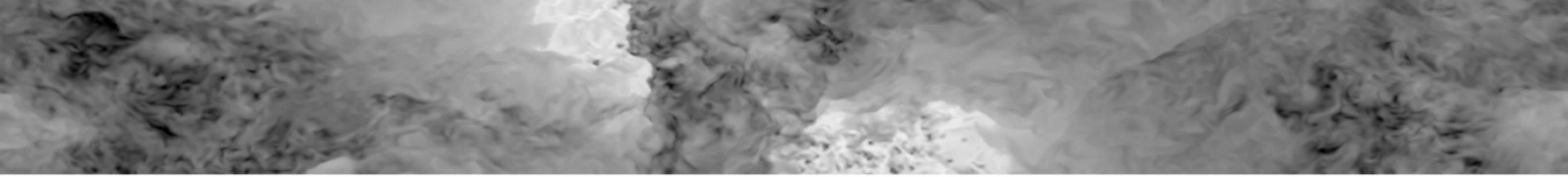

Numerical Simulations of Relativistic Turbulence

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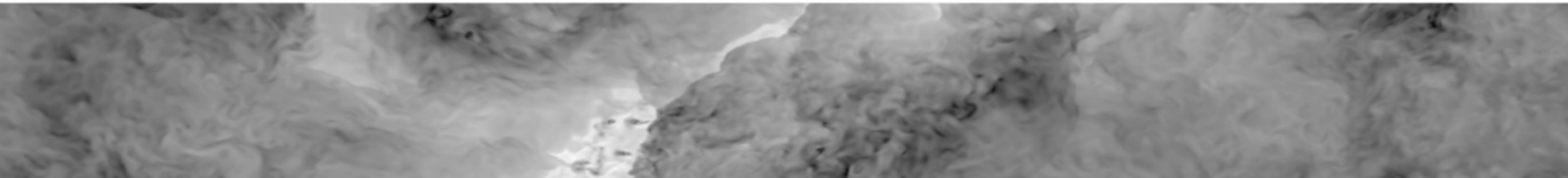
Collaborator: Luciano Rezzolla

A&A 547:A26 (2012), ApJL 766:L10 (2013)



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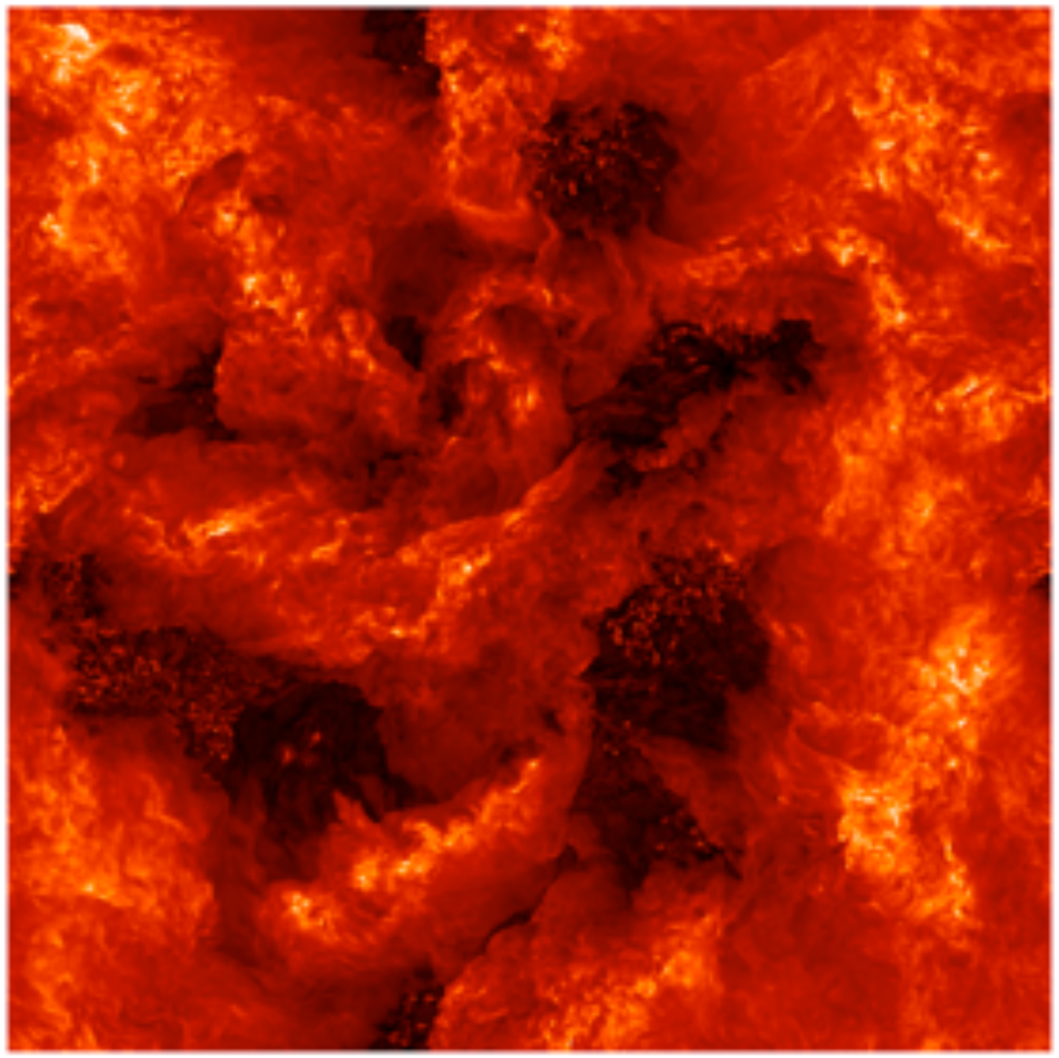
2. Model and methods
3. Results and conclusions

Relativistic Turbulence



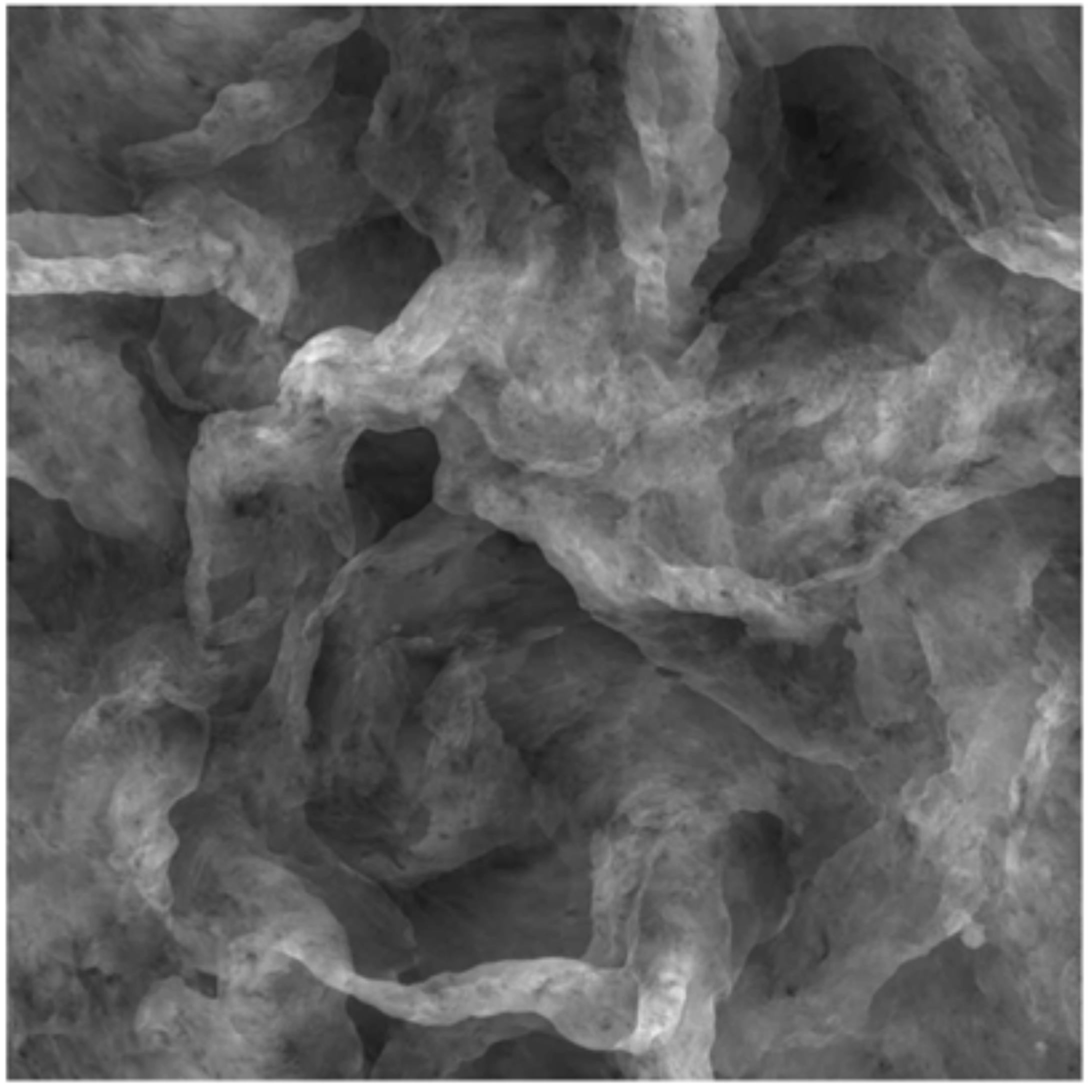
Applications

- GRB jets
- Binary neutron star merger
- Force-free magnetospheres
- Fluid/gravity correspondence
- …

Studies of Relativistic Turbulence

Numerical studies

- Chou: 2005, Chou & Lazarian 2014
- Zhang & MacFadyen: 2009
- Zrake & MacFadyen: 2012, 2013ab
- Inoue et al.: 2011
- Green et al.: 2012, 2014



From Zrake & MacFadyen 2013a

(Some) Previous Results

- Transport properties expected to change
- Power-spectrum of the velocity: Kolmogorov?
- Decay is exponentially fast

Open Questions

- To what extend can Kolmogorov's phenomenology be applied to the relativistic regime?
- What is the efficiency of relativistic turbulence?
- How does relativity impact the statistical properties of the velocity field?

Contents

Model

- Stress energy tensor:

$$\mathbf{T} = e\,\mathbf{u} \otimes \mathbf{u} + p\,\perp$$

- Euler equations:

$$\nabla \cdot \mathbf{T} = \mathbf{F}$$

- Equation of state:

$$p = \frac{1}{3}e$$

Statistics of Turbulence

- Velocity in the bulk frame $\{\mathbf{e}_a\}$

$$v_i = \frac{\mathbf{u}\cdot\mathbf{e}_i}{-\mathbf{u}\cdot\mathbf{e}_0} \qquad W = -\mathbf{u}\cdot\mathbf{e}_0 \qquad v_i \sim \mathcal{N}(0,\sigma)$$

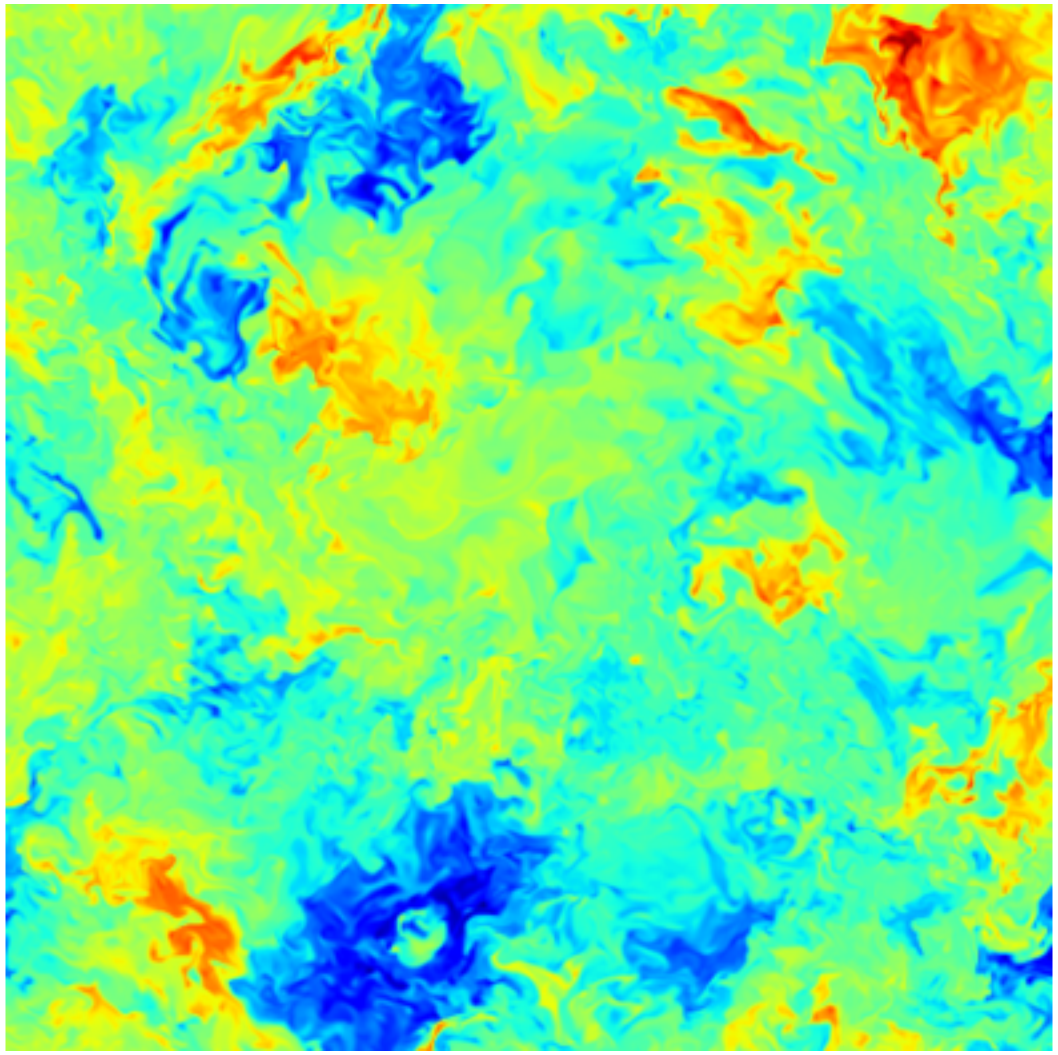
- Power spectrum of the velocity

$$E(k) = \langle \bar{v}_i(k) v^i(k)\rangle = \ldots = \mathcal{F}\big[\langle v_i(x) v^i(x+r)\rangle\big]$$

- Structure functions

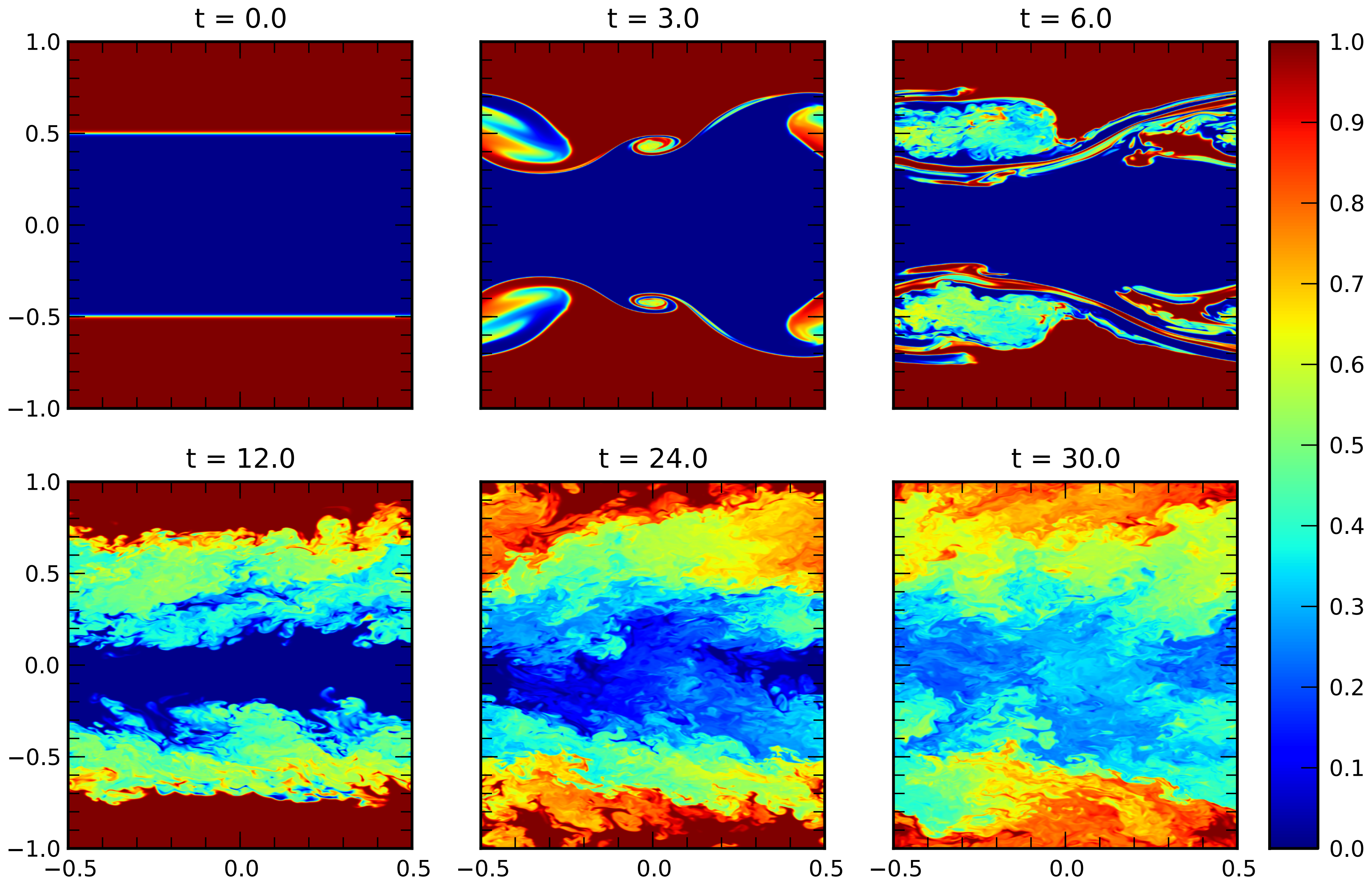
$$S_p^{||} = \left\langle \left| [\mathbf{v}(\mathbf{x}+\mathbf{r}) - \mathbf{v}(\mathbf{x})]\cdot\frac{\mathbf{r}}{|\mathbf{r}|}\right|^p \right\rangle =: \langle |\delta v|^p\rangle$$

Templated Hydrodynamics Code

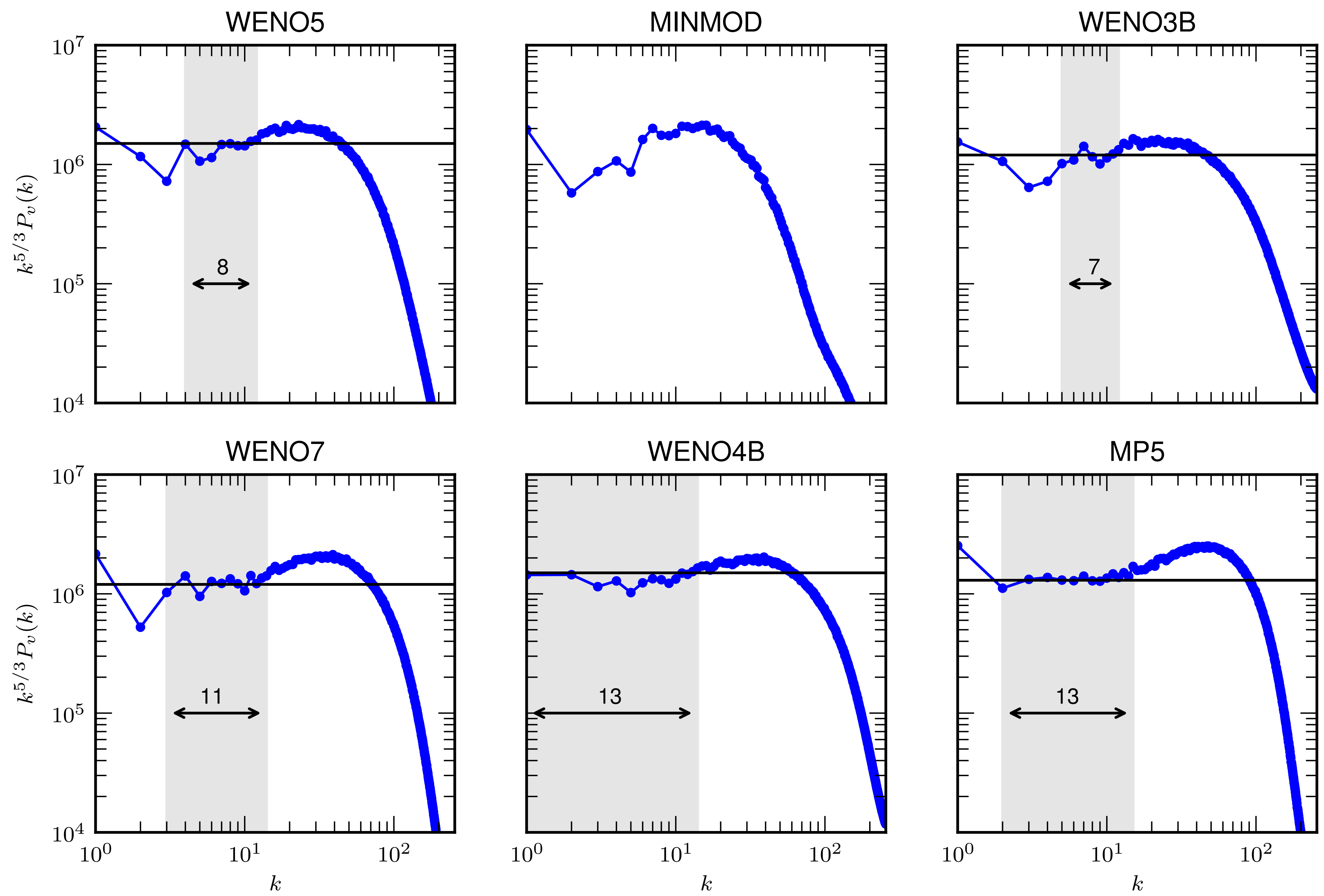


- GRHD
- Based on the Einstein Toolkit
- Nuclear equation of state
- High-order HRSC methods
- In progress: discontinuous Galerkin methods and neutrino radiation

Warm-up: Relativistic KH (I)



Warm-up: Relativistic KH (II)



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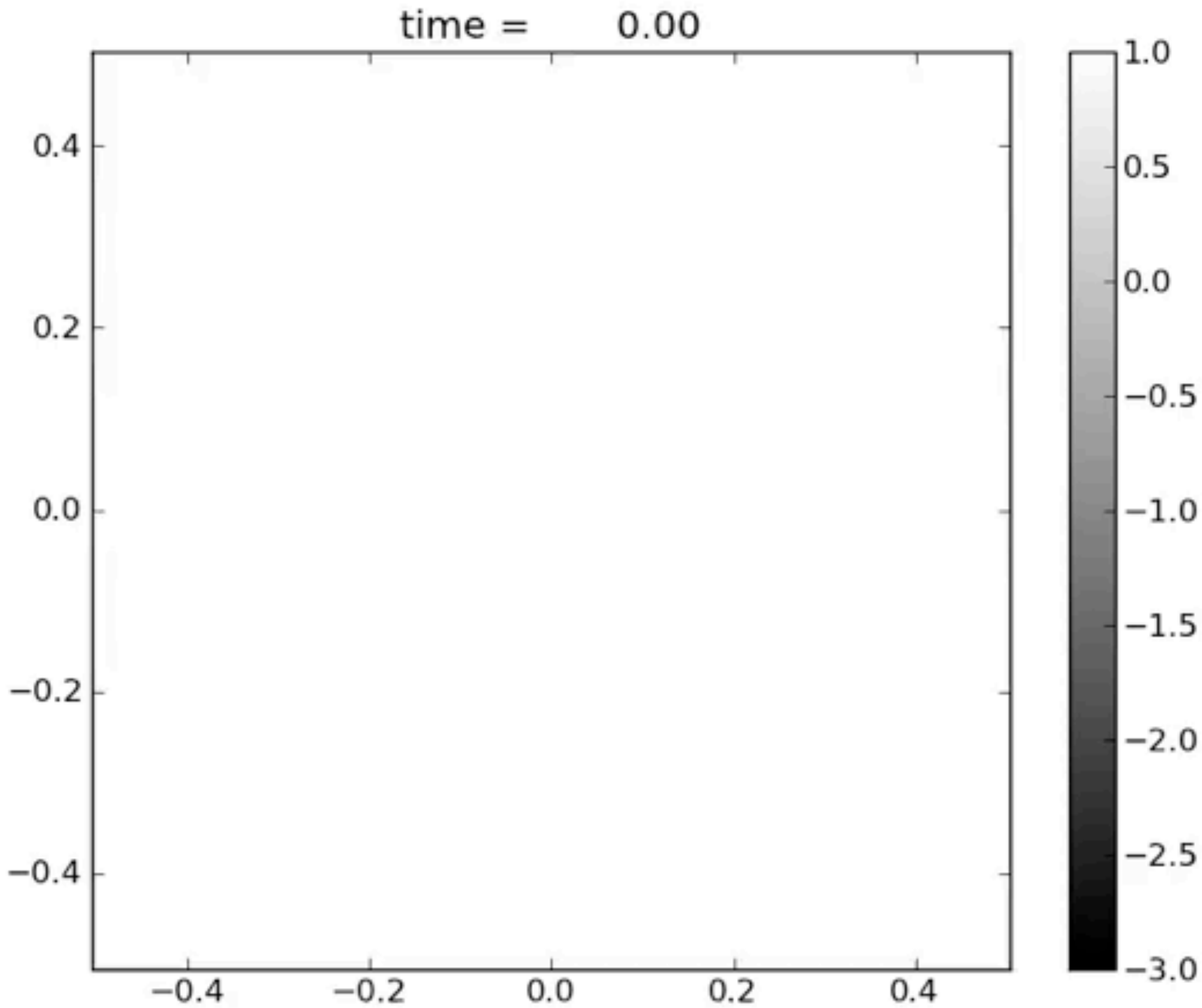
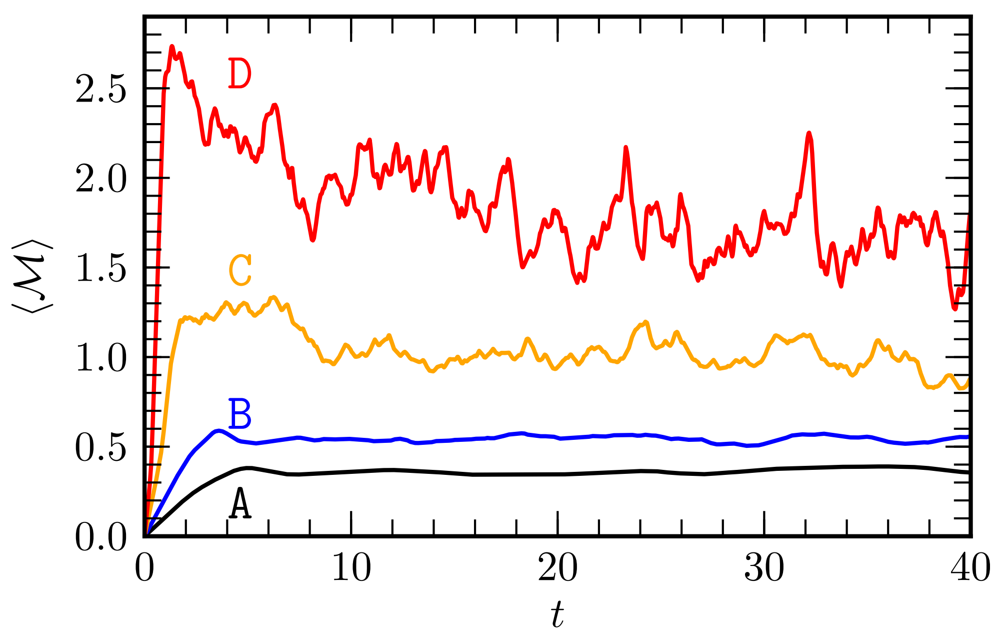
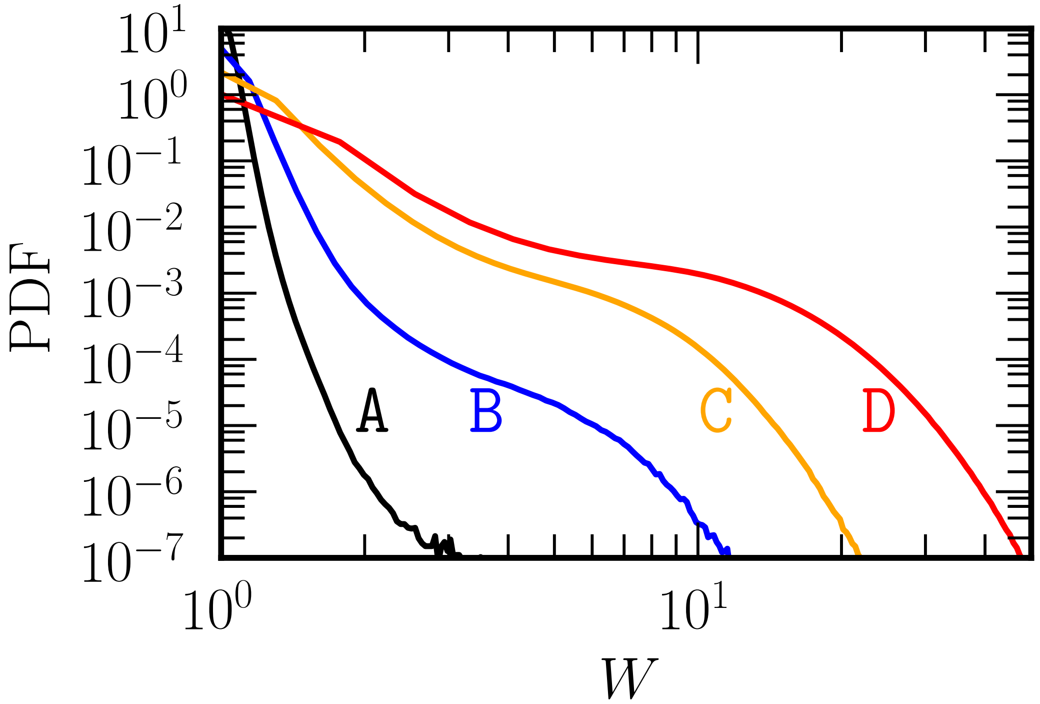
Relativistic Turbulence with THC

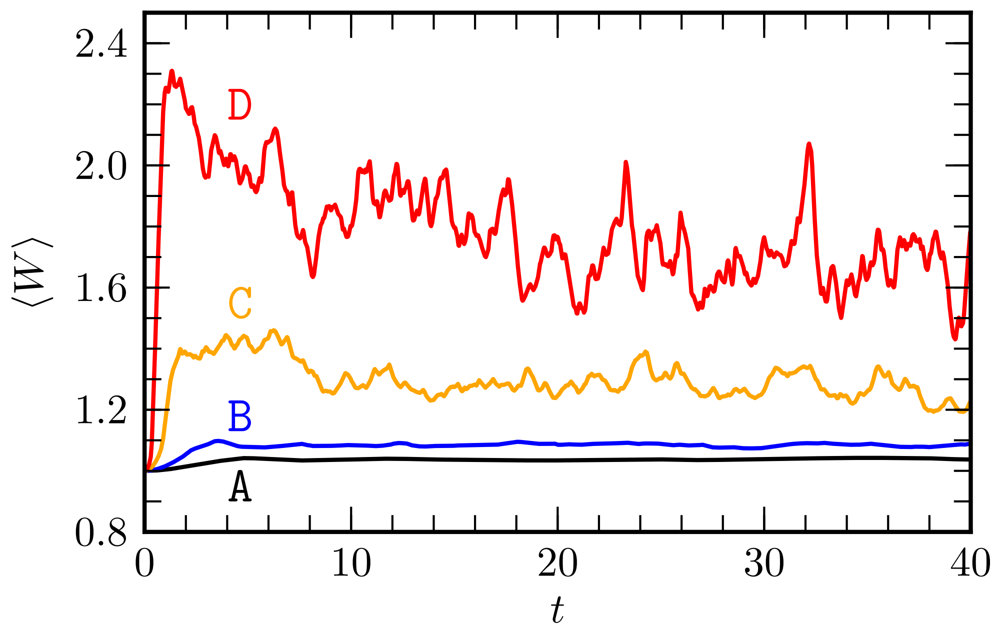


$$\log_{10}(\Gamma - 1)$$

Basic Flow Properties





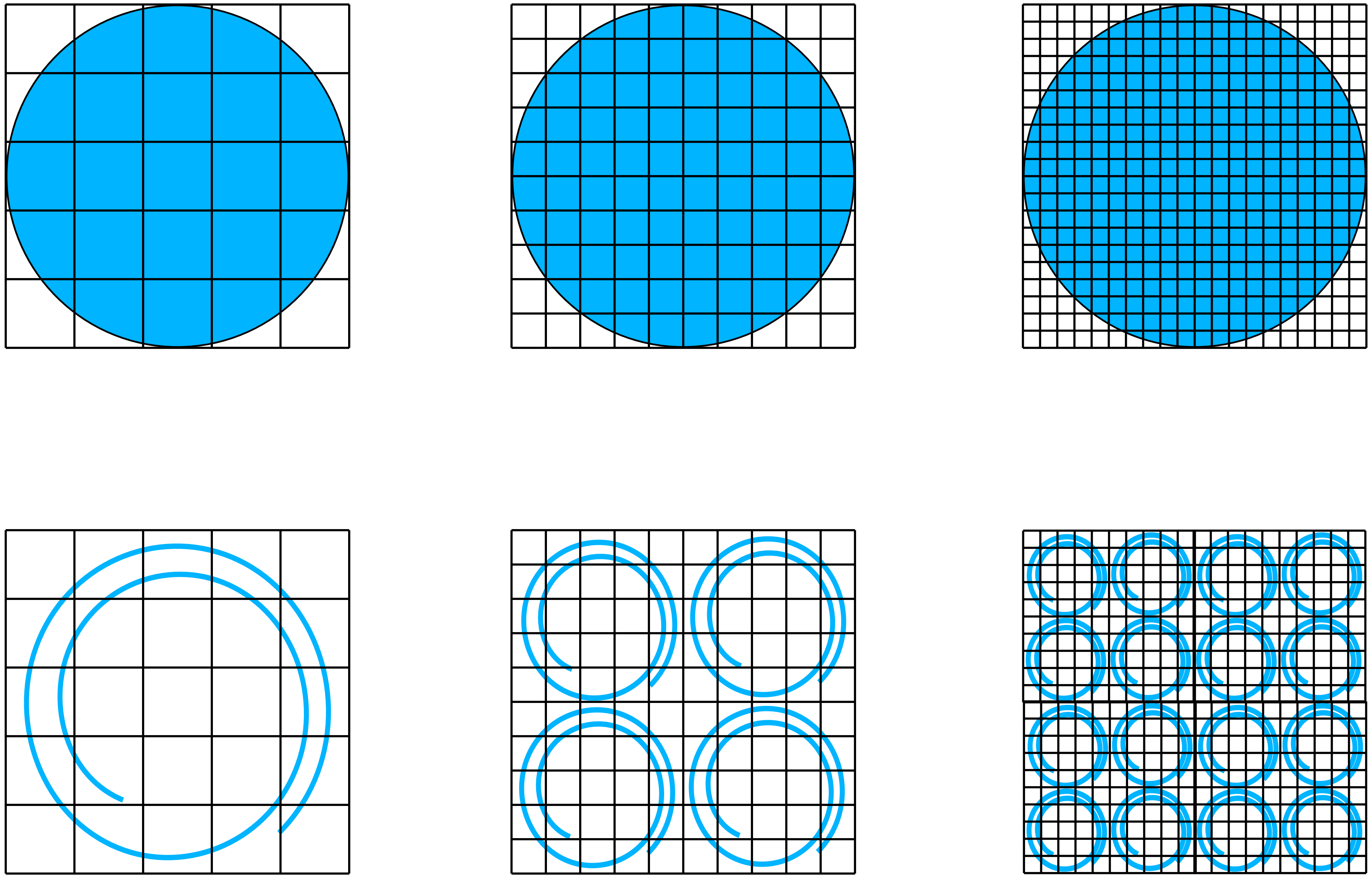


- Transonic turbulence
- Moderate average Lorentz factors
- Wide distribution in Lorentz factors

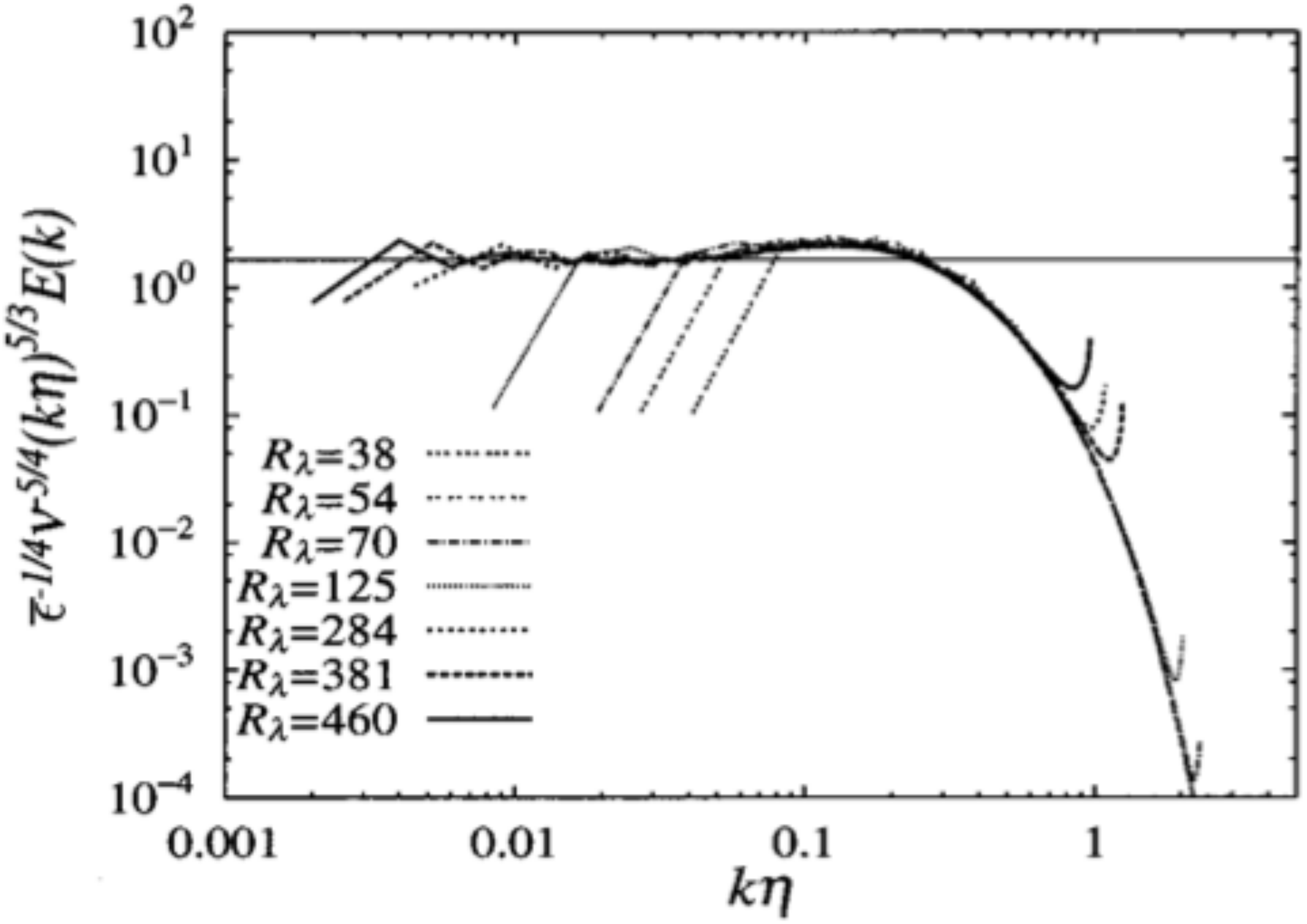
The Meaning of Convergence (I)

Global simulations:

Local simulations:

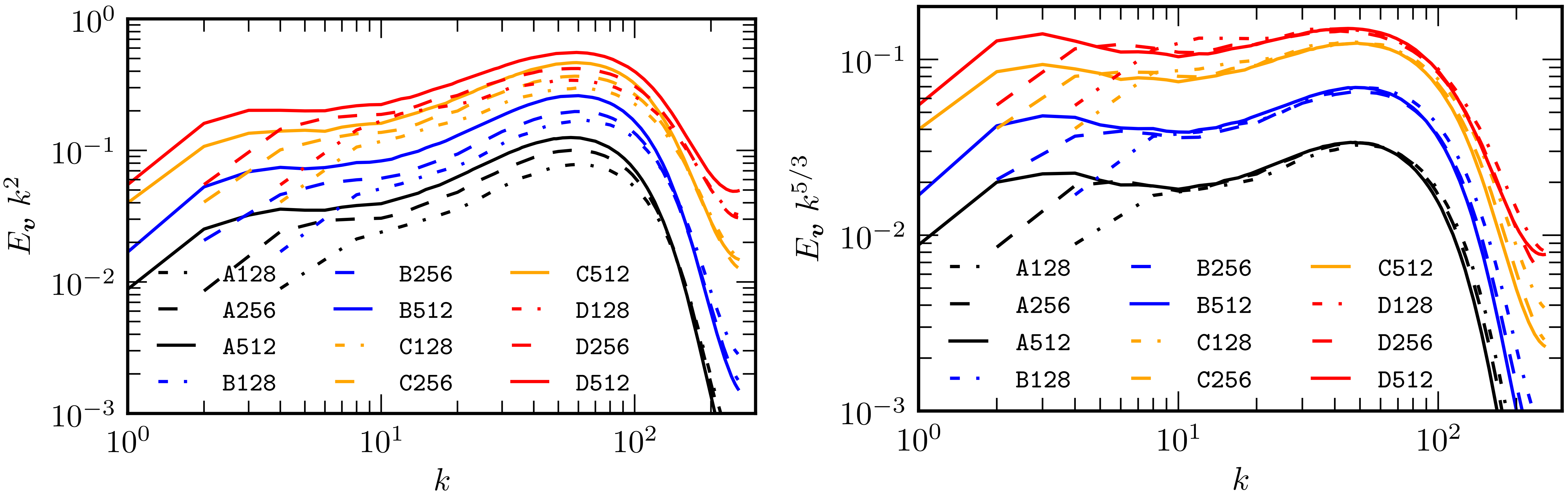


The Meaning of Convergence (II)



From Gotoh et al. 2002

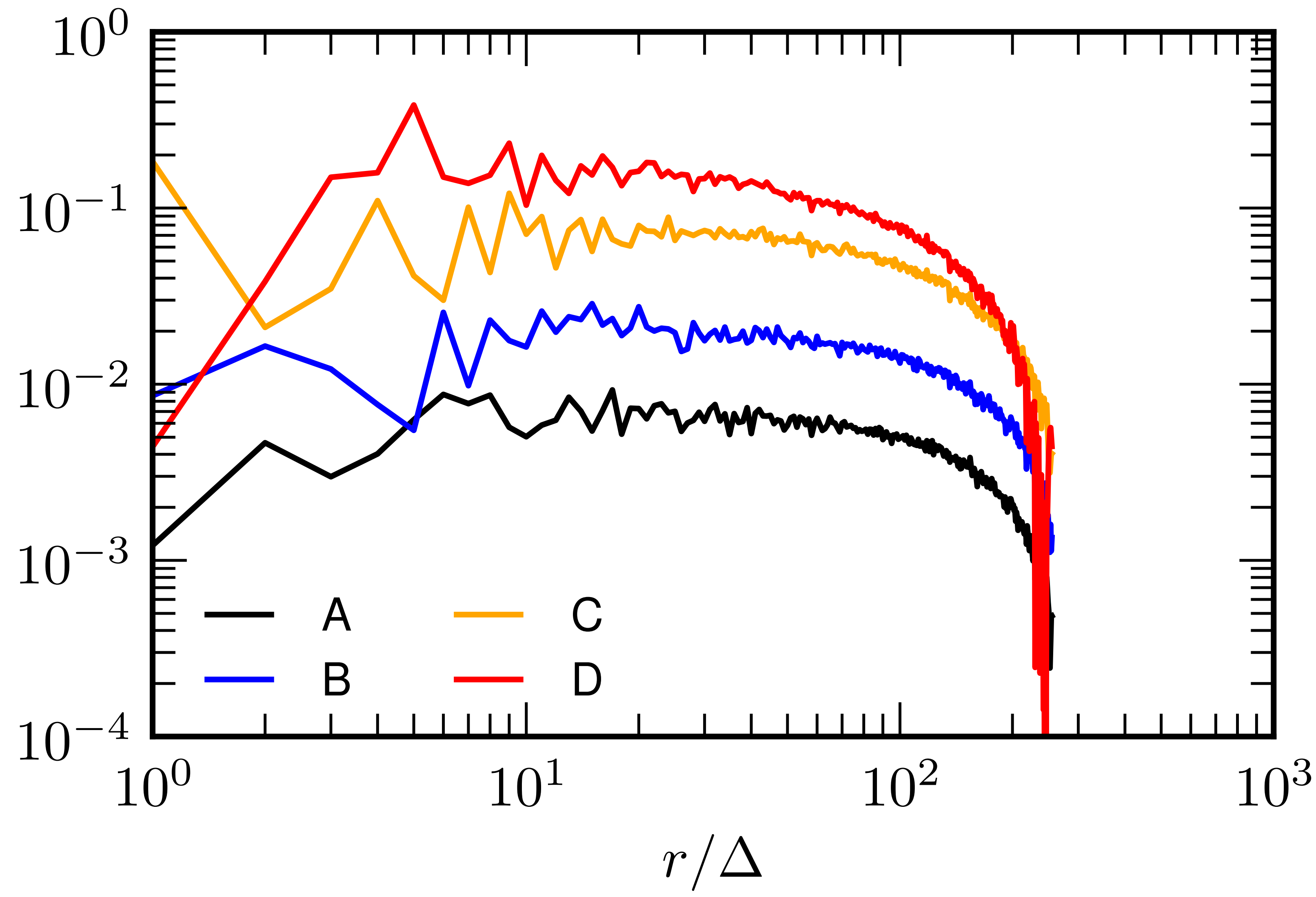
Velocity Spectrum



- The spectra are well converged
- Data consistent with Kolmogorov (-5/3) slope!

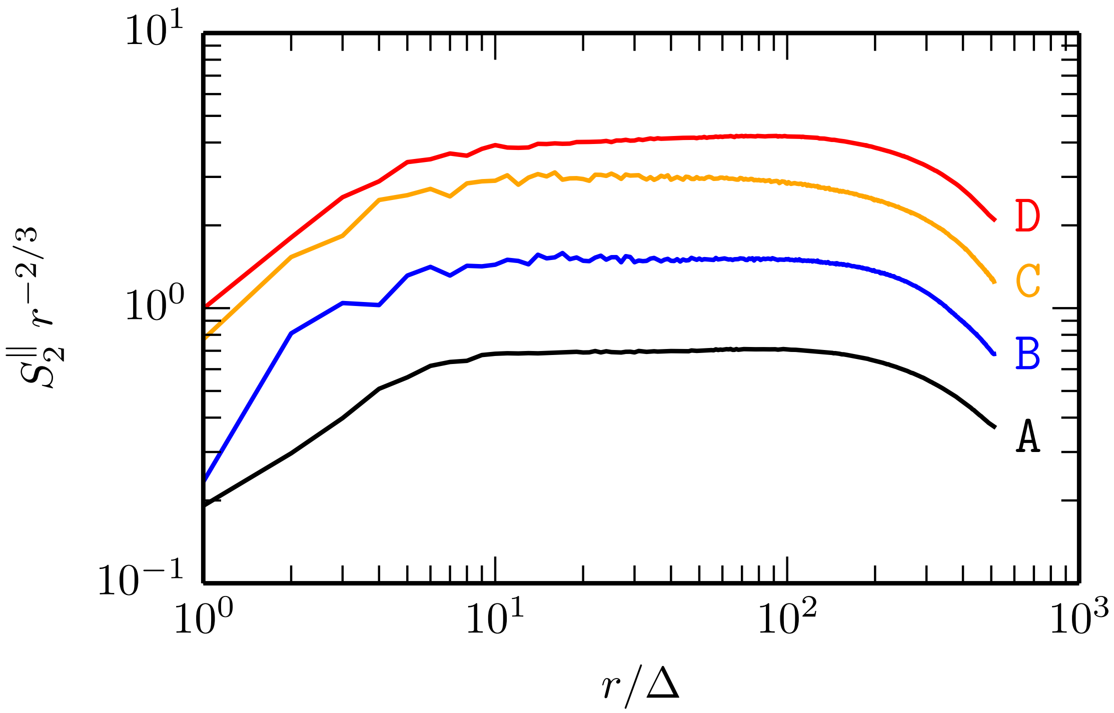
Enhanced Dissipation

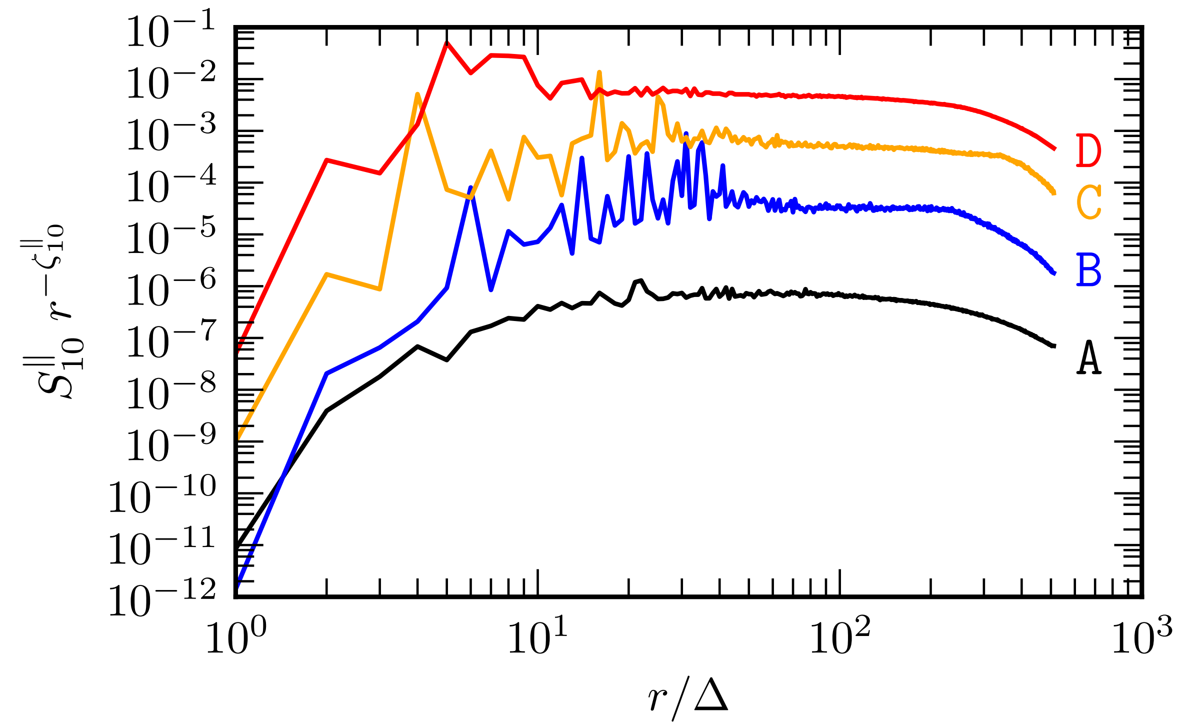
$$\langle \epsilon \rangle \sim \langle (\delta v)^3 \rangle r^{-1}$$

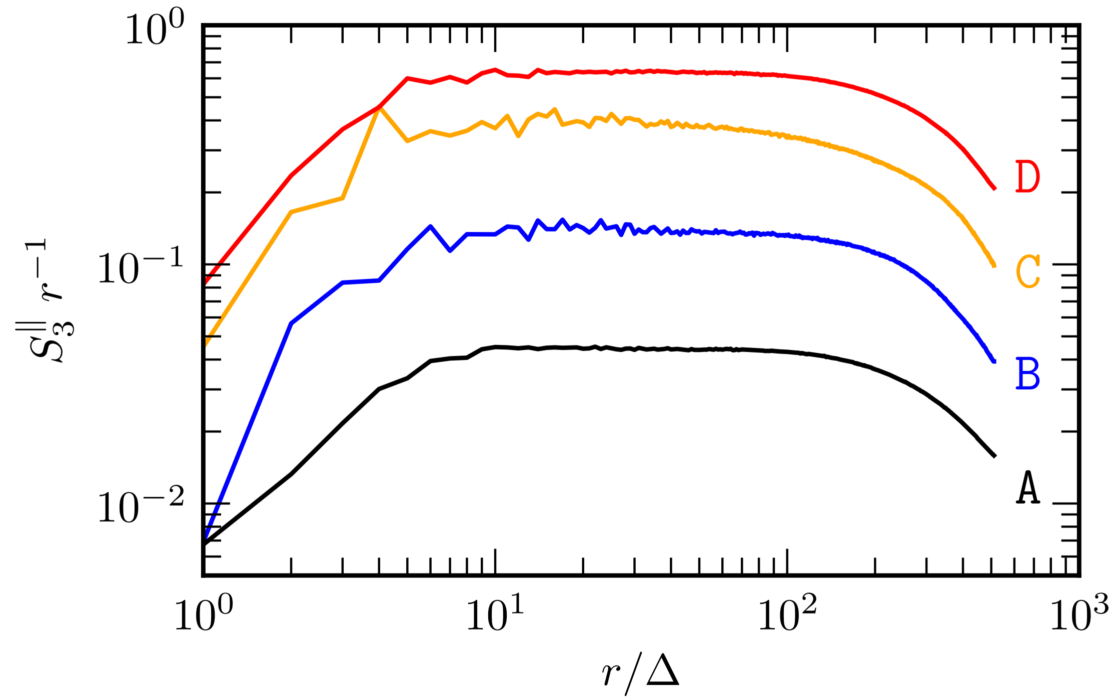


Dissipation increases (roughly) linearly with Lorentz factor

Structure Functions



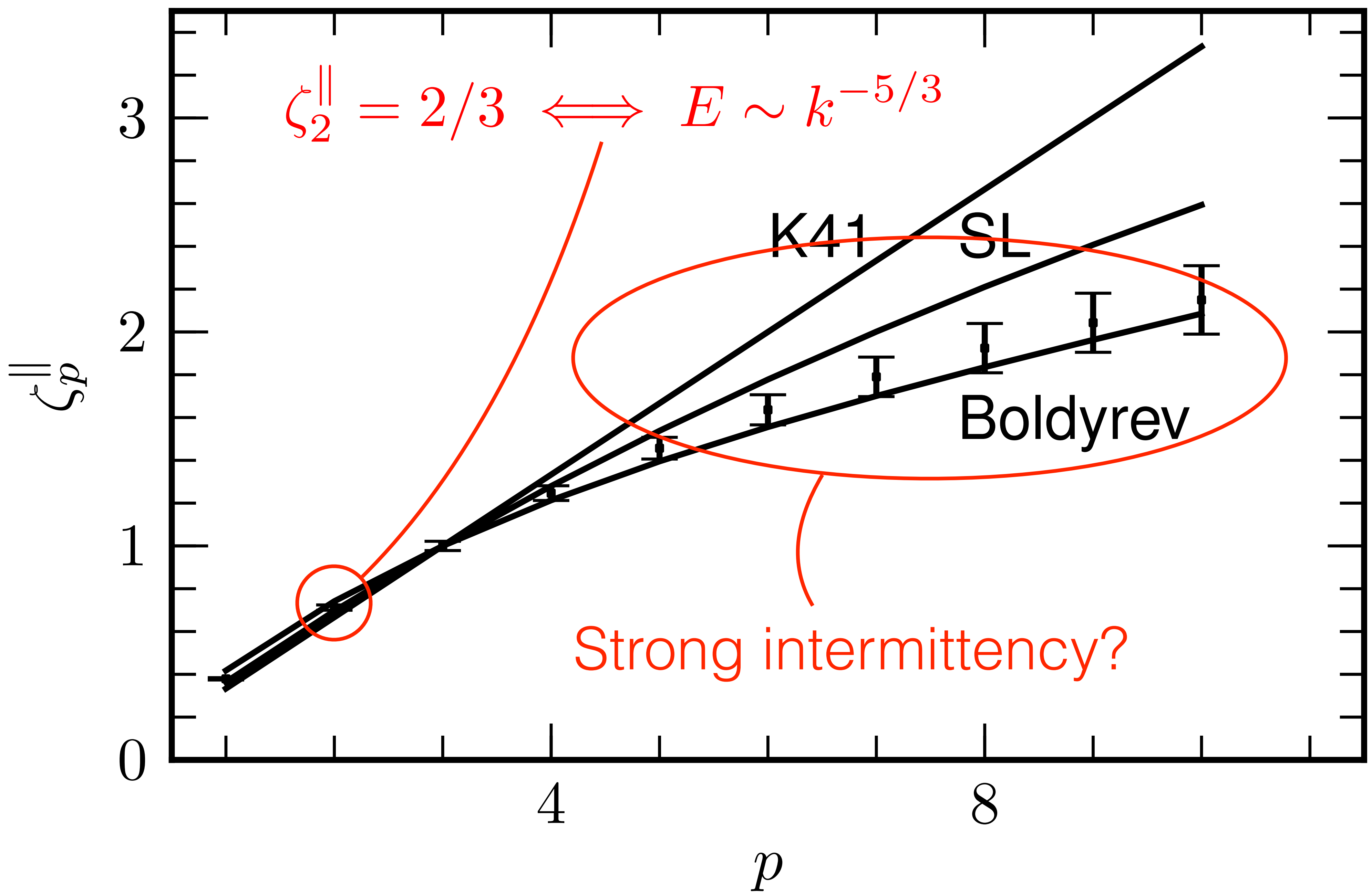




$$S_p^{\|} = \langle |\delta v|^p \rangle \qquad S_p^{\|} \sim r^{\zeta_p^{\|}}$$

- $S_2^{\|}(r)$ is the Fourier transform of $E_{\mathbf{v}}$
- $S_p^{\|}(r) \sim r^{p/3}$ if δv is Gaussian

Structure Exponents



Conclusions

- Kolmogorov’s phenomenology seems applicable*
- Kinetic energy dissipation increases with Lorentz factor
- Intermittency is not understood

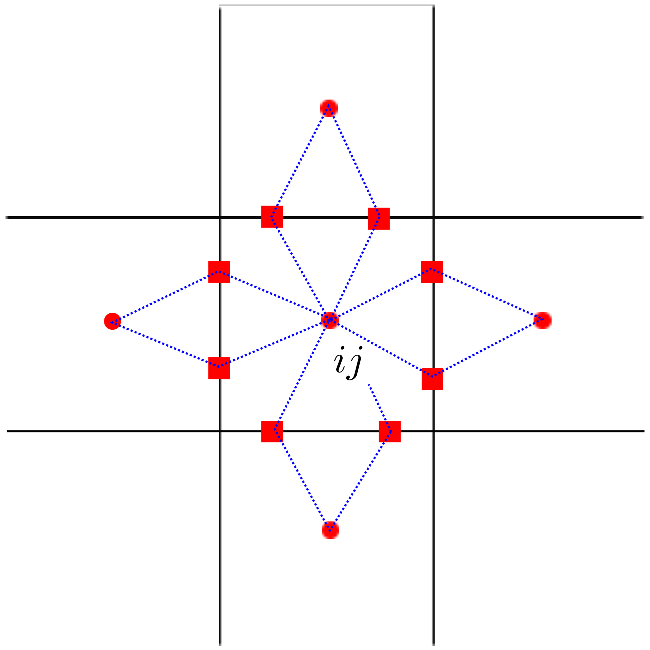
* to low order statistics of the velocity

High-Order HRSC Schemes

Finite Volumes

- Complex to implement
- Large comp. costs
- Conservative
- General grids

$$\frac{\mathrm{d}\langle \mathbf{U} \rangle_{ij}}{\mathrm{d}t} = -\frac{1}{V_{ij}} \int_{\partial V_{ij}} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$$



$$\partial_t \mathbf{u} + \nabla \cdot \mathbf{f}(\mathbf{u}) = 0$$

Finite Differences

- Simple to implement
- Low comp. costs
- Discrete conservation
- Tensor product grids

$$\frac{\mathrm{d}\mathbf{U}_{ij}}{\mathrm{d}t} = -\left[D \cdot \mathbf{F} \right]_{ij}$$