Asymmetric diffusion of Cosmic Rays

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Motivation

transport of Cosmic Rays in Galaxy

and Heliosphere
Regimes of transport: streaming

Transport of particles in magnetic fields:

Regular magnetic field → streaming along field lines

credit: JPL
Regimes of transport: pitch-angle diffusion

Transport of particles in magnetic fields:

- Regular magnetic field → streaming along field lines
- + Random fields/waves → standard diffusion

**pitch-angle diffusion**

\[
\frac{1}{\sin \alpha} \frac{\partial}{\partial \alpha} \left[ D(\alpha) \sin \alpha \frac{\partial f_e}{\partial \alpha} \right] = S_e(\alpha, \nu)
\]
Regimes of transport: trapping

Transport of particles in magnetic fields:

- Regular magnetic field
  → streaming along field lines

- + Random fields/waves
  → standard diffusion

- + Strong fluctuations (\(\delta B \sim B\))
  → diffusion with
    magnetic mirroring & trapping
Diffusion of particles through magnetic traps

Multi-mirror magnetic trap (Budker, Mirnov, Pastukhov, ...)

\[
N \text{ traps}
\]

\[ x^2 \sim D t, \quad D \sim v^2 \tau_i \]

Efficient transport suppression is when:

\[ l \lesssim \frac{v_i \tau_i}{R} \ll L \]

\[ D \sim \frac{v_i^2 \tau_i}{R^2} \]
Regimes of transport: asymmetric trapping

Transport of particles in magnetic fields:

- Regular magnetic field
  - streaming along field lines

- Random fields/waves
  - standard diffusion

- Strong fluctuations \((\delta B \sim B)\)
  - diffusion with
    - magnetic mirroring & trapping

- Field gradient, non-zero \(\langle \mu \cdot \nabla B \rangle\)
  - asymmetric diffusion
Model of turbulence

Processes:
- continuous trapping and de-trapping in magnetic irregularities (traps)
- Particle scattering:
  - perpendicular diffusion
  - pitch-angle scattering (V-diffusion)
  - leaking from traps

Large-scale nonlinear turbulence --- magnetic trapping
Small-scale turbulence --- wave-particle interactions, pitch-angle diffusion
Particle distribution in each trap

- Pitch-angle scattering
- Free streaming
- Leaking out
- Trapped
(1) Trapped population plays no role in particle transport

(2) Population of particles that can escape to the left is that in the loss-cone-1

(3) Population of particles that can escape to the right is the sum of the loss-cone-2 population and the leaking population

The left-escaping population is proportional to the loss-cone-1 volume

\[ \propto \left( \frac{2}{\pi/3} \right) [1 - \cos(\theta_1)] = 1 - \left( \frac{\Delta B_1}{B_1} \right)^{1/2} \sim \frac{1}{R_1} \]

\[ \Delta B_1 = B_1 - B_0 \]

The right-escaping population is proportional to the loss-cone-2 +leaking volume

\[ \propto \left( \frac{2}{\pi/3} \right) [1 - \cos(\theta_2) + (\cos(\theta_1) - \cos(\theta_2))] = 1 + \left( \frac{\Delta B_1}{B_1} \right)^{1/2} - 2 \left( \frac{\Delta B_2}{B_2} \right)^{1/2} \sim \frac{2}{R_2} - \frac{1}{R_1} \]
Particle escape rates

Particle escape rate is proportional to the collision frequency and the ratio of the loss-cone volumes.

Thus, probabilities (per pitch-angle scattering event) to escape to the left and to the right:

\[ r = 1/2 - \epsilon, \quad g = 1/2 + \epsilon \]

\[ \epsilon = \frac{\delta B}{4B_2} \frac{B_0}{(B_2 \Delta B_2)^{1/2}} \left[ 1 - \left( \frac{\Delta B_2}{B_2} \right)^{1/2} \right] \]

\[ \delta B = B_1 - B_2 = \lambda |\nabla B| \]

Characteristic scale of the magnetic mirrors, i.e., the scale of the outer range of turbulence.
Markov chain toy model (Helio)

\[ \lambda - \text{mean-free-path} \]

\[ J_{in} \] (particle in-flux)

Heliosphere

Sun \quad 1 \quad 2 \quad \ldots -m \quad N/2 \quad \ldots +1 \quad \ldots +m \quad N-1 \quad N \quad ISM

system size
Markov chain toy model (Galaxy)

Galactic mid-plane

$\lambda$ – mean-free-path

System size

$\frac{1}{2}$ Milky Way

$J_{in}$ (particle in-flux)

Galactic mid-plane

1 2 

..-m N/2 ..+1 ..+m N-1 N

$r_-$ $r_-$ $r_-$ $r_-$ $r_+$ $r_+$ $r_+$ $r_+$ $r_+$

$g_-$ $g_-$ $g_-$ $g_-$ $g_+$ $g_+$ $g_+$ $g_+$ $g_+$

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Markov chain

Rules

\[ \begin{align*}
    d_t n_1 &= n_2 r_2 - n_1 (g_1 + r_1) \\
    d_t n_j &= n_{j-1} g_{j-1} + n_{j+1} r_{j+1} - n_j (g_j + r_j) \\
    d_t n_N &= J_{in} + n_{N-1} g_{N-1} - n_N (g_N + r_N)
\end{align*} \]

Left margin

Internal site \( (j\text{-th}) \)

Right margin

Steady state

\[ d_t n_j = 0 \]

Generating function

\[ F(\xi) \equiv \sum_{j=0}^{N} \xi^j n_j = \frac{\xi(\xi^N J_{in}/r - \xi^{N+1} \alpha n_N - n_1)}{[(1 + \alpha)\xi - 1 - \alpha \xi^2]} \]

Solution

Particle density is found everywhere: \( n_k, ~ k = 1 \ldots N \)

\[ n_k = \frac{1}{k!} \left. \frac{d^k F(\xi)}{d\xi^k} \right|_{\xi=0} \]

where \( \alpha = g/r \)
Solution (CR density)

\[ n_k = (1 + \alpha + \alpha^2 + \cdots + \alpha^{k-1}) n_1 = \frac{\alpha^k - 1}{\alpha - 1} n_1, \quad 1 \leq k \leq N \]

\[ n_{N+1} = -J_{in} + \frac{\alpha^{N+1} - 1}{\alpha - 1} n_1 \equiv 0 \]

where \( \alpha = \frac{g}{r} = \frac{p_{right}}{p_{left}} \)
Bonus slide: Continuous limit

\[ n_{j-1}g_{j-1} \rightarrow n(x)g(x) - \frac{d(ng)}{dx} + \frac{1}{2!} \frac{d^2(ng)}{dx^2} - \frac{1}{3!} \frac{d^3(ng)}{dx^3} + \ldots \]

Markov equation

\[ \frac{dn_j}{dt} = n_{j-1}g_{j-1} + n_{j+1}r_{j+1} - n_j(g_j + r_j) \]

becomes...

\[ \frac{\partial n}{\partial t} = (r - g) \frac{\partial n}{\partial x} + \frac{1}{2} (r + g) \frac{\partial^2 n}{\partial x^2} + \frac{1}{3!} (r - g) \frac{\partial^3 n}{\partial x^3} + \ldots \]

convection-diffusion equation
Conclusions

Diffusion of particles in high-amplitude MHD turbulence with B-field gradient is asymmetric.

It results in non-zero flux through the system due to the $\langle \mu \cdot \nabla B \rangle$-force.

Propagation equation is *convection-diffusion* equation only in the second order expansion.