Production and decay of magnetic energy in a relativistic fluid

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Tuesday May 13, 2014

Why turbulence?

arth … oceans, rivers, atmosphere, geo-dynamo

pace … sun, solar wind, heliopause (MHD)

terstellar medium … supersonic, giant radio lobes

upernovae … mixing / nuclear burning

eutron stars … superfluid, relativistically warm, magnetized

ternal shocks … GRB prompt, Blazar emission (kinematically
lativistic)

xternal shock … GRB afterglow (magnetic field decay)

Outline

Relativistic hydrodynamic turbulence

Magnetic energy production by turbulent dynamo

Magnetic energy decay in a relativistic fluid: universal

Characterizing relativistic turbulence

- Must use gamma-beta, not v/c!

- Kolmogorov 1941: $P_v(k) \sim k^{-5/3}$

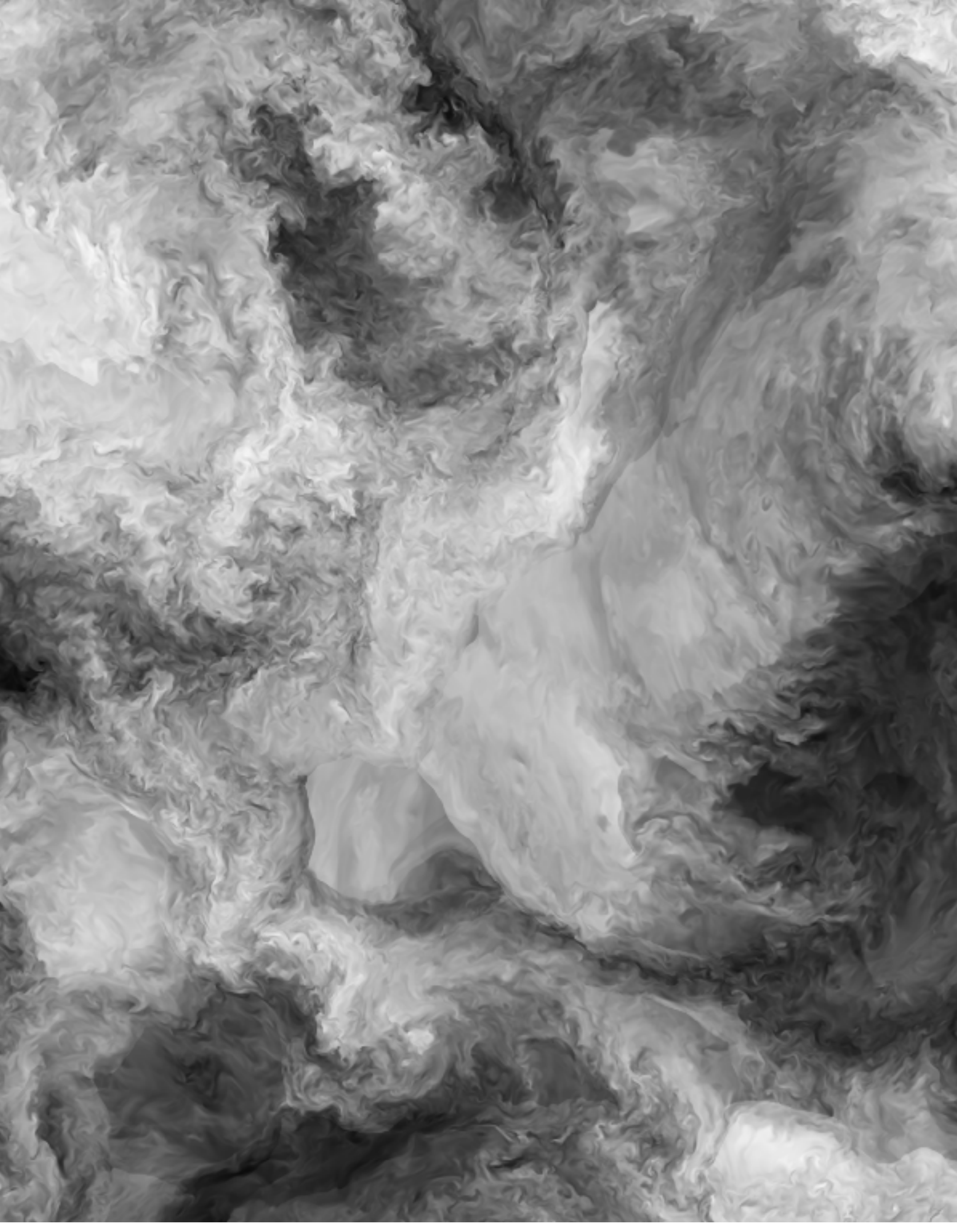
- She-Leveque 1994: intermittency

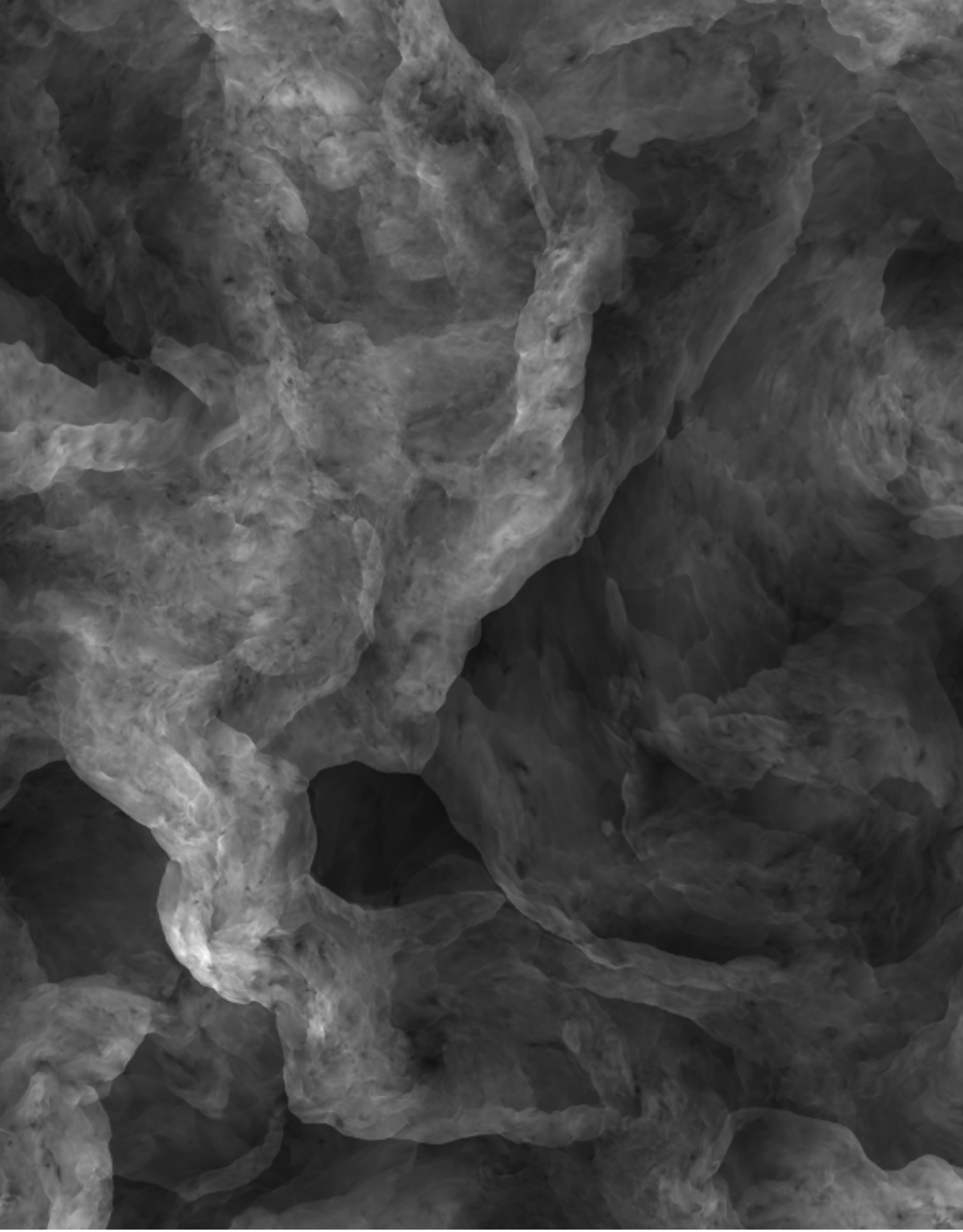
Numerical calculations

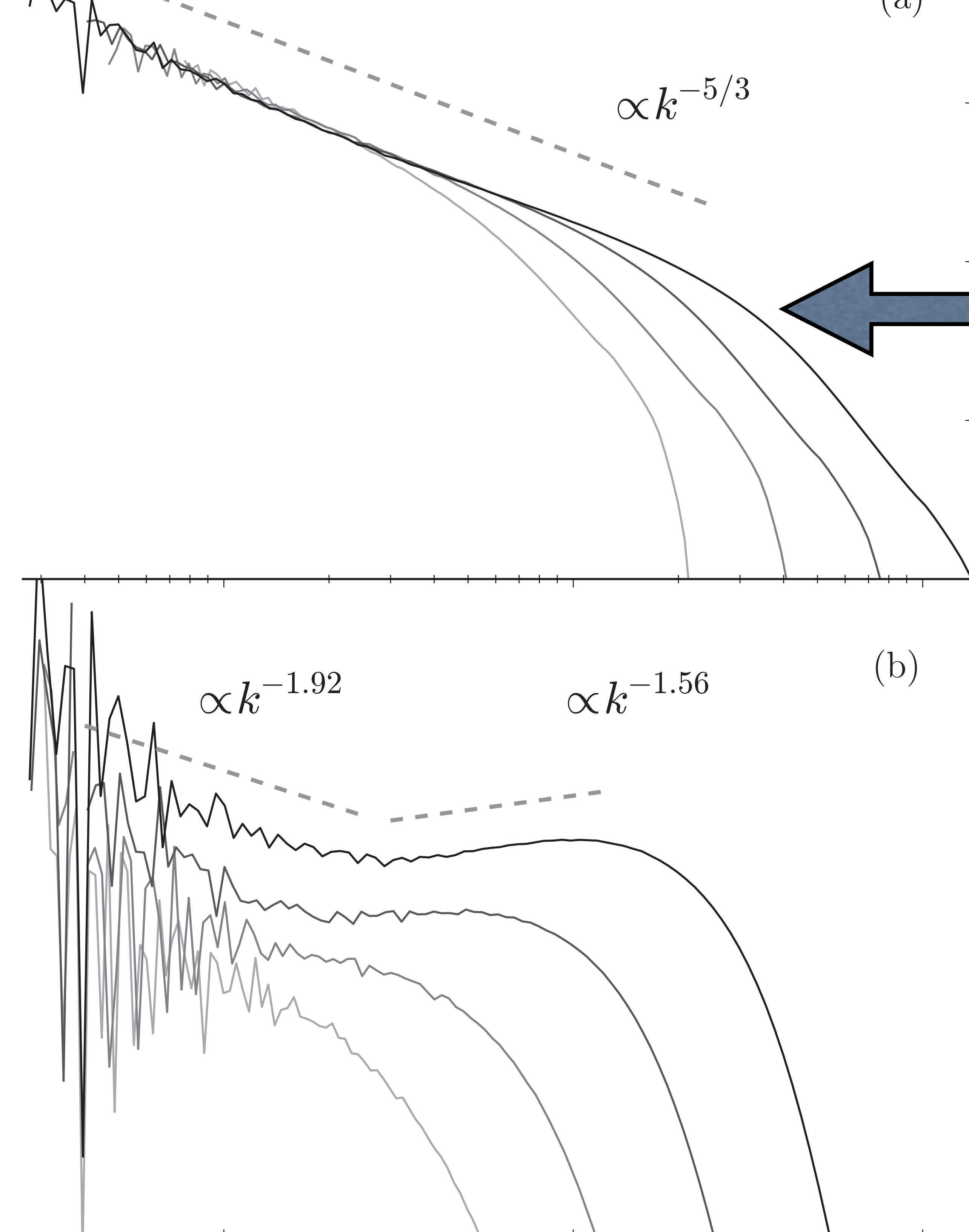
Driven turbulence (stirred “by hand” at large scales)

Periodic box … resolutions up to 2048^3

Relativistic turbulent cascade, fluctuating γ~3



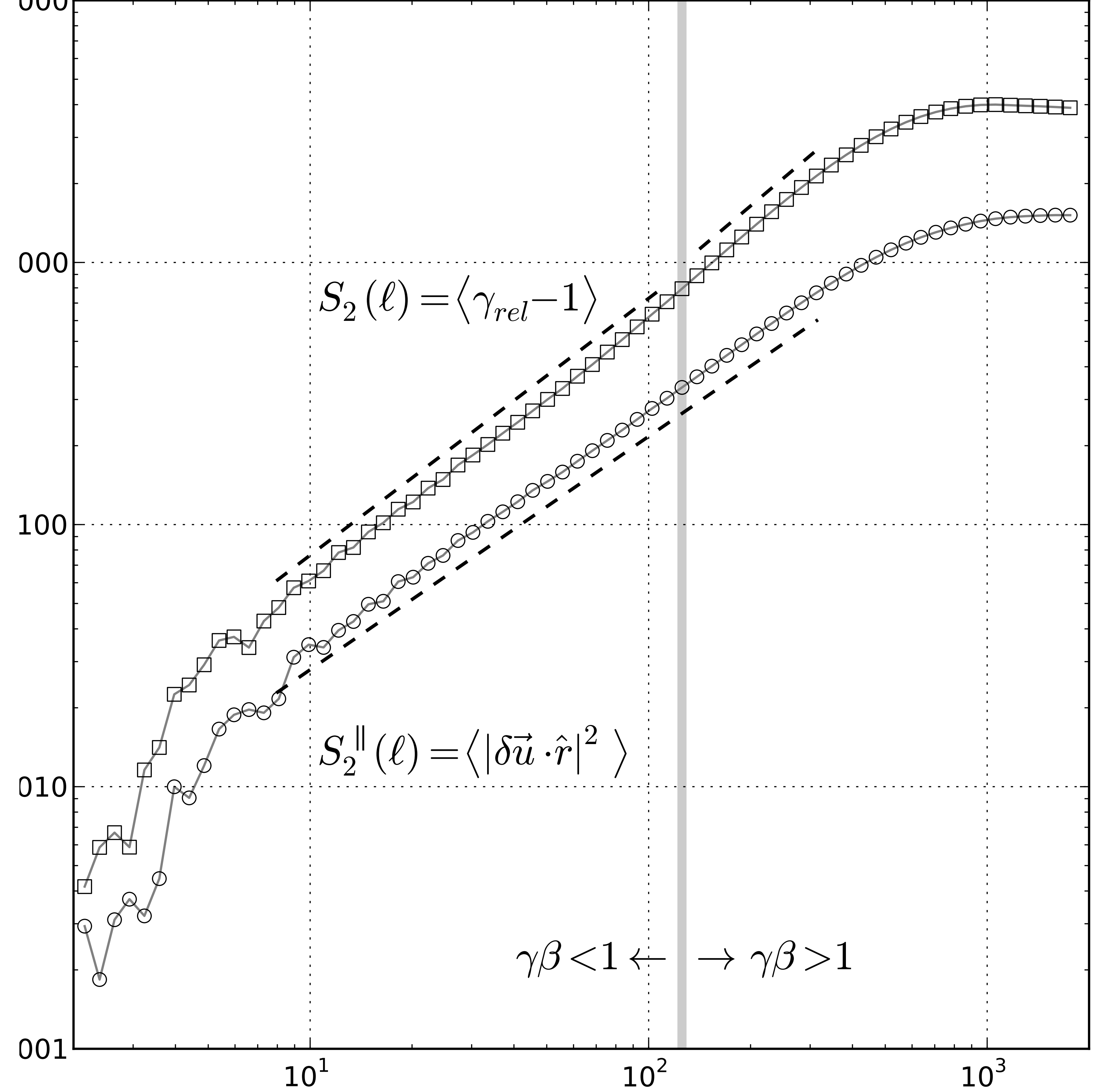




2048³

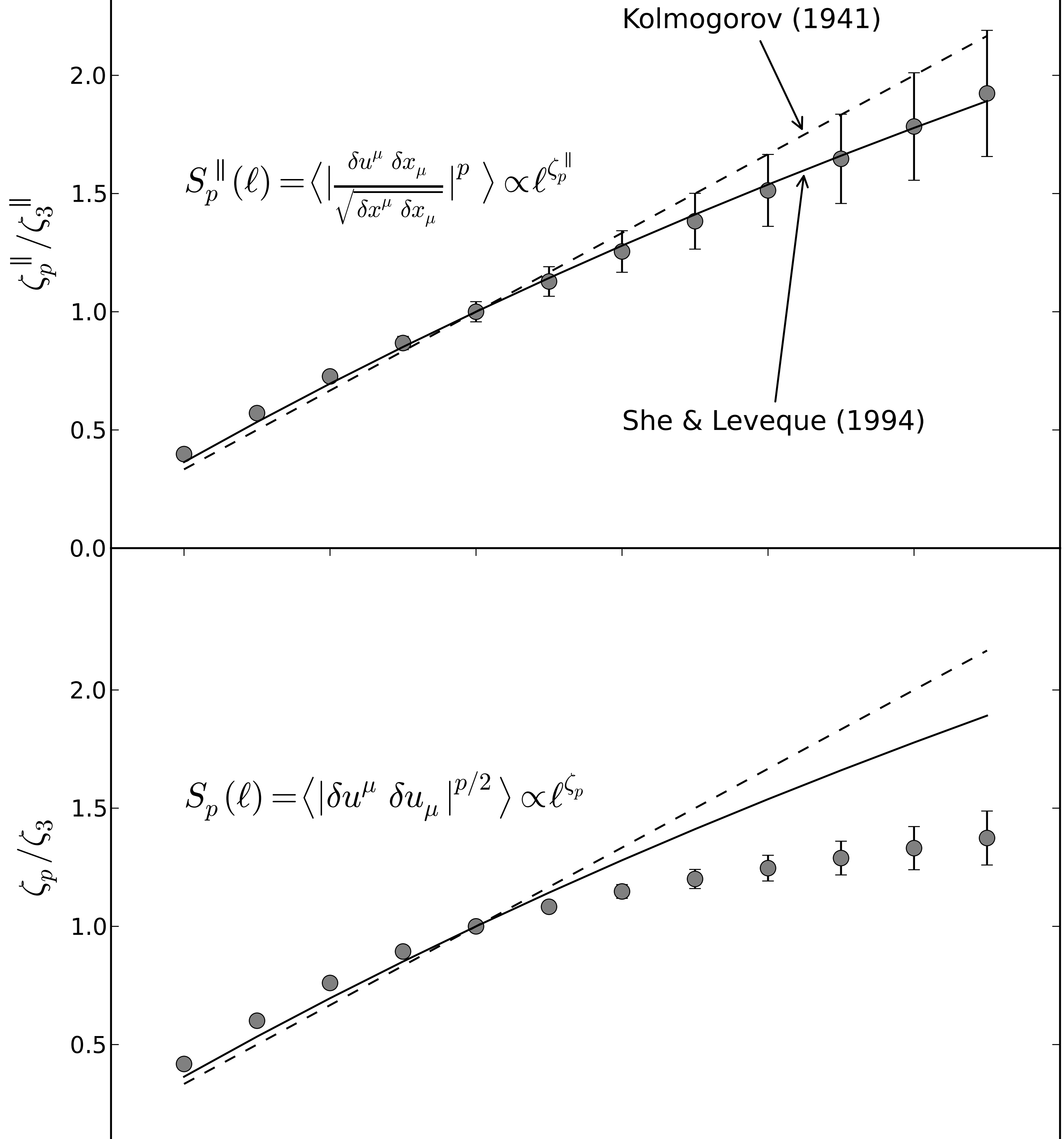
Relativistic
turbulence ha
roughly 5/3 slo
Kolmogorov-li

Zrake and



000
000
100
010
001
10^1
10^2
10^3
$S_2(\ell) = \langle \gamma_{rel} - 1 \rangle$
$S_2^{\parallel}(\ell) = \langle |\delta\vec{u} \cdot \hat{r}|^2 \rangle$
$\gamma\beta < 1 \leftarrow \ \rightarrow \gamma\beta > 1$

Zrak
MacF
(20



Kolmogorov (1941)
She & Leveque (1994)
$S_p^{\parallel}(\ell) = \left\langle \left| \frac{\delta u^\mu \, \delta x_\mu}{\sqrt{\delta x^\mu \, \delta x_\mu}} \right|^p \right\rangle \propto \ell^{\zeta_p^{\parallel}}$
$\zeta_p^{\parallel} / \zeta_3^{\parallel}$
$S_p(\ell) = \left\langle \left| \delta u^\mu \, \delta u_\mu \right|^{p/2} \right\rangle \propto \ell^{\zeta_p}$
ζ_p / ζ_3
0.0
0.5
1.0
1.5
2.0

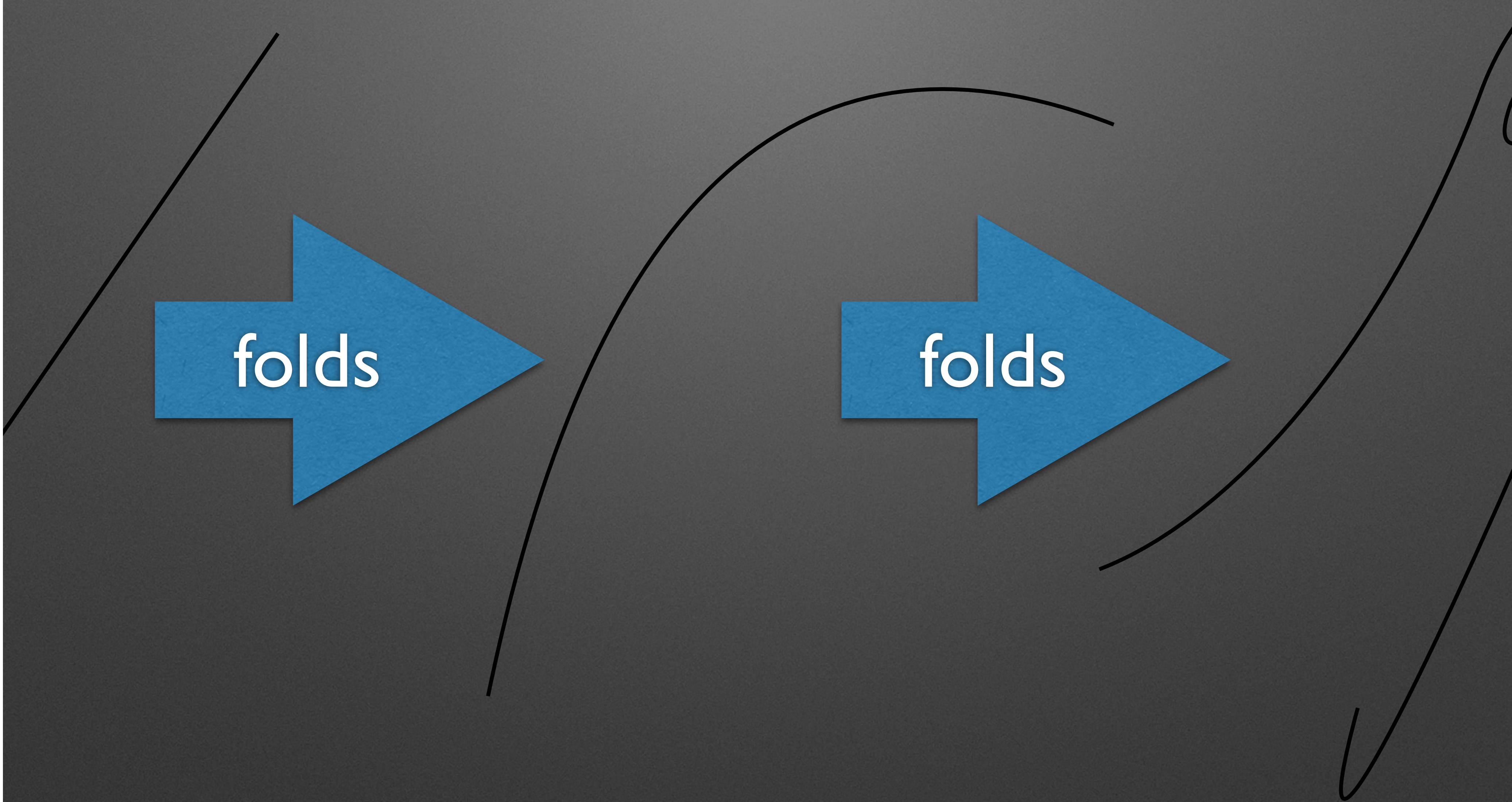
Summary

elativistic hydrodynamic turbulence *must be*
haracterized relativistically

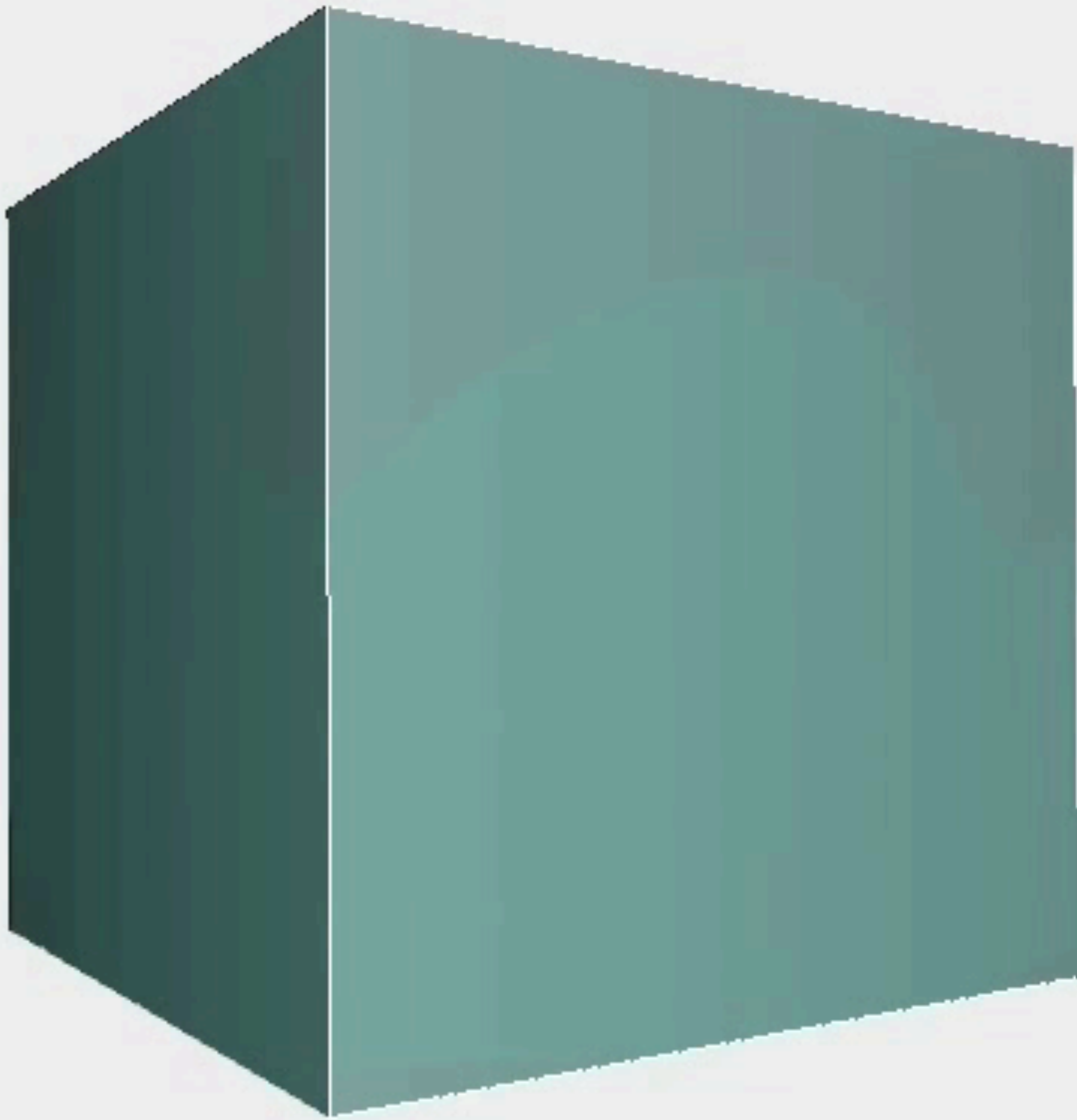
caling is *nearly K41*, but at sufficiently high resoluti
ignificant deviations

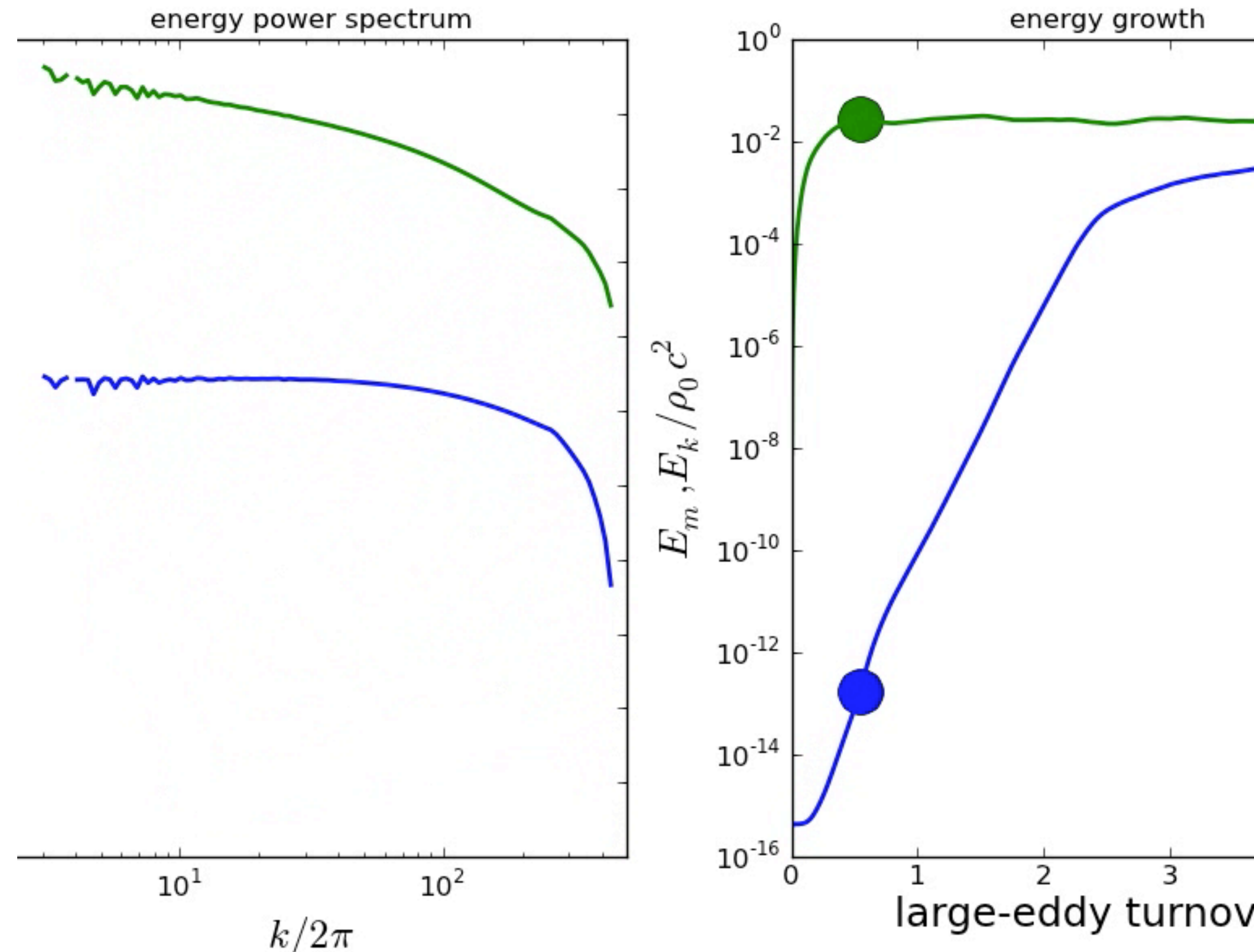
ntermittency *consistent with She-Leveque* if properl
haracterized

Turbulent dynamo



Perfect MHD: magnetic field frozen into the fluid




energy power spectrum
10^1
10^2
$k/2\pi$
energy growth
$E_m, E_k/\rho_0 c^2$
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
0
1
2
3
large-eddy turnov

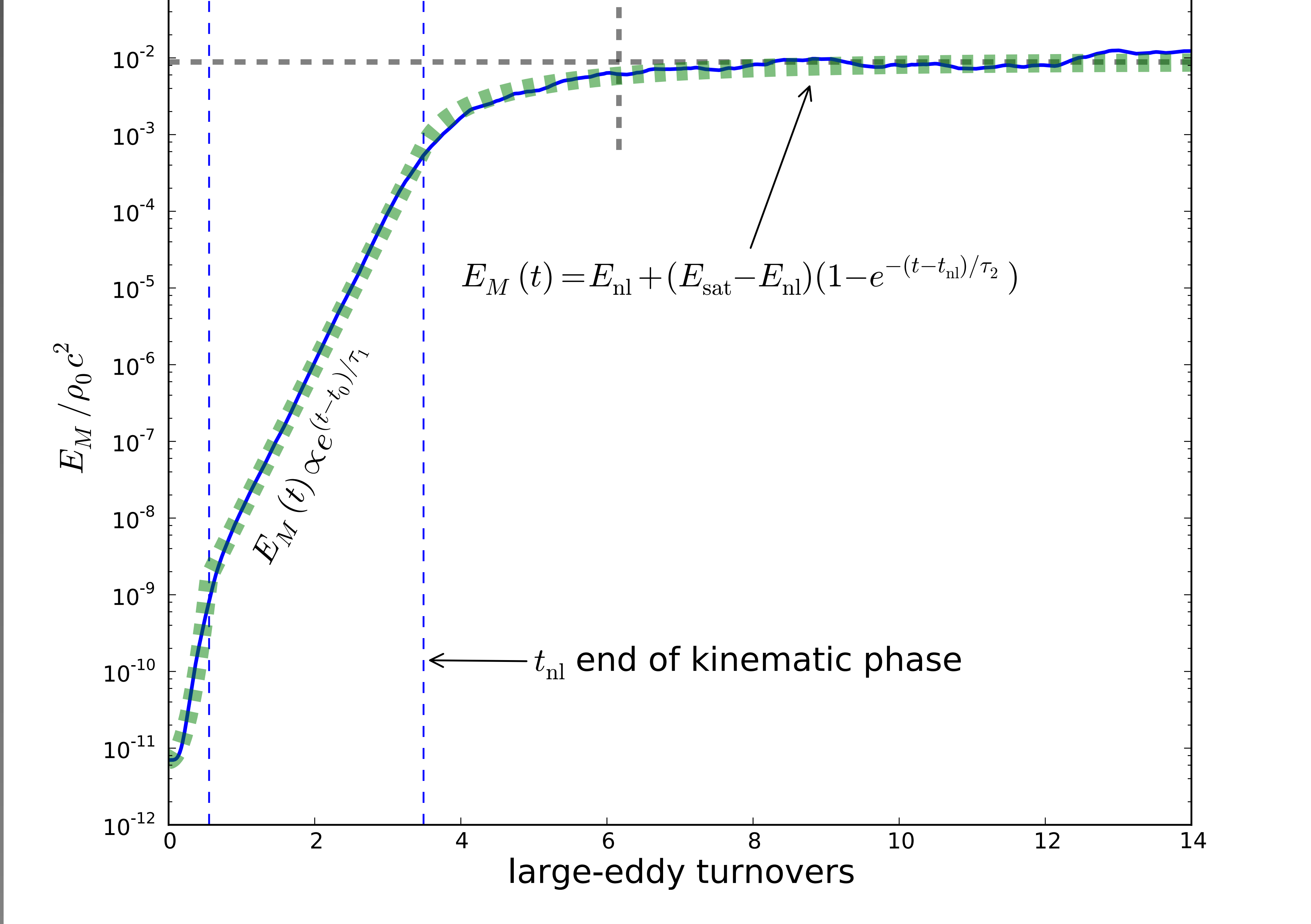
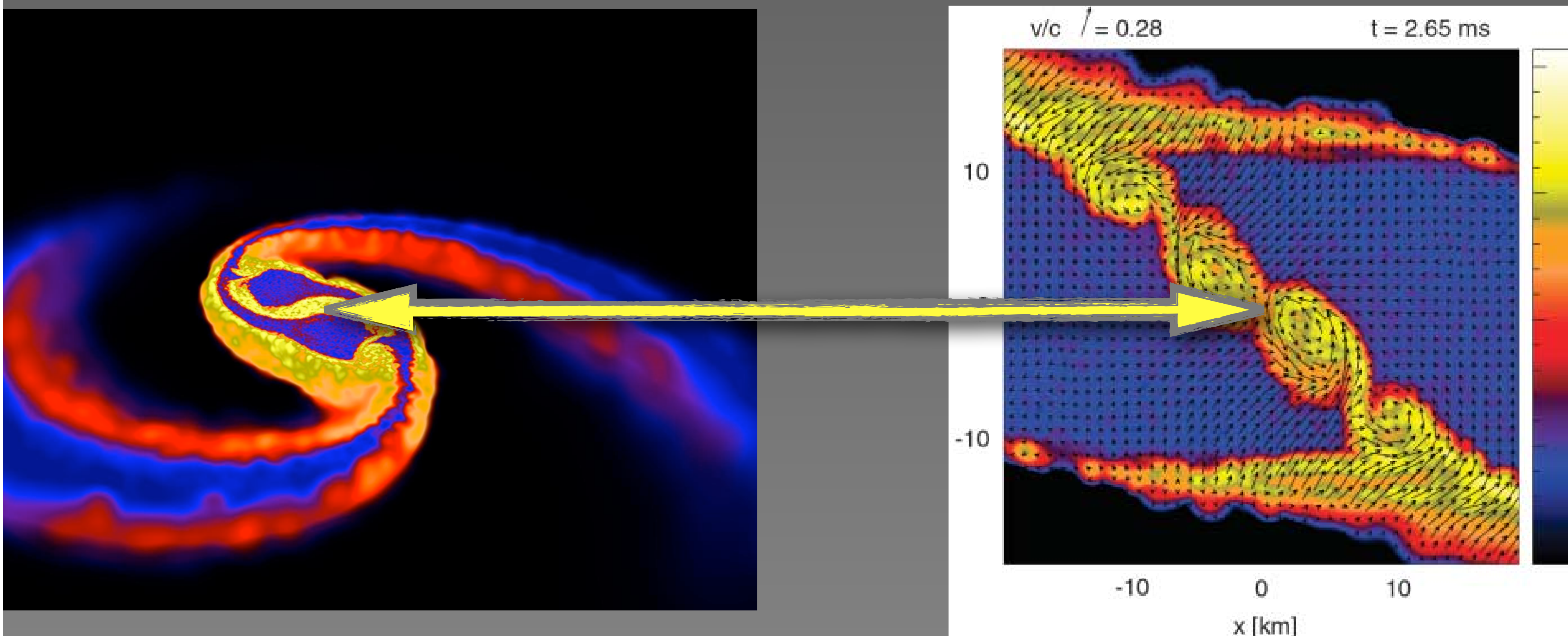


Figure 3. Time history of the magnetic energy for a representative run at 128^3, together with the empirical model (Equation 3) with best-fit parameters. The horizontal dashed line indicates the magnetic energy, $E_{\rm sat}$ at the dynamo completion. From left to right, the vertical dashed lines mark the end of the startup,

Neutron star merger

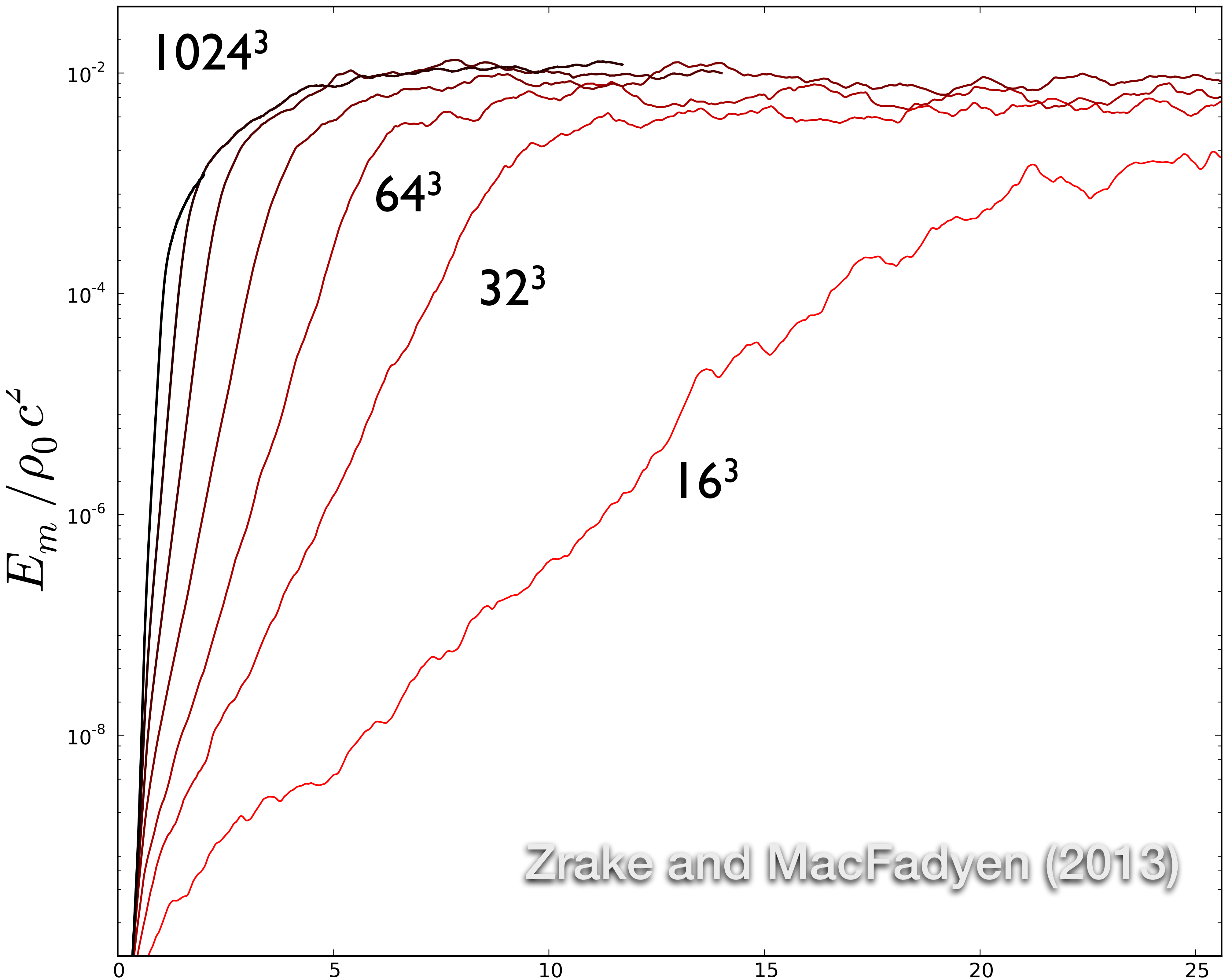


See also:

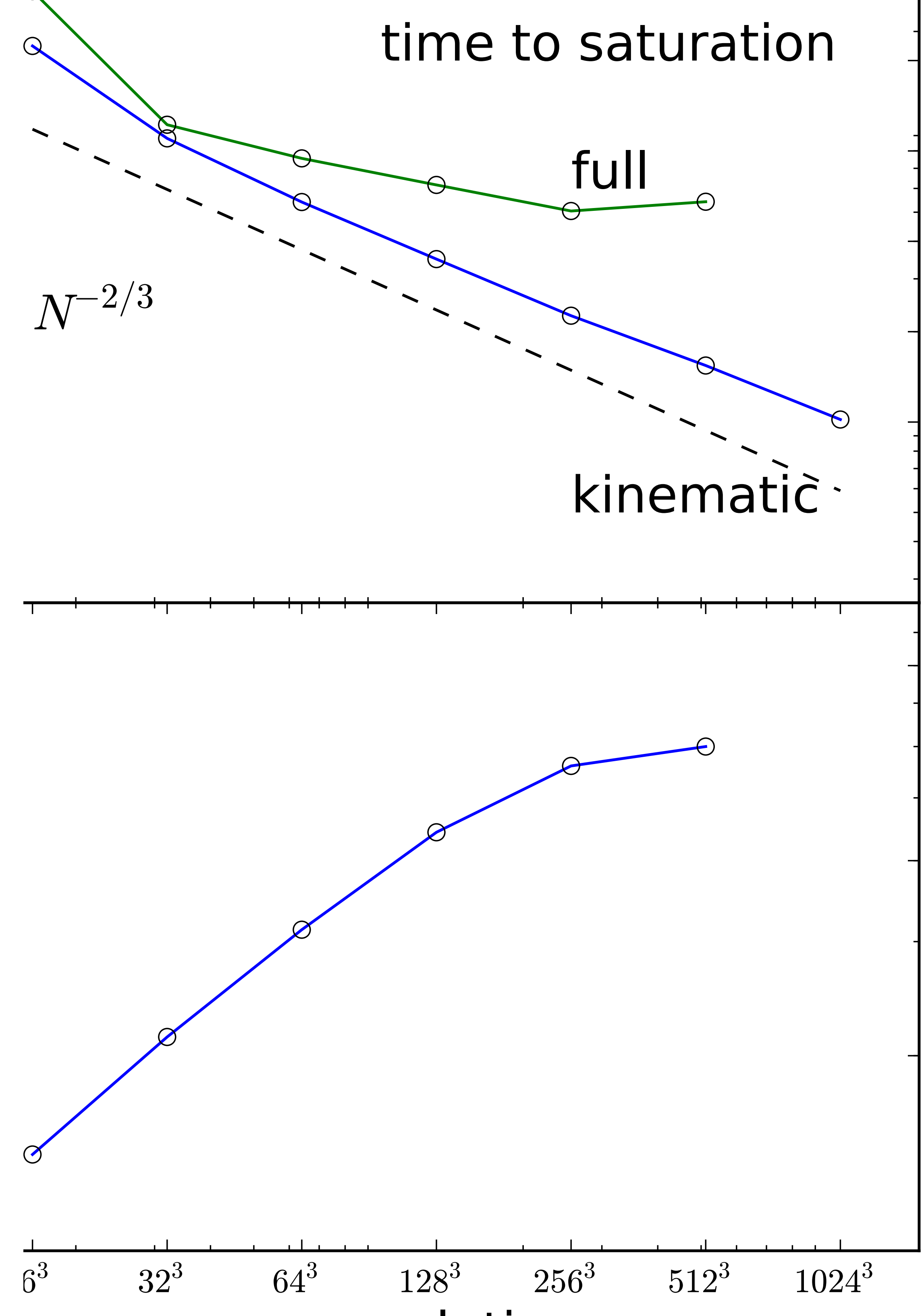
Liu et al. (2008)
Anderson et al.
Giacomazzo (20
Shibata et al. (20

Equipartition

$$\mathrm{MS} \gtrsim 10^{16}\mathrm{G}\left(\frac{\rho}{10^{13}\mathrm{g/cm^3}}\right)^{1/2}\left(\frac{v_{\mathrm{ec}}}{0.}\right.$$



1024³
64³
32³
16³
Zrake and MacFadyen (2013)
$E_m \,/\rho_0 c^2$
10^-2
10^-4
10^-6
10^-8
0
5
10
15
20
25



Zrake and MacFadye

Figure 4. *Top*: Convergence study of the k
growth time τ_1 (*blue*) and the dynamo completio
defined as $t_{\rm nl} + \tau_2$. *Bottom*: Convergence st
fit model parameter $E_{\rm sat}$ expressed as the rati
kinetic energy E_M/E_K. The converged valu
averaged $E_M/E_K \approx 0.6$. Nevertheless, at in

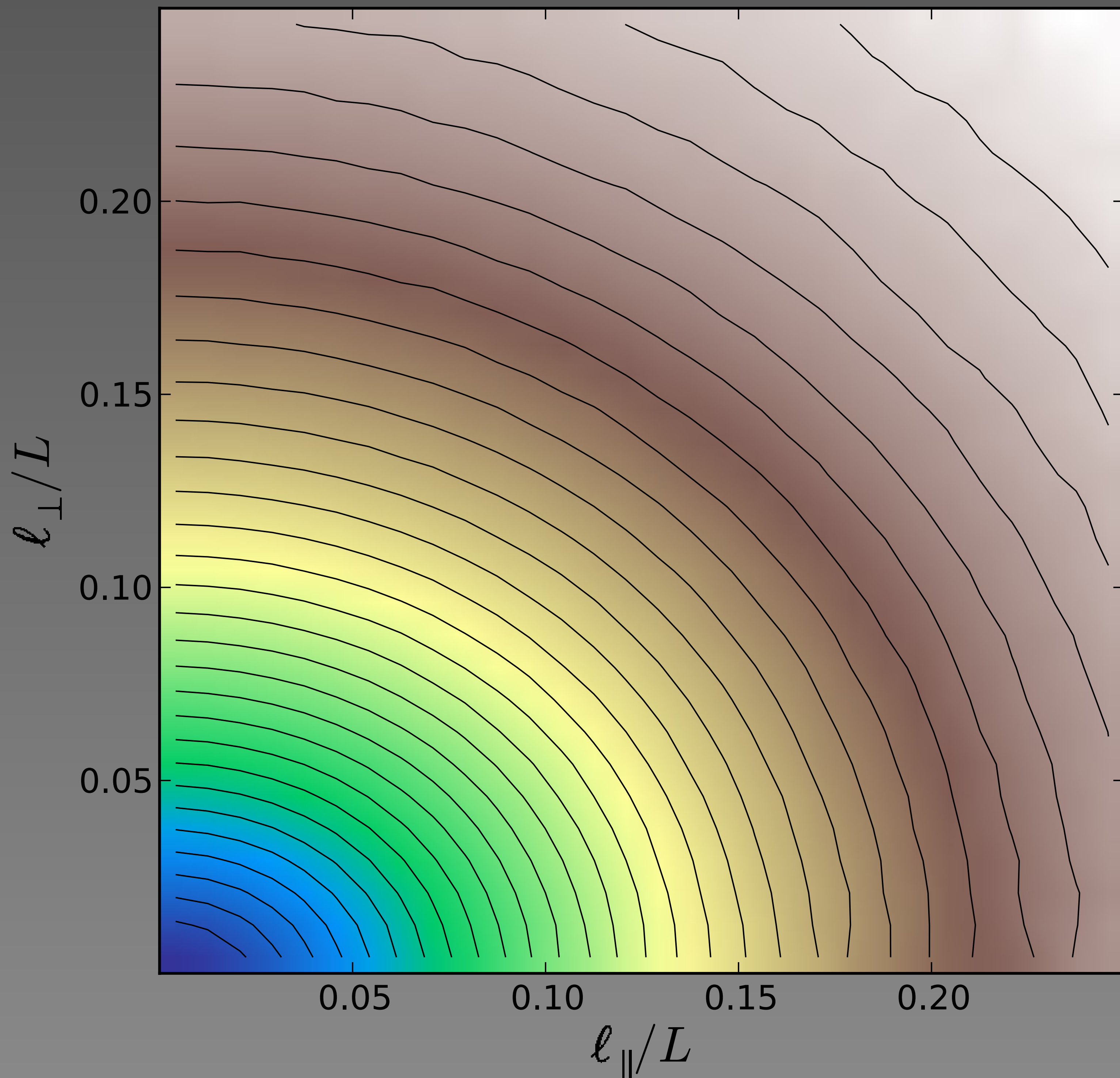


Figure 9. Shown is the two-dimensional, second-order structure function of the velocity field $S_2^v(\ell_\perp, \ell_\parallel)$. The offset vector $\boldsymbol{\ell}$ is decomposed into

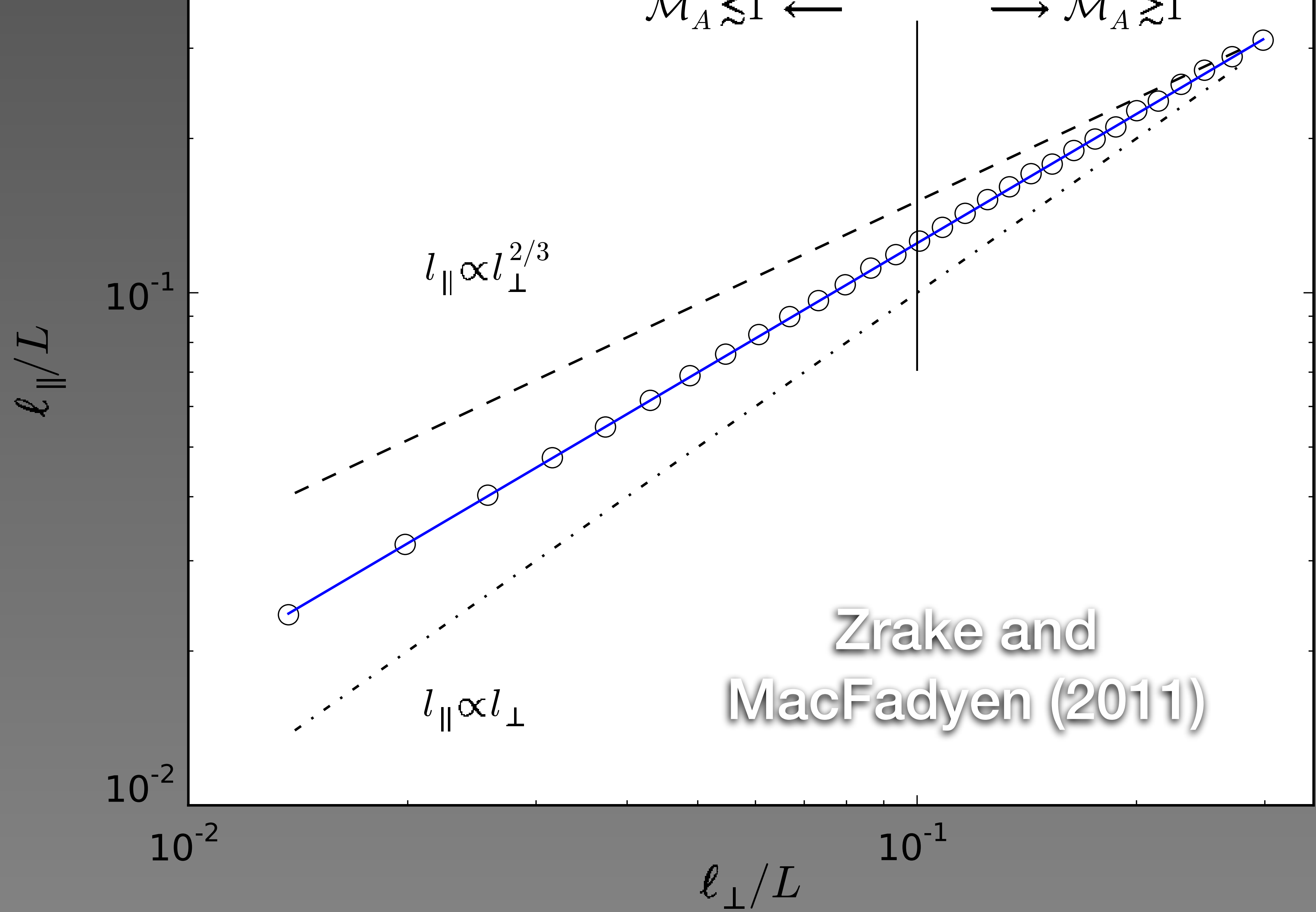


Figure 10. Semi-major ($\ell_{\|}$) and semi-minor ($\ell_{\perp}$) axes of the elliptical contours obtained from Figure 9 (circles), and the best-fit line (solid) $\ell_{\|} \propto \ell_{\perp}^{0.84}$. For comparison we provide the prediction of Goldreich & Sridhar (1995) $\ell_{\|} \propto \ell_{\perp}^{2/3}$ (dashed) and a slope of unity (dash-dotted). The vertical line marks the scale

Summary

Driven relativistic MHD turbulence achieves *quipartition with turbulent kinetic energy*

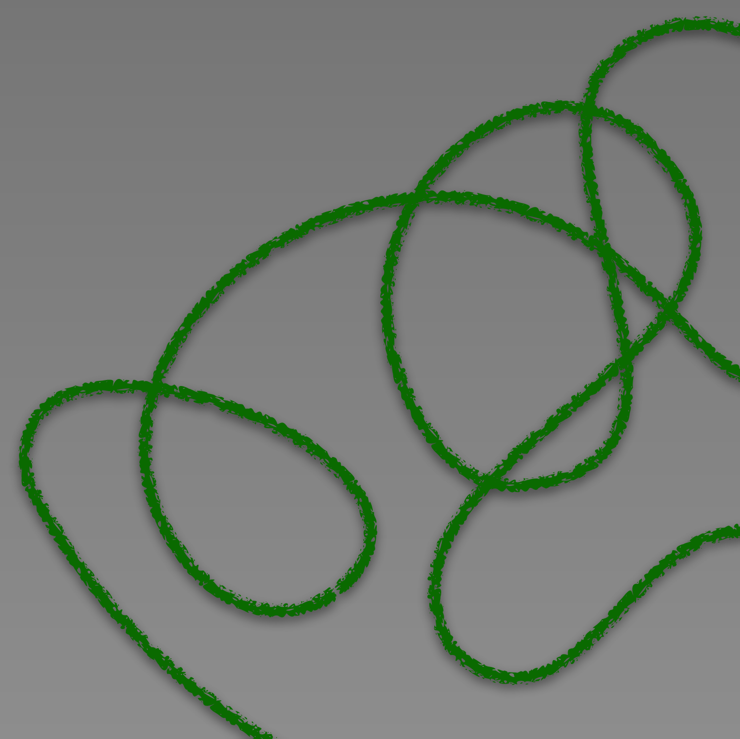
oldreich-Sriedhar scaling evident, but anisotropy is nore weakly dependent on the scale

IS-NS mergers *produce copious magnetic energy*, 10^{50} ergs if turbulence is prevalent

Question:

w do magnetic field fluctuations decay in a
ativistic fluid?

$$_{\mathbf{k}}(t = 0) \propto \delta(k - k_0)$$



In particular

lassical MHD flow problem

there an “inverse cascade” in MHD?

o current sheets emerge from generic initial data?

the dissipation “bursty”?

nagnetic energy loss mediated by resistivity, or sor
nlinear process?

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

Simplest possible model

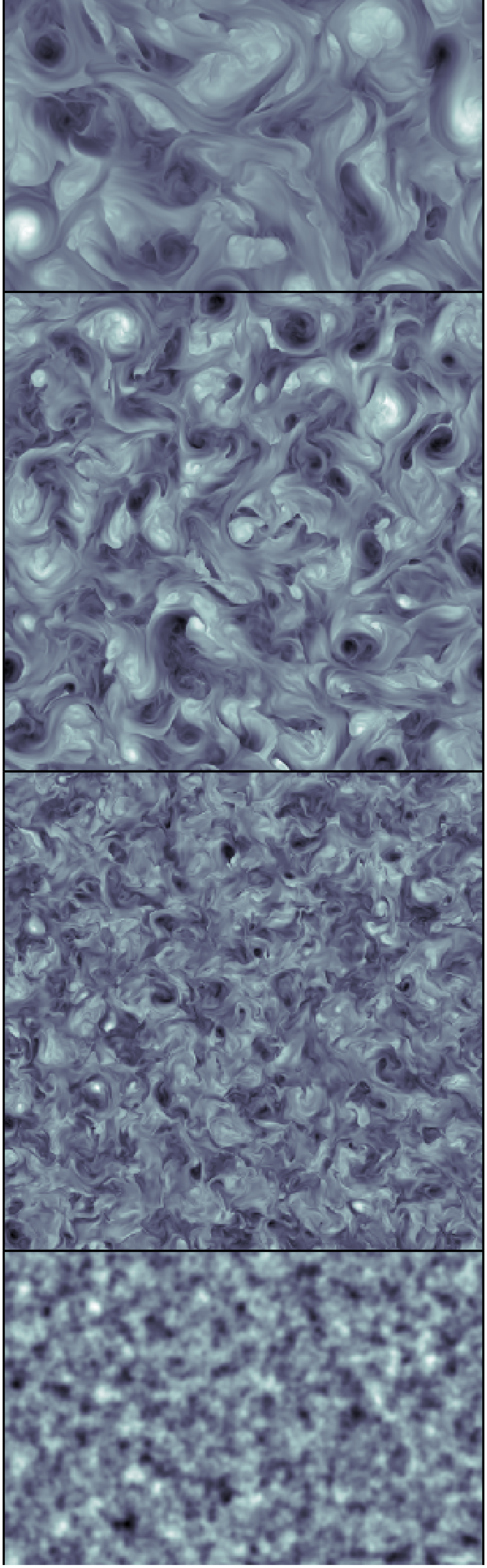
Relativistic MHD equations $B^2 \sim p_{gas} \sim$

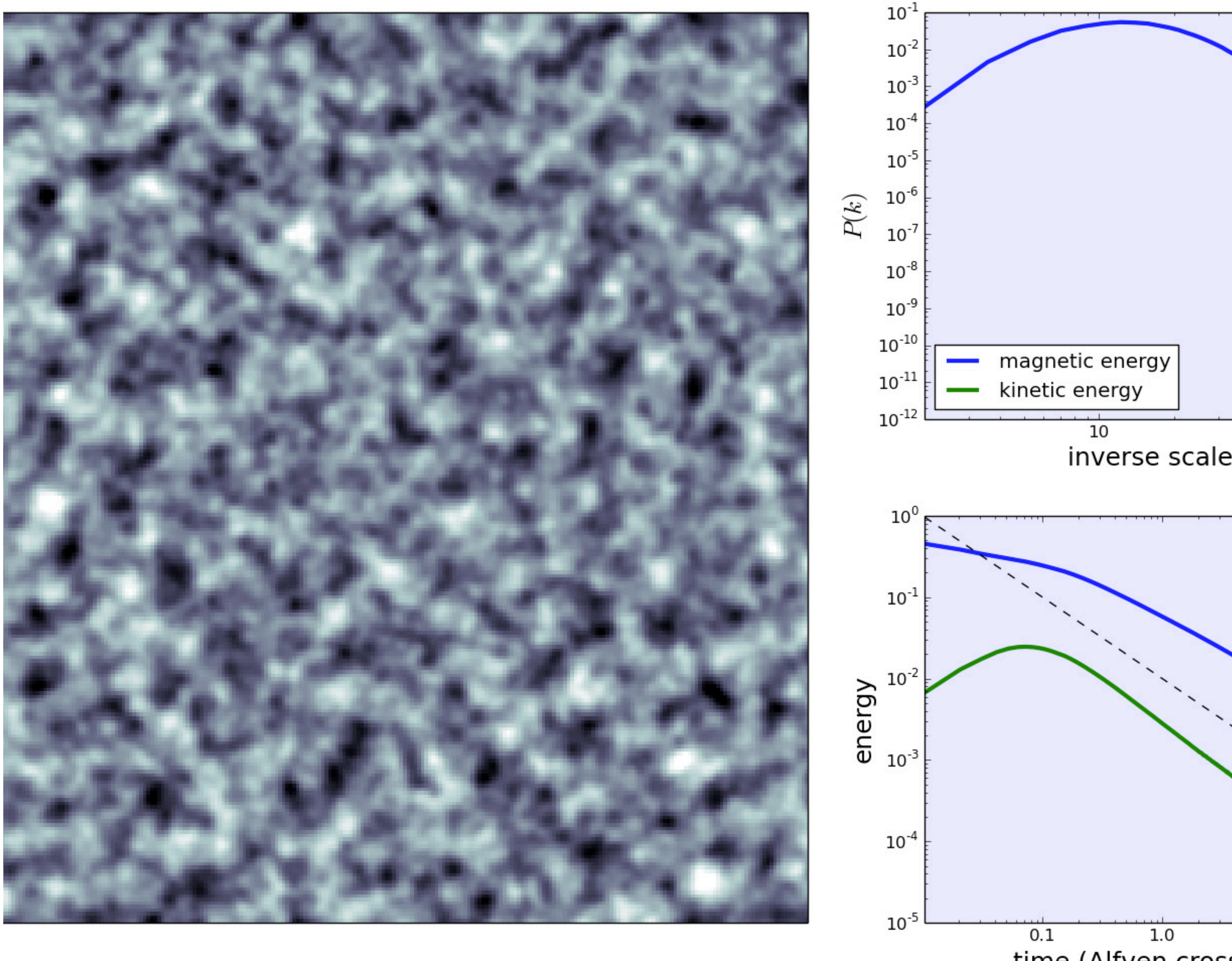
Ideal (artificial resistivity and viscosity)

Three dimensions, periodic, cartesian

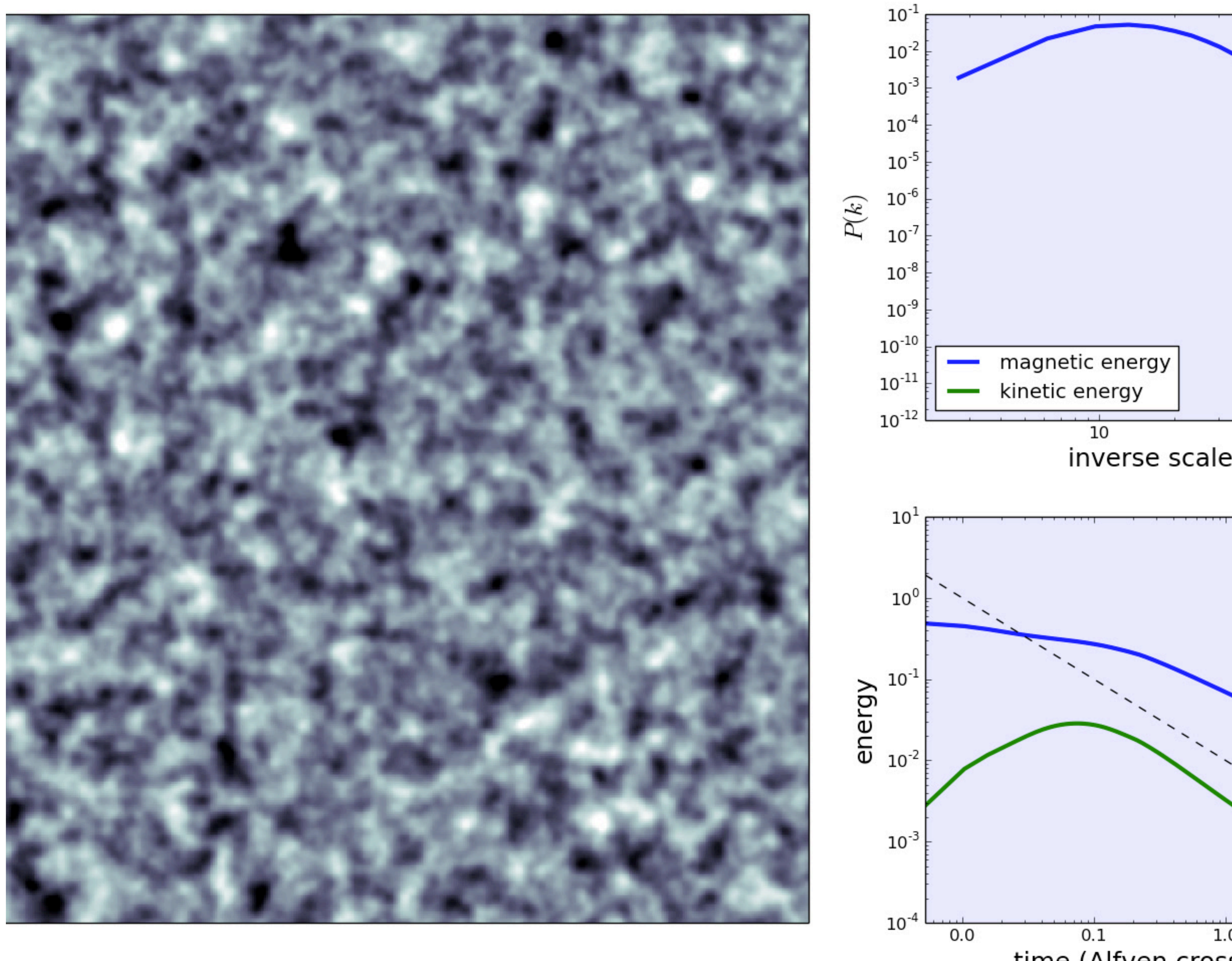
imulate stationary gas embedding a tangled
nagnetic field, let it go

Magnetic field is initially Gaussian random

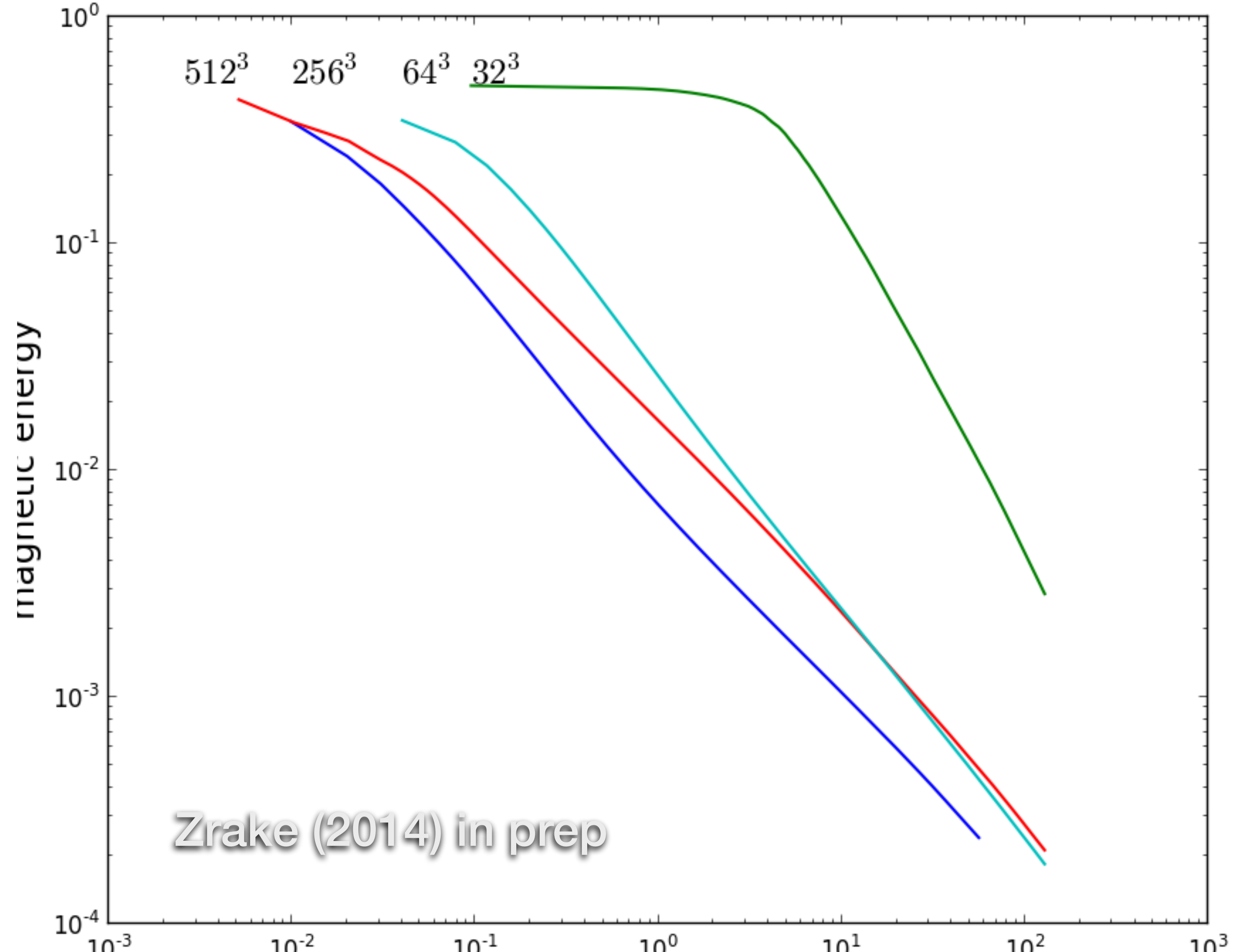




$P(k)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
magnetic energy
kinetic energy
10
inverse scale
energy
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0.1
1.0
time (Alfven



magnetic energy
kinetic energy
$P(k)$
inverse scale
10
energy
0.0
0.1

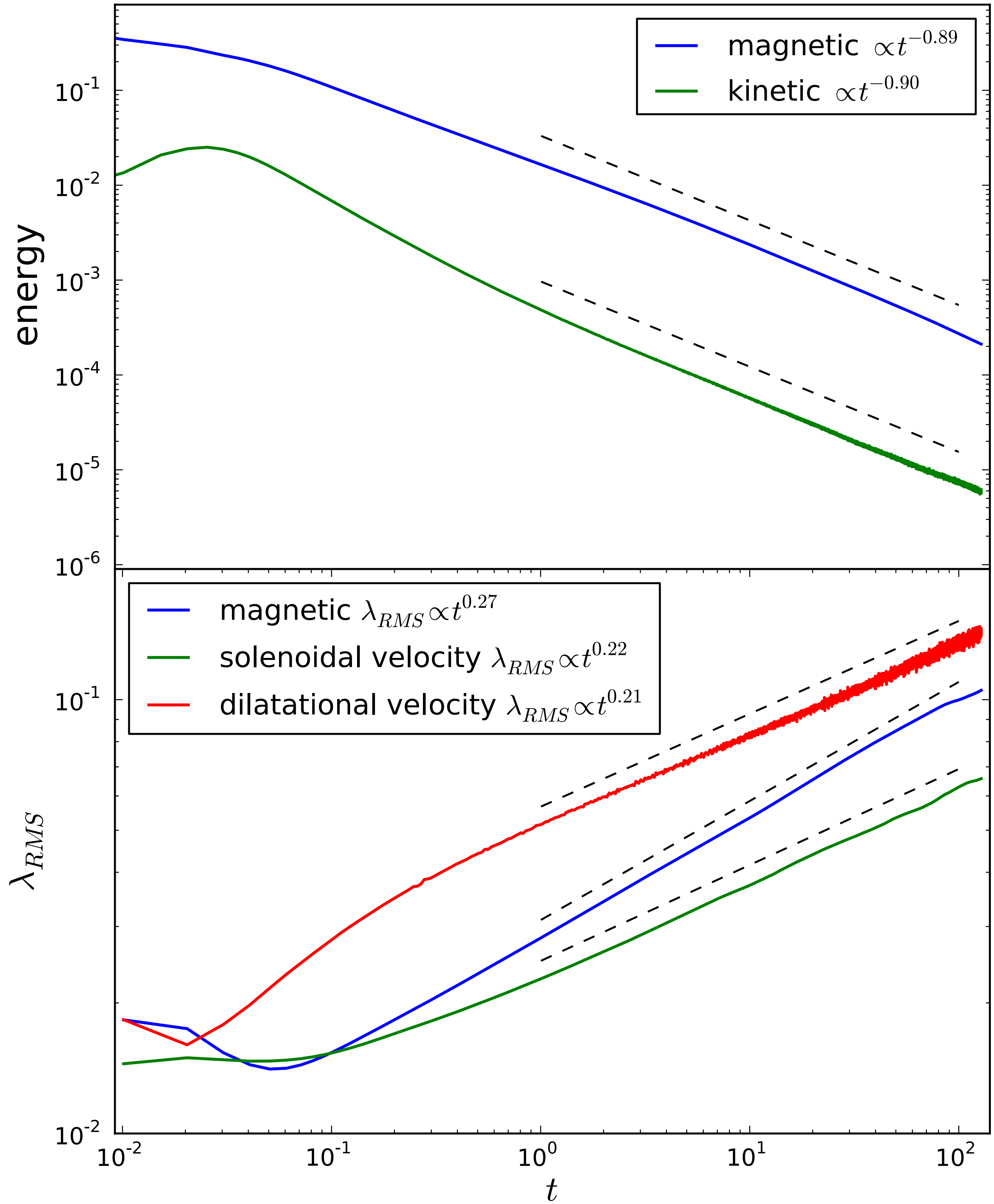


magnetic energy
512^3
256^3
64^3
32^3
Zrake (2014) in prep
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

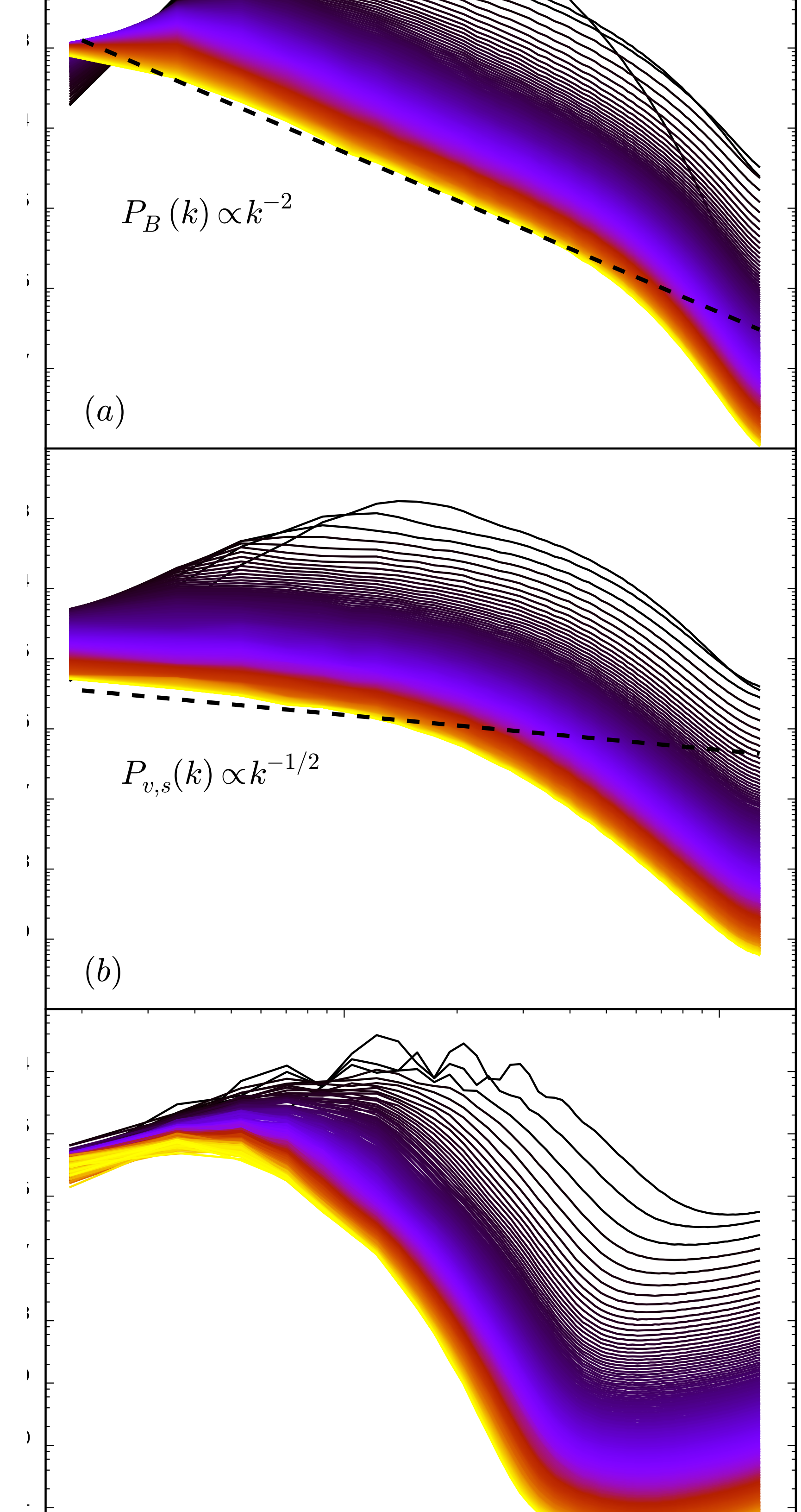
Why power law decay?

$$\dot{E}(t) \sim E(t)/t_A(t)$$

$$t_A \sim \lambda(t)/v_A(t)$$



magnetic $\propto t^{-0.89}$
kinetic $\propto t^{-0.90}$
energy
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
magnetic $\lambda_{RMS} \propto t^{0.27}$
solenoidal velocity $\lambda_{RMS} \propto t^{0.22}$
dilatational velocity $\lambda_{RMS} \propto t^{0.21}$
λ_{RMS}
10^{-1}
10^{-2}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
t



Power spectrum
B is consistent w
current sheets

Summary

agnetic energy decay self-similar ~ t^{-1} *independent of grid resolu*

o explosive dissipation seen with beta ~ 1 fluid

onsistent with Alfven wave cascade, but also with current sheet
rmation: *which controls the decay?*

eed to quantify the “current-sheet-ness” of 3d field configuration
eas?