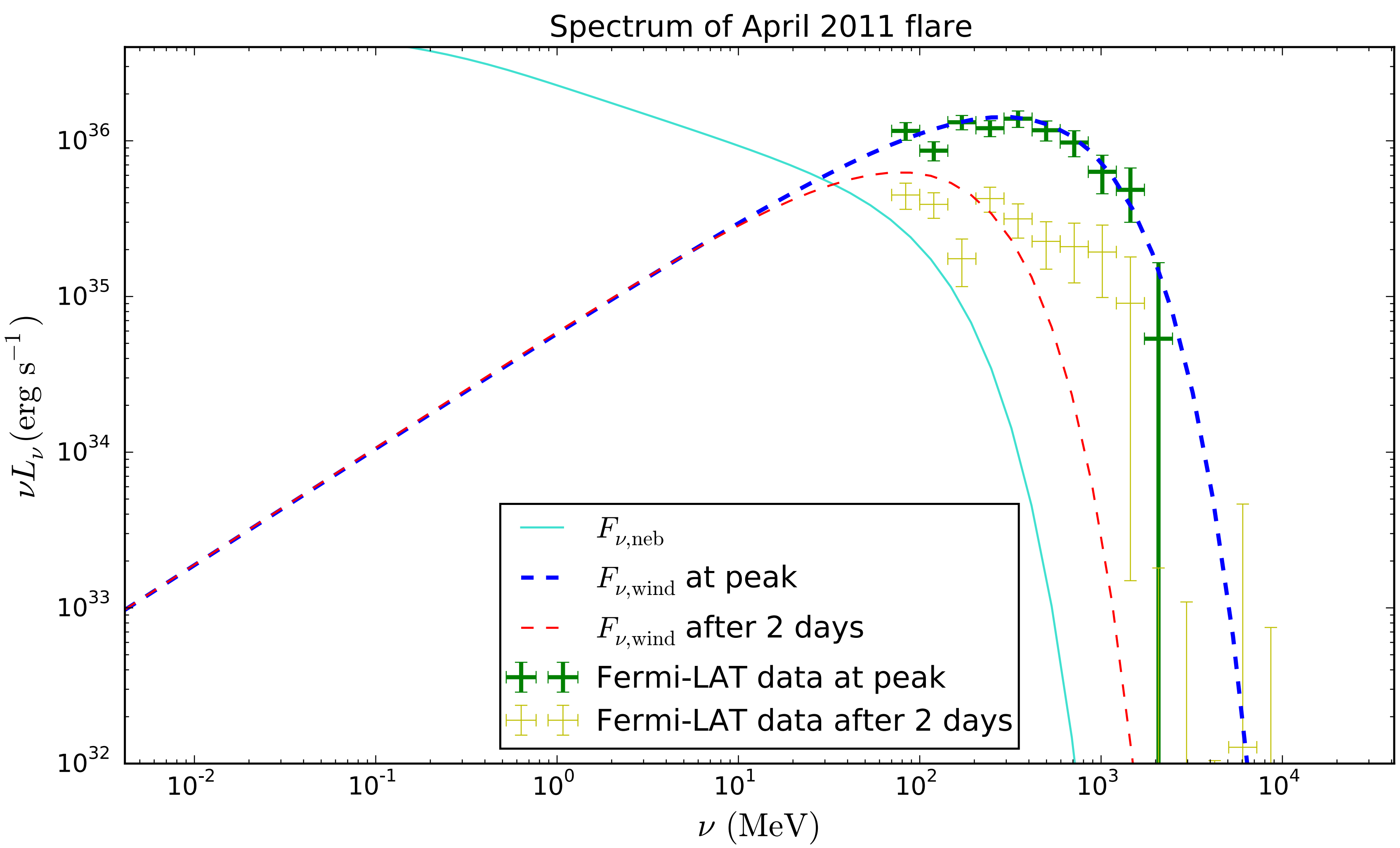


$$B_s = 655\,\mu\mathrm{G}$$

More generally,

- Crab flares due to “blob” of high energy particles injected into stronger B field
- Variability time ~ size of the blob
- Radiation can exceed 100 MeV for electron cooling times
- *Bayesian block time-series analysis w / Jeff Scargle*

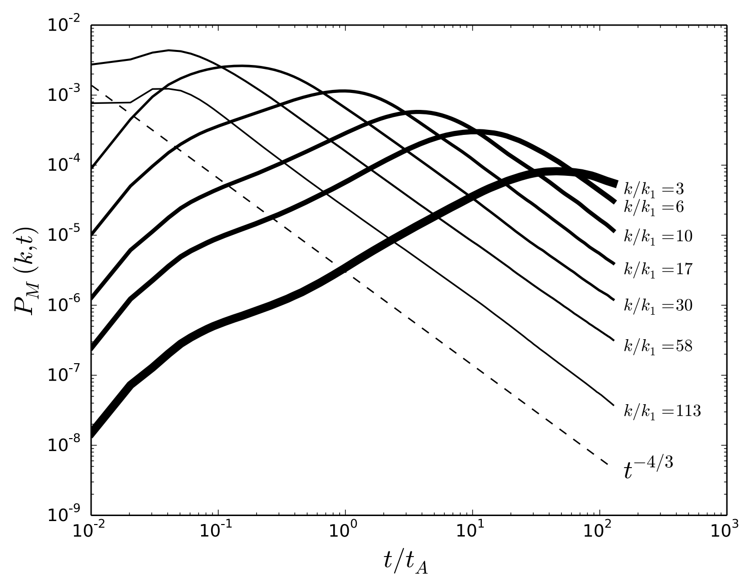
Fermi-LAT spectrum

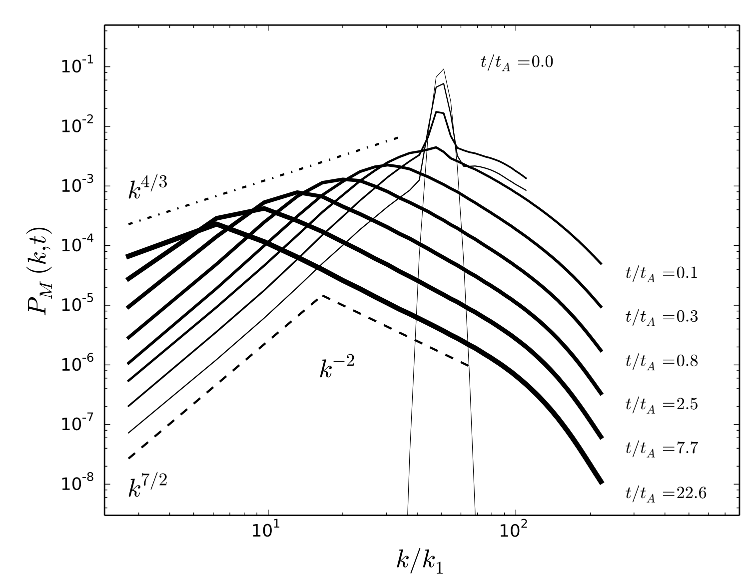


$$\gamma_{\mathrm{max}} = 7 \times 10^9$$

Magnetic self-assembly is generic…

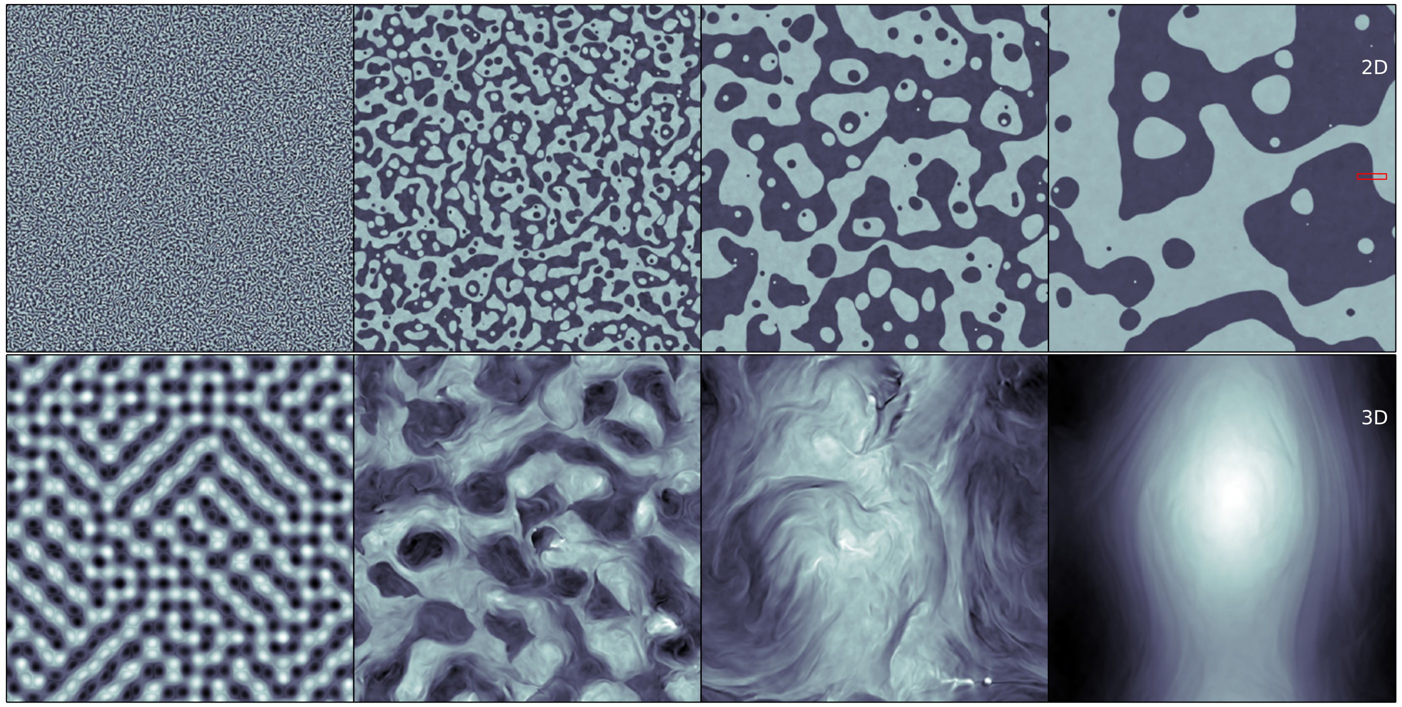
Not limited to 2D or high helicity.





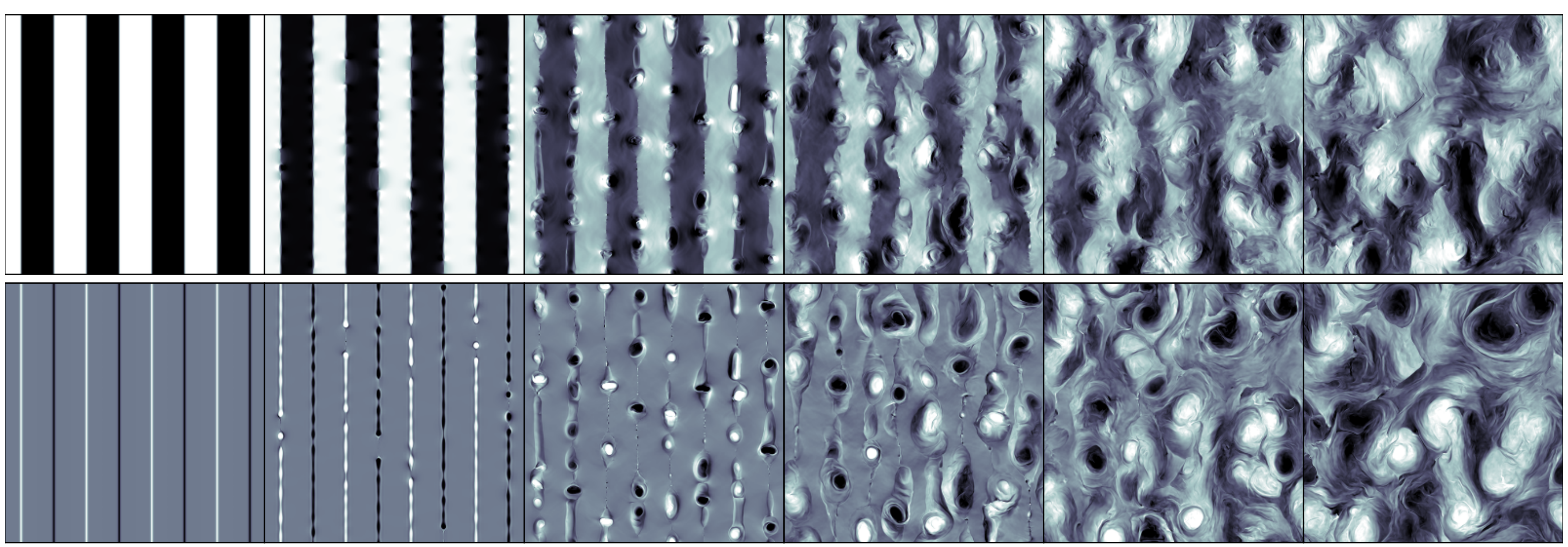
Taylor's hypothesis not satisfied in 2D.

Be careful with 2D!



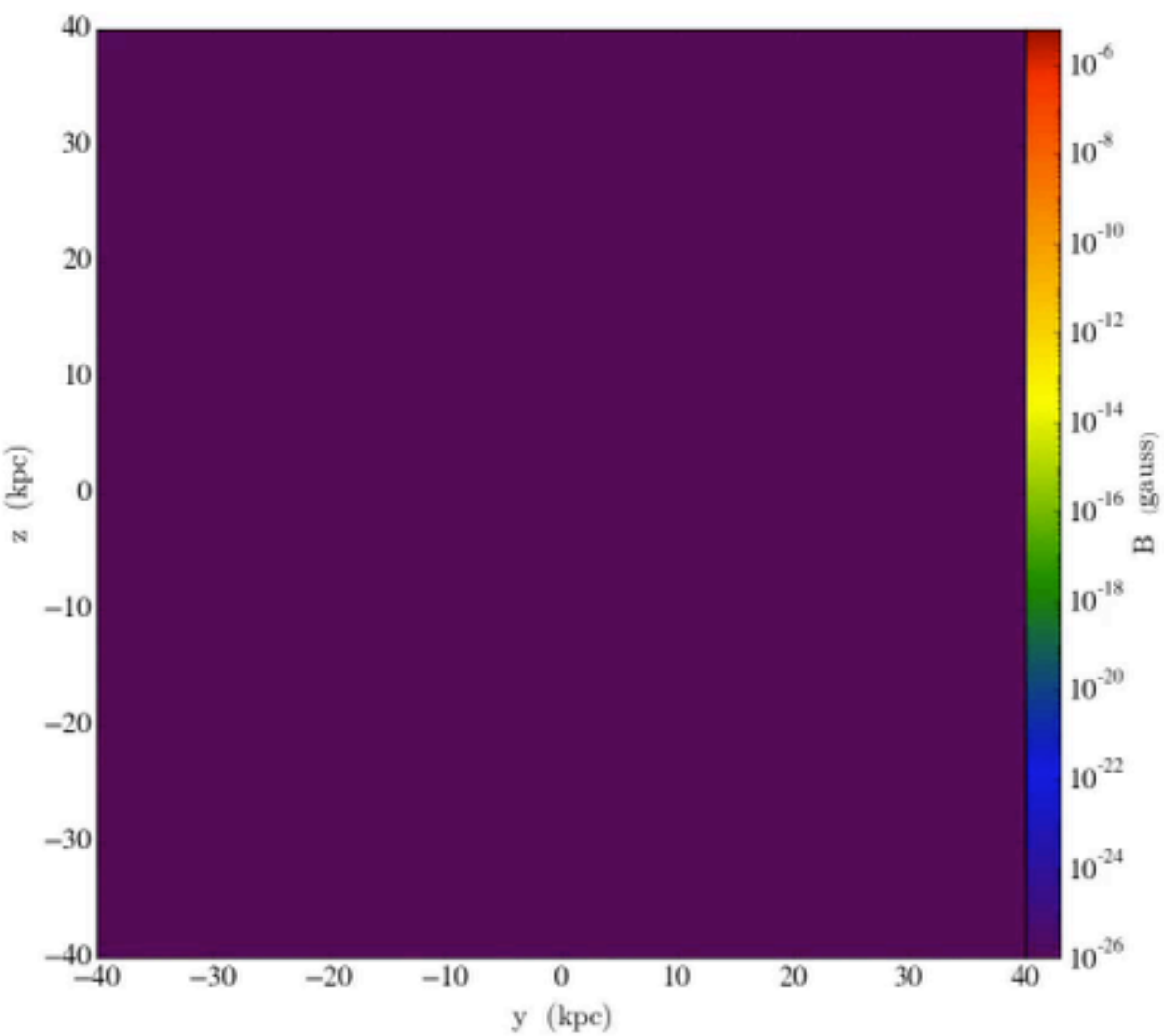
Pulsar striped wind converts to isotropic turbulence (if mass loading is high)…

Crab Flares may be attributed to large *upstream* coherent structures interacting with wind-termination shock.

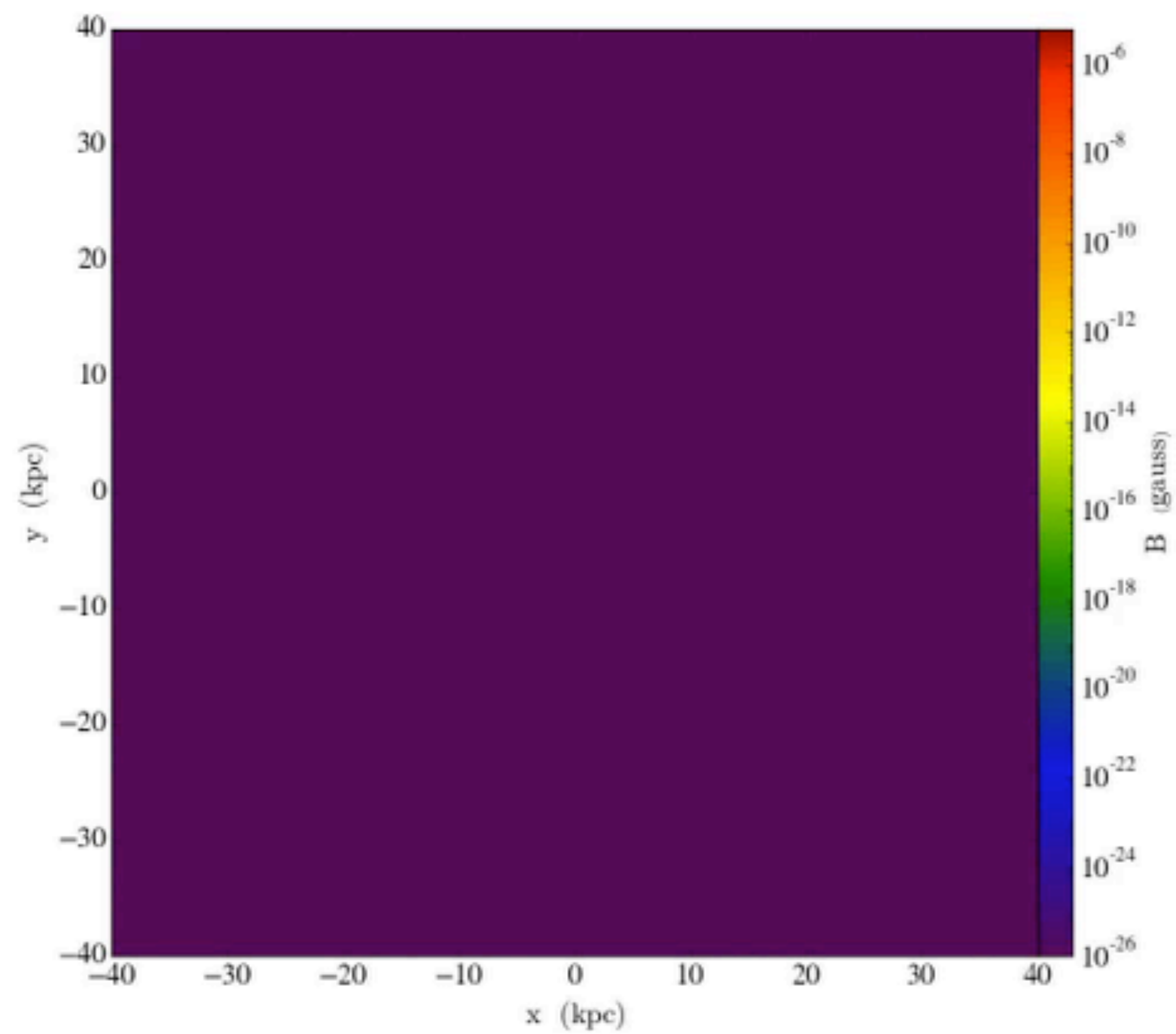


SN driven galactic dynamo

Iryna Butsky, JZ, Ji-Hoon Kim, Eric Yang, Tom Abel



edge-on



top-down