Asymmetric diffusion of Cosmic Rays

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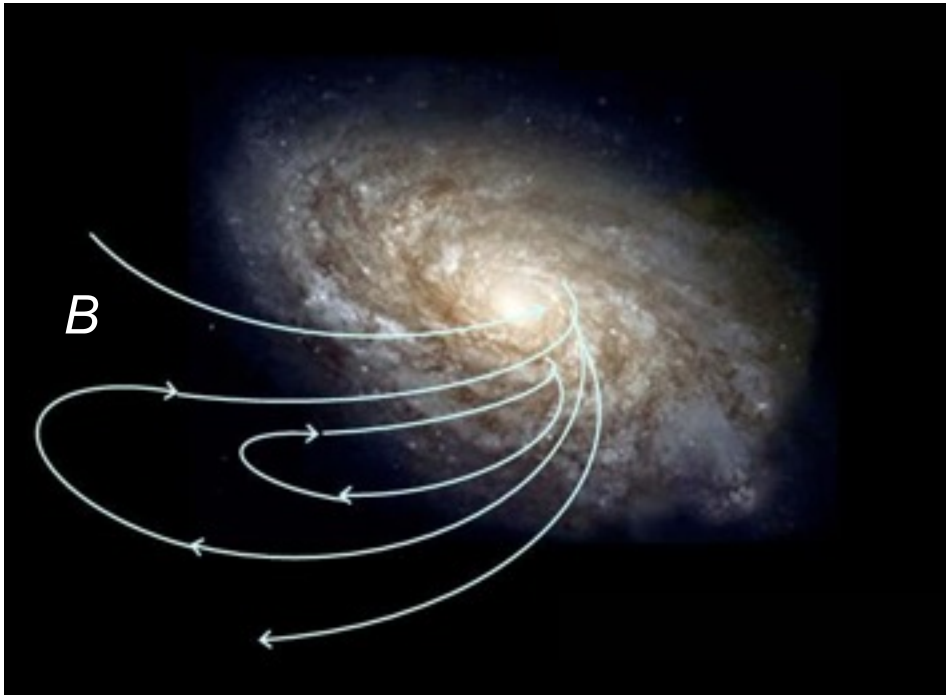
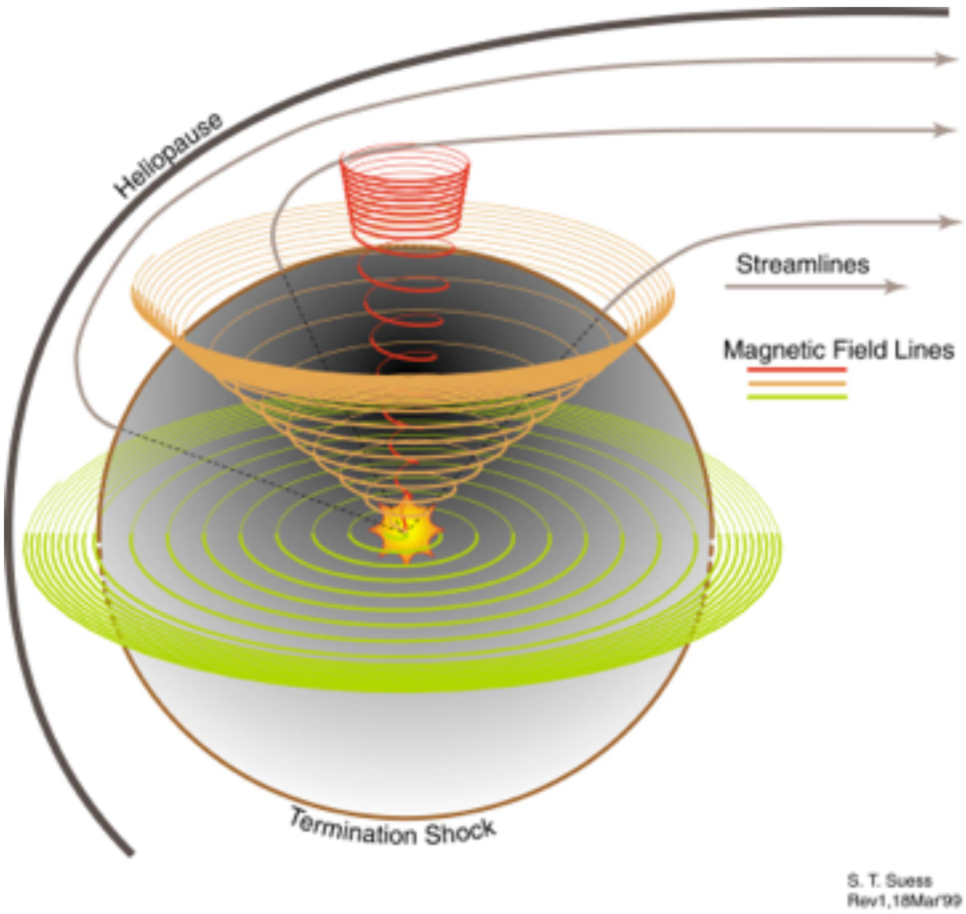
Relativistic Plasma Astrophysics workshop — Purdue U. — May 12, 2014

Motivation

transport of Cosmic Rays in Galaxy



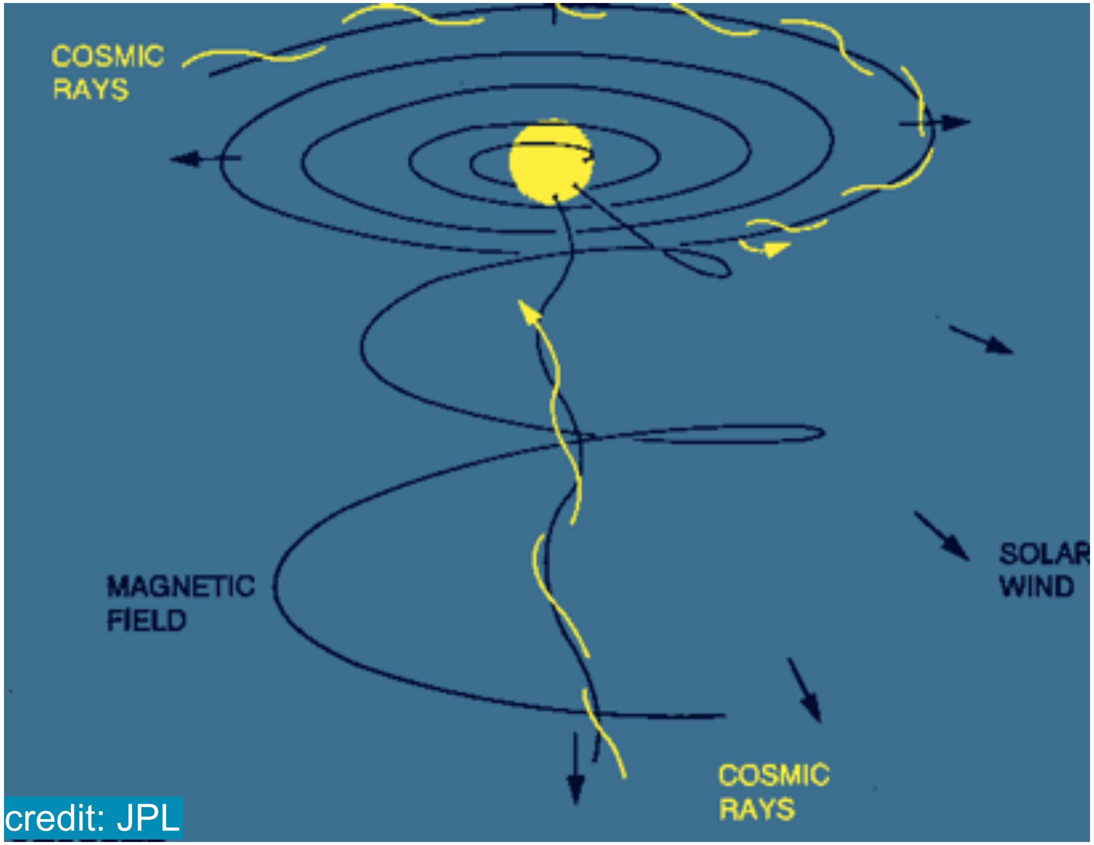


and Heliosphere

Regimes of transport: streaming

Transport of particles in magnetic fields:

Regular magnetic field
→ streaming along field lines



Regimes of transport: pitch-angle diffusion

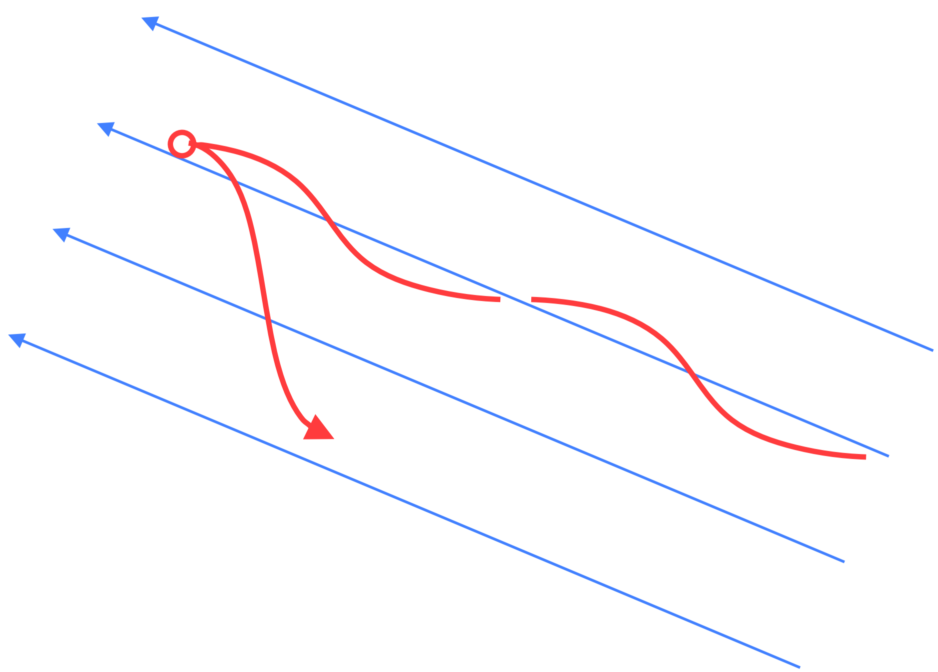
Transport of particles in magnetic fields:

Regular magnetic field
→ streaming along field lines

+ Random fields/waves
→ standard diffusion

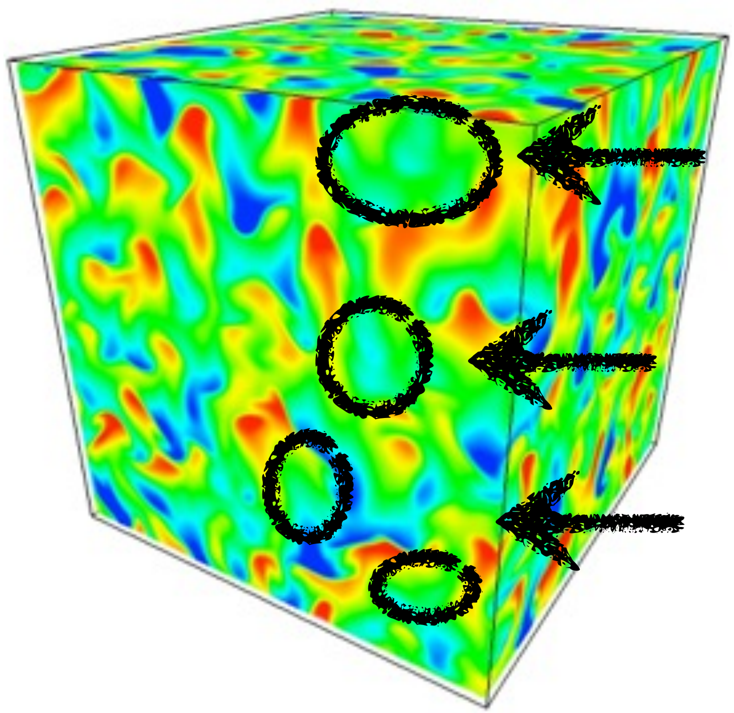
pitch-angle diffusion

$$\frac{1}{\sin\alpha}\frac{\partial}{\partial\alpha}\left[D(\alpha)\sin\alpha\frac{\partial f_e}{\partial\alpha}\right] = S_e(\alpha, v)$$



Regimes of transport: trapping

Transport of particles in magnetic fields:



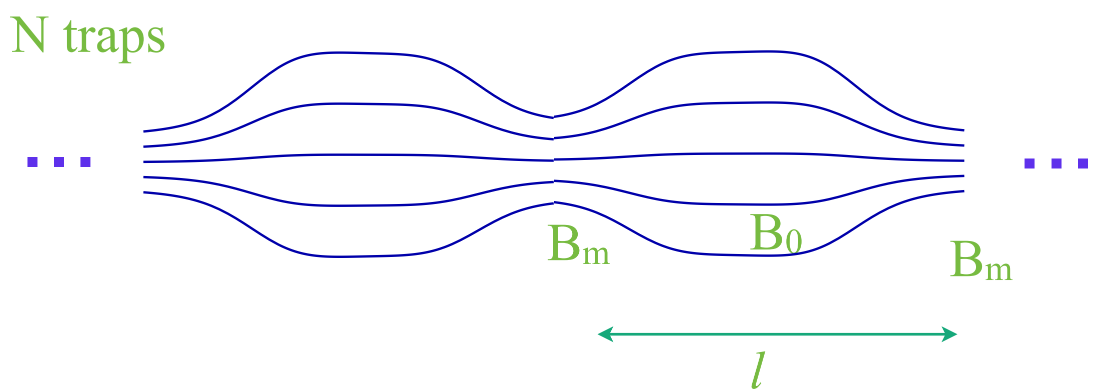
Regular magnetic field
→ streaming along field lines

+ Random fields/waves
→ standard diffusion

+ Strong fluctuations ($\delta B \sim B$)
→ diffusion with
magnetic mirroring & trapping

Diffusion of particles through magnetic traps

Multi-mirror magnetic trap (Budker, Mirnov, Pastukhov,...)



$$R = \frac{B_m}{B_0}$$

$$\boxed{x^2 \sim Dt,} \quad D \sim v^2 \tau_i$$

Efficient transport suppression is when: $l \lesssim \frac{v_i \tau_i}{R} \ll L$

$$D \sim \frac{v_i^2 \tau_i}{R^2}$$

Regimes of transport: asymmetric trapping

Transport of particles in magnetic fields:

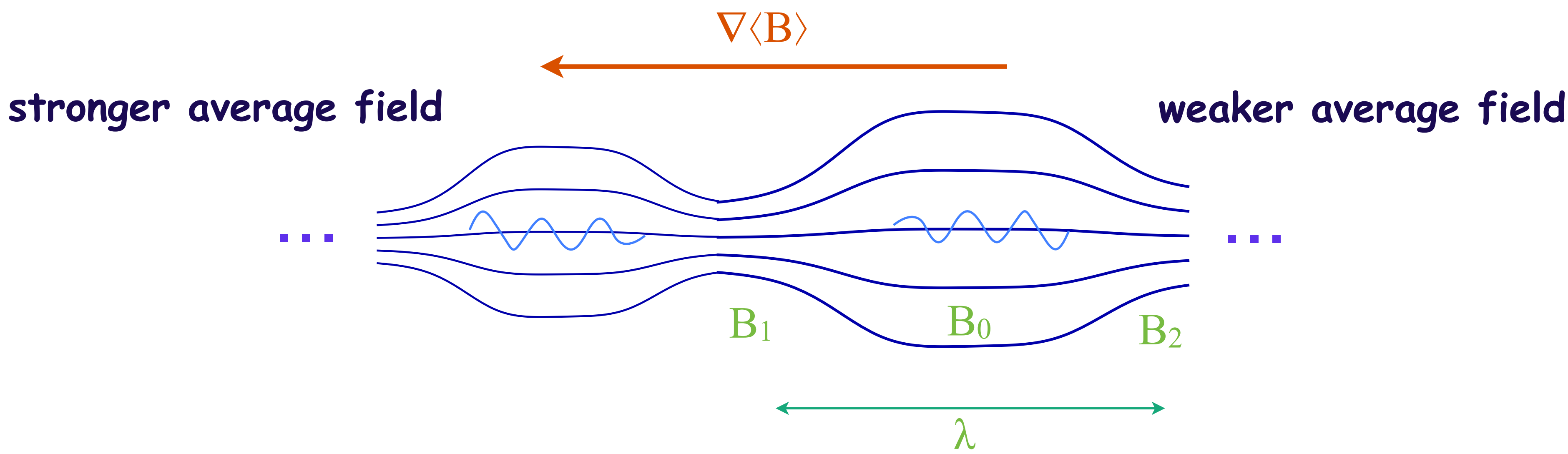
Regular magnetic field
→ streaming along field lines

+ Random fields/waves
→ standard diffusion

+ Strong fluctuations ($\delta B \sim B$)
→ diffusion with magnetic mirroring & trapping

+ Field gradient, non-zero $\langle \mu \cdot \nabla B \rangle$
→ asymmetric diffusion

Model of turbulence

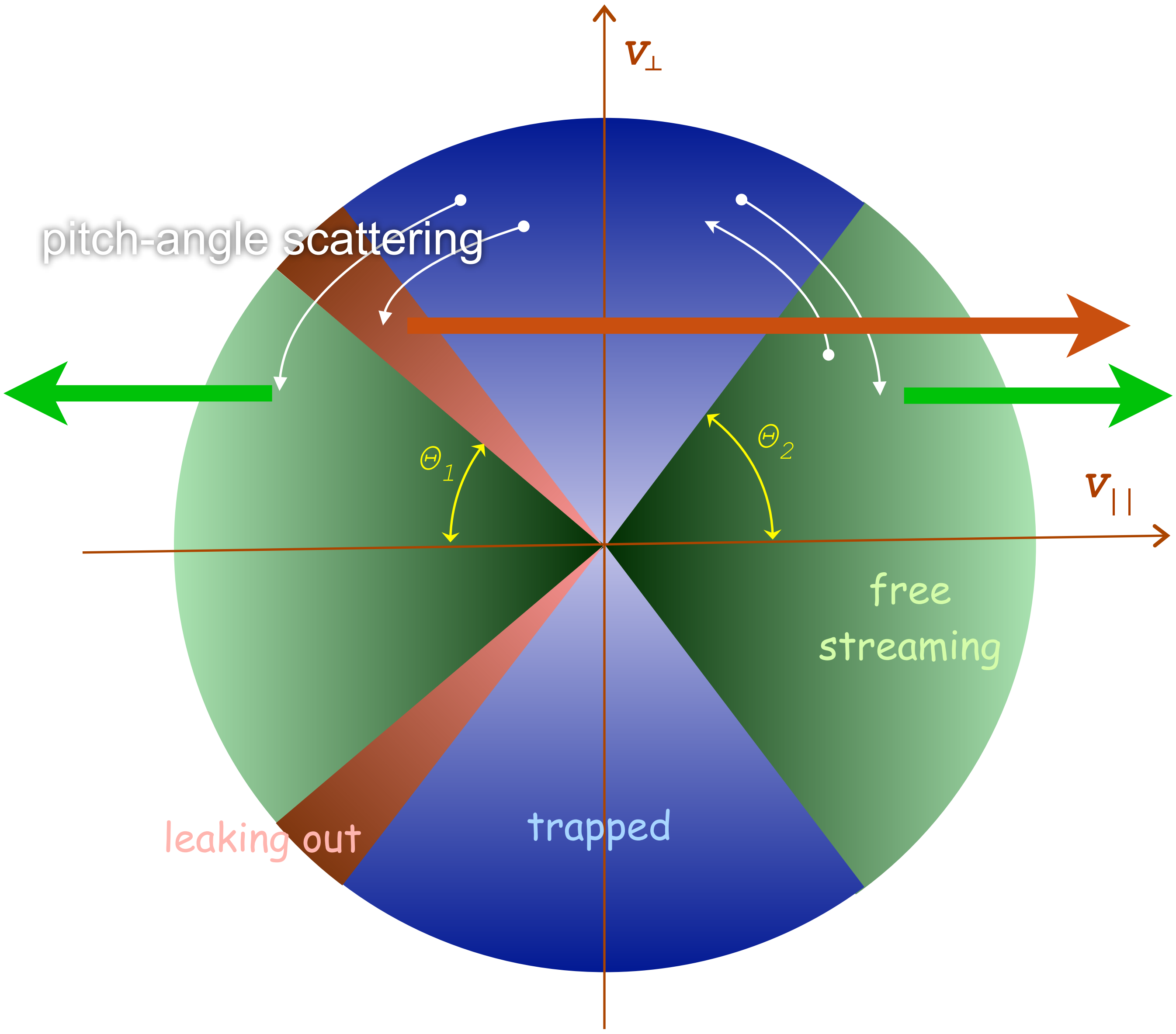


Large-scale nonlinear turbulence --- *magnetic trapping*
Small-scale turbulence --- *wave-particle interactions, pitch-angle diffusion*

Processes: → continuous trapping and de-trapping in magnetic irregularities (traps)

→ Particle scattering:
+ perpendicular diffusion
+ pitch-angle scattering (V-diffusion)
+ leaking from traps

Particle distribution in each trap



Particle populations in each trap

(1) Trapped population plays no role in particle transport

(2) Population of particles that can escape to the left is that in the loss-cone-1

(3) Population of particles that can escape to the right is the *sum* of the loss-cone-2 population and the leaking population

The left-escaping population is proportional to the loss-cone-1 volume

$$\propto \left(\frac{2}{\pi/3}\right)[1-\cos(\theta_1)] = 1 - \left(\frac{\Delta B_1}{B_1}\right)^{1/2} \sim \frac{1}{R_1} \qquad \Delta B_1 = B_1 - B_0$$

The right-escaping population is proportional to the loss-cone-2 +leaking volume

$$\propto \left(\frac{2}{\pi/3}\right)[1-\cos(\theta_2) + (\cos(\theta_1) - \cos(\theta_2))] = 1 + \left(\frac{\Delta B_1}{B_1}\right)^{1/2} - 2\left(\frac{\Delta B_2}{B_2}\right)^{1/2} \sim \frac{2}{R_2} - \frac{1}{R_1}$$

Particle escape rates

Particle escape rate is proportional to the collision frequency and the ratio of the loss-cone volumes

Thus, probabilities (per pitch-angle scattering event) to escape to the left and to the right:

$$r = 1/2 - \epsilon, \quad g = 1/2 + \epsilon$$

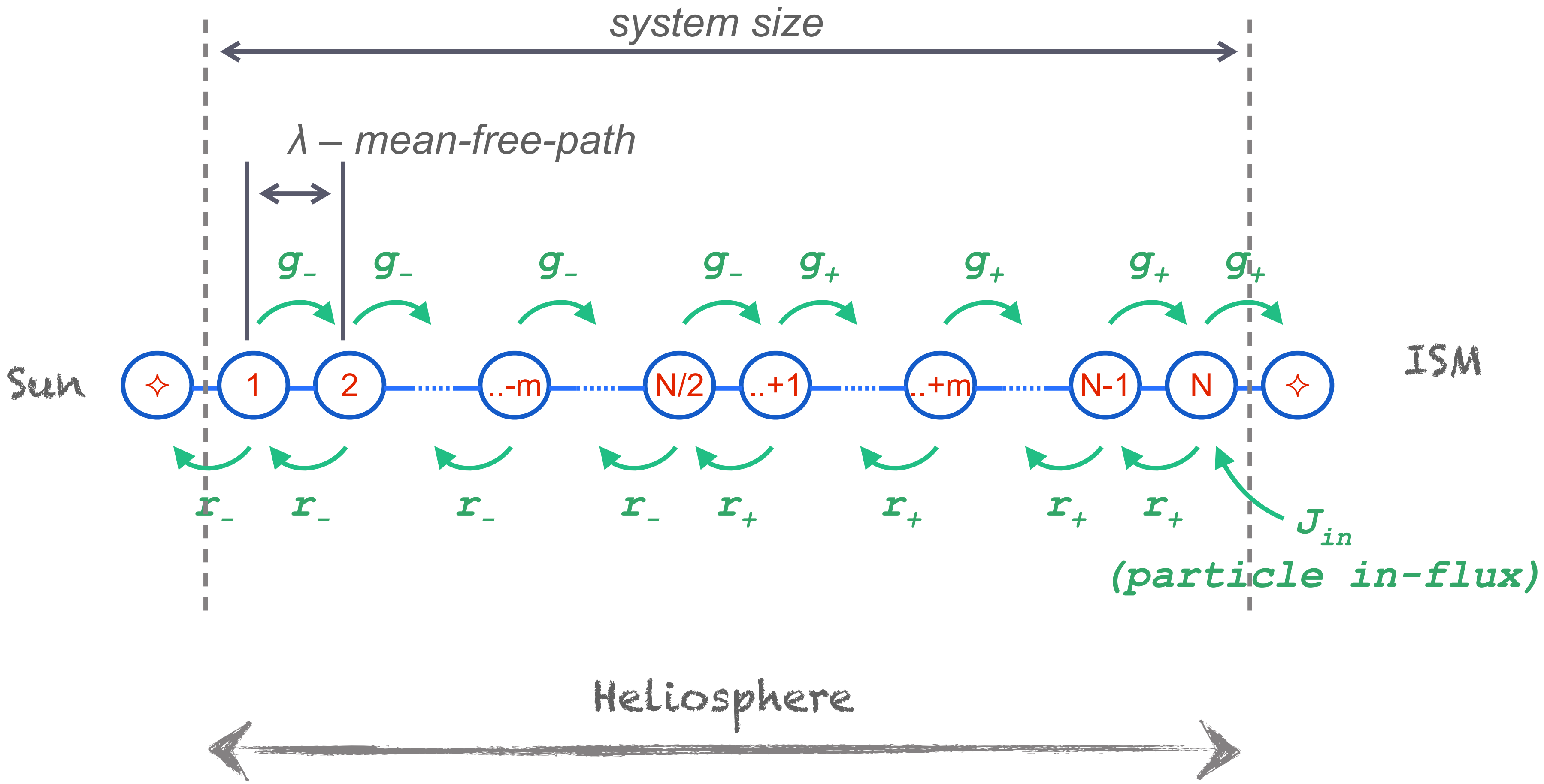
$$\epsilon = \frac{\delta B}{4B_2} \frac{B_0}{(B_2\,\Delta B_2)^{1/2}} \left[1 - \left(\frac{\Delta B_2}{B_2}\right)^{1/2}\right]$$

$$\delta B = B_1 - B_2 = \lambda |\nabla B|$$

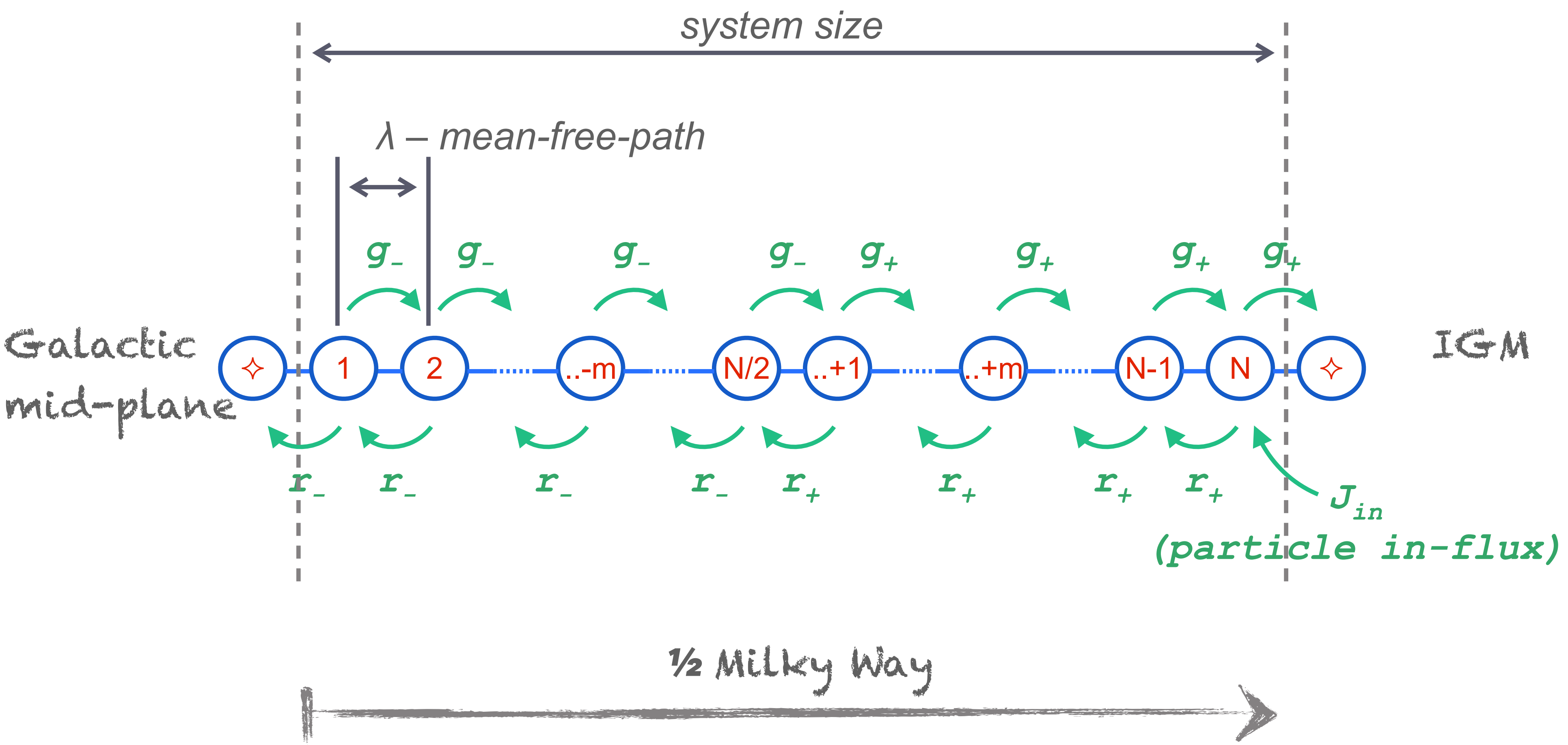
Characteristic scale of the magnetic mirrors, i.e., the scale of the outer range of turbulence

$$\epsilon \sim \frac{1}{8R^2}\,\lambda\,\frac{|\nabla B_0|}{B_0}$$

Markov chain toy model (Helio)



Markov chain toy model (Galaxy)



Markov chain

Rules

$d_t\ n_1 = n_2 r_2 - n_1(g_1+r_1)$ ⬅ Left margin

$d_t\ n_j = n_{j-1} g_{j-1} + n_{j+1} r_{j+1} - n_j(g_j+r_j)$ ⬅ Internal site (*j*-th)

$d_t\ n_N = J_{in} + n_{N-1} g_{N-1} - n_N(g_N+r_N)$ ⬅ Right margin

Steady state $d_t\ n_j = 0$

Generating function

$$F(\xi) \equiv \sum_{j=0}^{N} \xi^j n_j = \frac{\xi(\xi^N J_{\text{in}}/r - \xi^{N+1}\alpha n_N - n_1)}{[(1+\alpha)\xi - 1 - \alpha\xi^2]}$$

Solution

Particle density is found everywhere: $n_k,\ k = 1 \ldots N$

$$n_k = \frac{1}{k!} \left. \frac{d^k F(\xi)}{d\xi^k} \right|_{\xi=0}$$

where $\alpha = g/r$

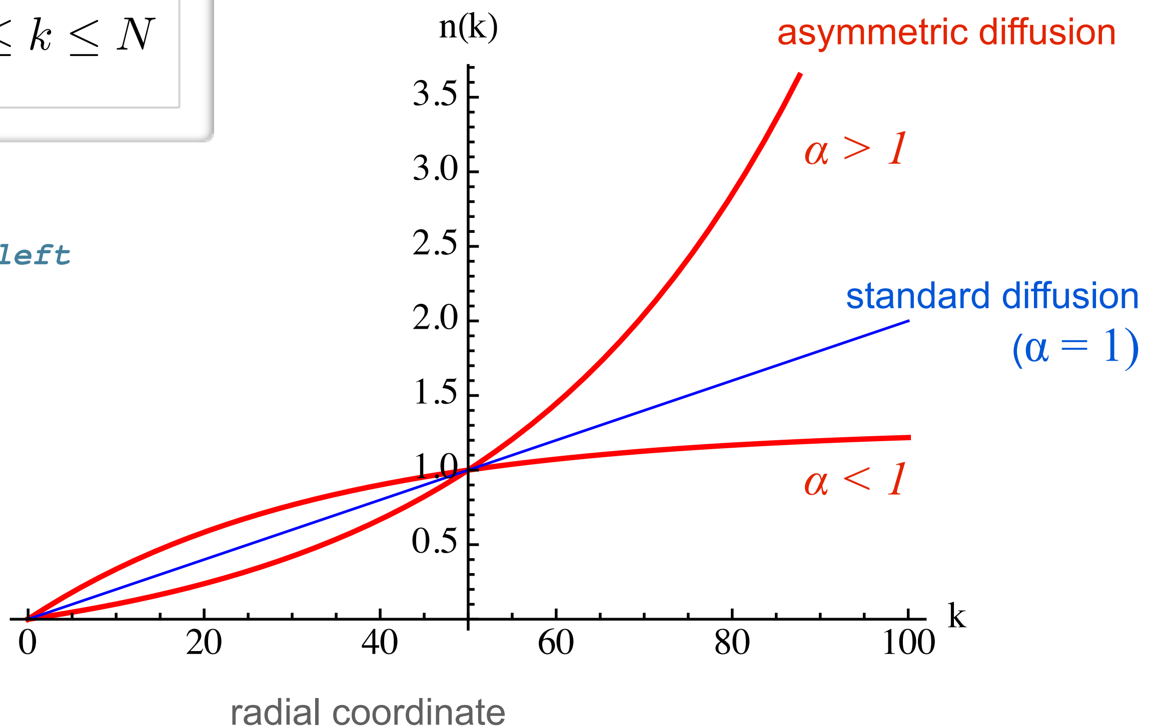
Solution (CR density)

$$n_k = (1 + \alpha + \alpha^2 + \cdots + \alpha^{k-1})\, n_1 = \frac{\alpha^k - 1}{\alpha - 1}\, n_1\,, \quad 1 \le k \le N$$

$$n_{N+1} = -J_{in} + \frac{\alpha^{N+1} - 1}{\alpha - 1}\, n_1 \equiv 0$$

$$n_k = J_{in}\left(\frac{\alpha^k - 1}{\alpha^{N+1} - 1}\right), \quad 1 \le k \le N$$

where $\alpha = g/r = p_{right}/p_{left}$



Bonus slide: Continuous limit

$$n_{j-1}g_{j-1} \to n(x)g(x) - \frac{d(ng)}{dx} + \frac{1}{2!}\frac{d^2(ng)}{dx^2} - \frac{1}{3!}\frac{d^3(ng)}{dx^3} + \dots$$

Markov equation

$$\frac{dn_j}{dt} = n_{j-1}g_{j-1} + n_{j+1}r_{j+1} - n_j(g_j + r_j)$$

becomes...

$$\frac{\partial n}{\partial t} = (r-g)\frac{\partial n}{\partial x} + \frac{(r+g)}{2}\frac{\partial^2 n}{\partial x^2} + \frac{(r-g)}{3!}\frac{\partial^3 n}{\partial x^3} + \dots$$

convection-diffusion equation

Conclusions

Diffusion of particles in high-amplitude MHD turbulence with B-field gradient is *asymmetric*

It results in non-zero flux through the system due to the $\langle \mu \quad \nabla B \rangle$-force

Propagation equation is *convection-diffusion* equation only in the second order expansion