Pitch-Angle Diffusion of High Energy Cosmic Rays in Intermediate Turbulence

John Daniel Riordan, Asaf Pe'er - May 2016

UCC
Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

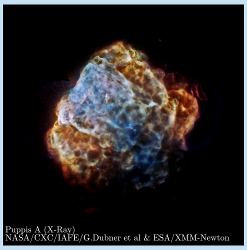
Abstract

How high-energy cosmic rays are accelerated, and at which sources they do this is a fundamental unanswered question in astrophysics. One possible explanation is given by the theory of Diffusive Shock Acceleration (DSA). In this context we investigate the pitch-angle diffusion of cosmic rays in astrophysical plasmas, using a novel code to examine turbulence levels beyond those accessible to analytic theories ($\delta B/B_0 \approx 1$). We achieve this by numerically solving the Newton-Lorentz equation for population of test-particles against a background field and population of turbulent Alfvén waves. Comparing our results with the predictions from quasilinear theory, we see that they agree within the weak turbulence range ($\delta B/B_0 < 0.3$), and that diffusion is suppressed above this level. Furthermore we demonstrate the time dependence of the pitch-angle distribution. Finally we quantify the limitations to the popular Bohm diffusion approximation and also show that significant current anisotropies occur in jet outflows. These results are significant in view of the high levels of magnetic turbulence expected to exist in jets.

Background

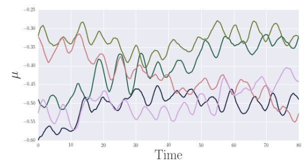
The Problem

- DSA is believed to play a significant role in the production of cosmic rays in GRBs/AGNs/SNRs.
- In these objects:
  - regions of strong turbulence are expected.
  - a significant fraction of the fluid energy may be injected, so the test-particle approximation fails.
  - space and time scales are too large for PIC.

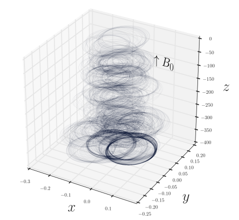


- Hence we have systems which don't obey the assumptions of Jokipii's quasilinear theory (QLT) and are difficult to simulate directly.

$$D_{\mu\mu}(\mu) = \frac{\langle \Delta\mu \rangle^2}{\Delta t} \text{ where } \mu = v_z/v = \cos\vartheta.$$



Particle trajectories in μ-space.



Example particle motion.

Quasilinear Theory

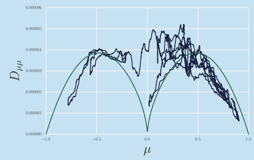
- In terms of μ, the Fokker-Planck equation is:

$$\partial_t f + \mu v \partial_z f = \partial_\mu (D_{\mu\mu} \partial_\mu f)$$

- Classic QLT (see e.g. Blandford & Eichler '87) predicts that particles interact with only resonant waves ($k_z = (v\mu)^{-1}$) and

$$D_{\mu\mu} = \left(\frac{\delta B}{B_0}\right)^2 \frac{2\pi^2 (1-\mu^2)\, v}{|\mu|\, r_g{}^2}$$

- Good for $\delta B/B_0 \ll 1$ but overestimates diffusion for stronger turbulence.
- This form for $D_{\mu\mu}$ is valid for low turbulence levels where QLT (green line) applies, as shown by our simulation (blue):



Results

Model

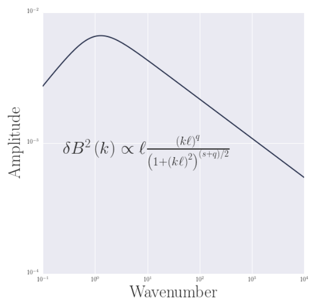
- Relativistic Newton-Lorentz motion:

$$\partial_t^2 \vec{x} = \frac{q}{\gamma m} \vec{v} \times \left(B_0 \hat{z} + \vec{\delta B}\right)$$

- Turbulence represented by $\sim 10^3$ slab Alfvén waves of random phase ψ and polarisation ϕ.

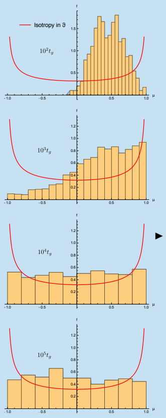
$$\delta B^2 = \sum_{k_z} e^{ik_z z} (\sin\phi \hat{x} + \cos\phi \hat{y})\, G(k)\, k_z \Delta \ln k$$

- For the spectrum $G(k)$ we use the Shalchi & Weinhorst generalised form. This encompasses an inertial range, turnover scale ($\approx r_g$), and a kinetic range e.g. Kolmogorov/Goldreich-Sridhar:



Result: Anisotropies

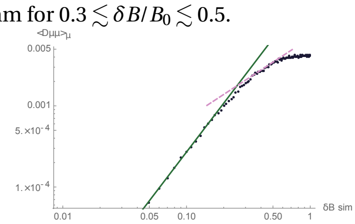
- We measure the reversal time t_{rev} - the time taken for μ (gyrotime-averaged) to change sign.
- PIC simulations indicate that particles can be accelerated to high energies after ≈ 10 shock crossings ($\gtrsim t_{rev}$). We find that the particle distribution can remain anisotropic over hundred/thousands of times t_{rev} depending on turbulence parameters.
- The injected distribution is not isotropic in the fluid frame, and there is insufficient time to isotropise before reaching high energy, hence the assumption of classic DSA that the distribution becomes isotropic after each crossing is invalid here.
- By the Sturm-Louiville theorem the distribution should reach a constant in μ steady state. This occurs after $\sim 1/\langle D_{\mu\mu} \rangle_\mu$.



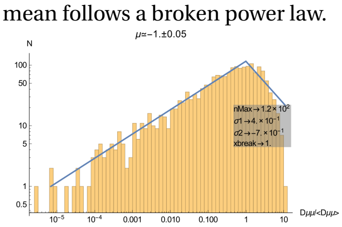
Relaxation to isotropy in μ.
$t_{rev} \approx 100 t_g$

Result: Deviation from QLT/Bohm

- Observed diffusion (dots) is suppressed relative to analytic prediction at higher turbulence. Bohm diffusion (dashed red) gives $D_{\mu\mu} \sim \delta B/B_0$ whereas QLT (solid green) gives $D_{\mu\mu} \sim (\delta B/B_0)^2$. As is demonstrated QLT is approximately valid for $\delta B/B_0 \lesssim 0.3$, and Bohm for $0.3 \lesssim \delta B/B_0 \lesssim 0.5$.



- The distribution of the measured $D_{\mu\mu}$ around their mean follows a broken power law.



Summary

- In accelerators of high energy cosmic rays strong turbulence is expected.
- This lies beyond the scope of QLT/Bohm diffusion. We give the valid regions for these approximations.
- Our results show anisotropic distributions after $\sim 10 t_{rev}$, enough time for acceleration to occur.
- With our improved model we can now simulate the whole acceleration region at intermediate turbulence.

Contact: J. D. Riordan, UCC. (dannyriordan@gmail.com)