A Few Words About AGNs and PSRs

V.S. Beskin

P.N.Lebedev Physical Institute

Moscow Institute of Physics and Technology

with

A.A.Philippov

Berkeley

V.I.Krauz

Kurchatov Institute

Ya.N.Istomin

P.N.Lebedev Physical Institute

A.V.Chernoglazov, A.Galishnikova, E.E. Nokhrina, D.Novoselov, M.Rashkovetskiy

Moscow Institute of Physics and Technology

Supported by

Russian Science Foundation (Grant No. 16-12-10051)

Astrophysical jets: from observations to theory and laboratory experiment

Government of the Russian Federation

(agreement No. 05.Y09.21.0018)

Laboratory of Fundamental and Applied Research of Relativistic Objects of the Universe in MIPT

Plan

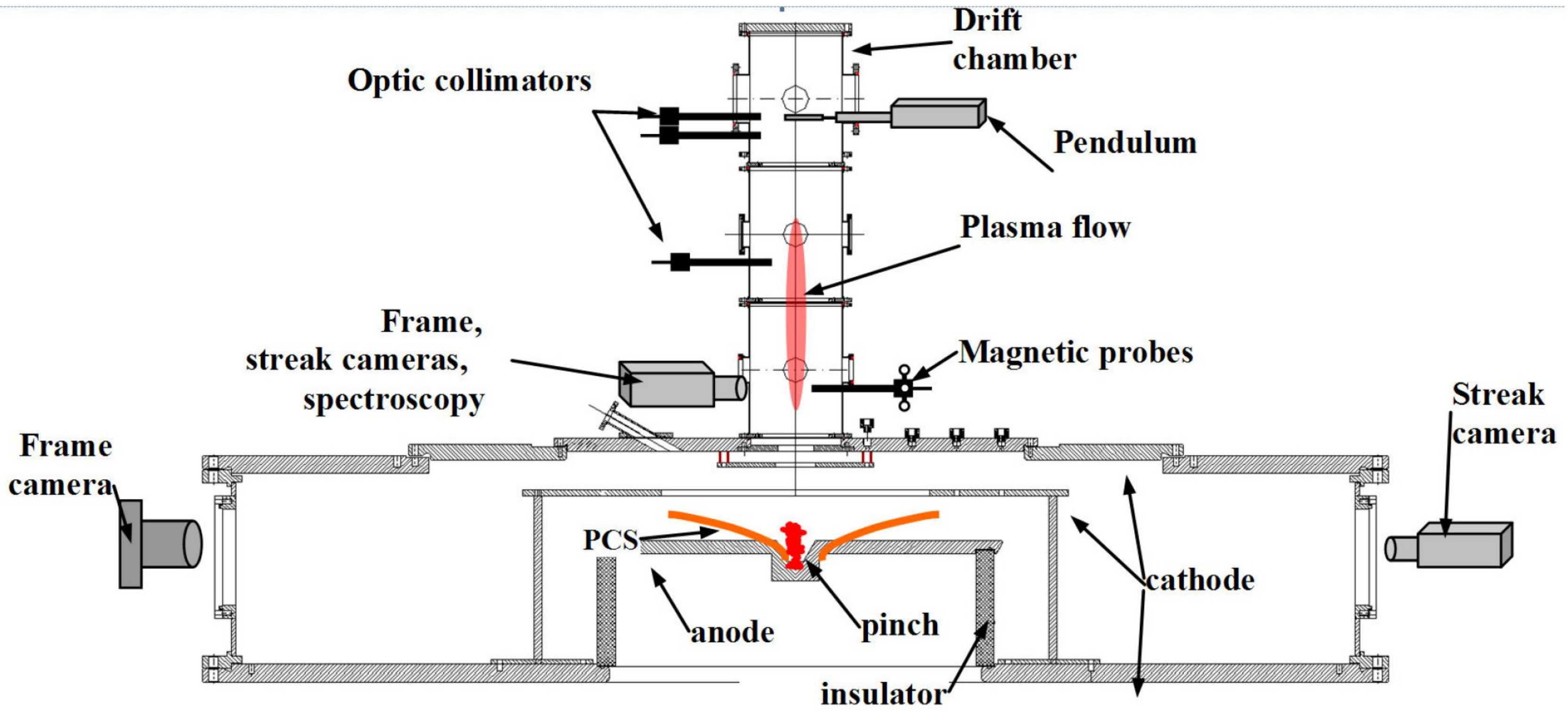
- AGNs (information only)
- PSRs (in more detail)

AGNs

- Laboratory experiment on plasma focus facility
- One (rather technical) theoretical result +

Laboratory experiment

Plasma focus facility in KI



Laboratory experiment

Plasma focus facility in KI

Laboratory experiment

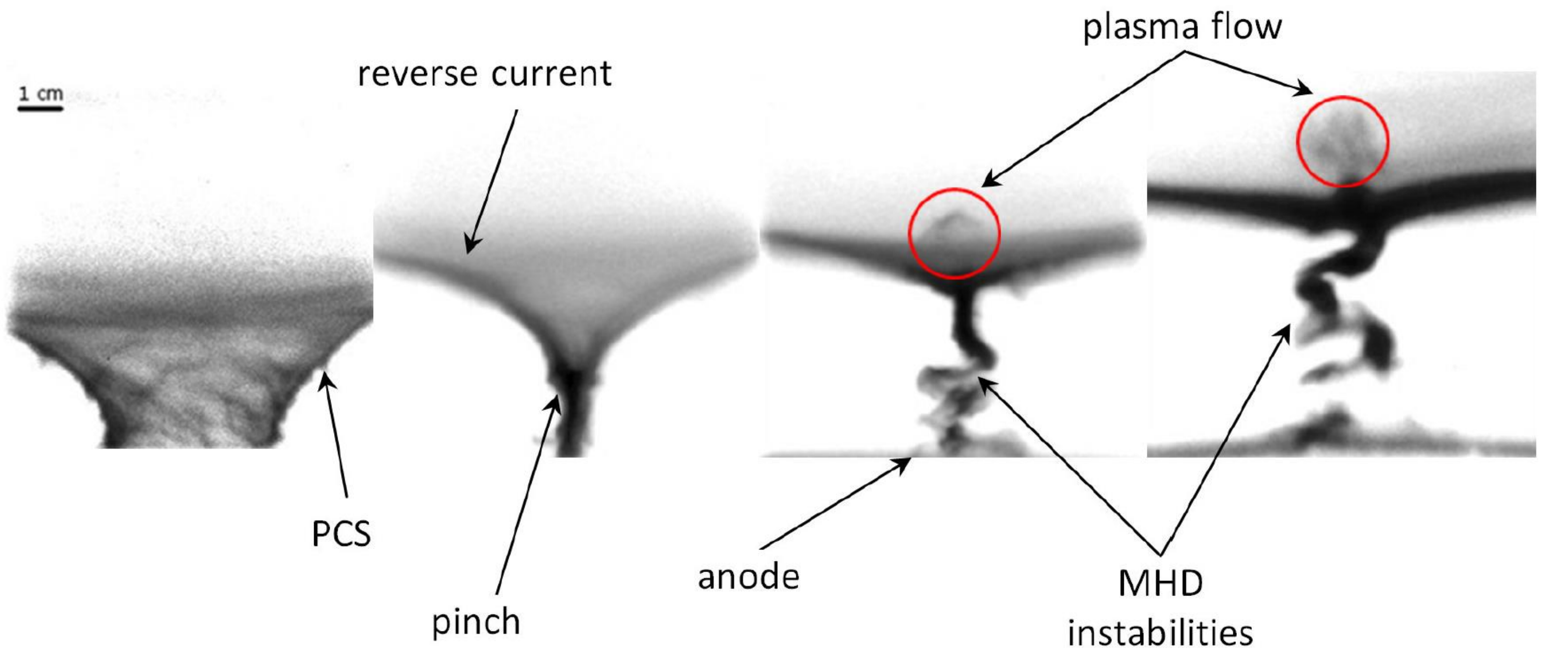
Plasma focus facility in KI

Table 1. Key dimensionless parameters

	YSO		PF-3 (35 cm above the anode)
Peclet	10^{11}	> 1, convective heat transfer	$> 10^7$
Reynolds	10^{13}	$\gg 1$, the viscosity is important	$10^4 - 10^5$
Magnetic Reynolds	10^{15}	> 1, magnetic field is frozen	~ 100
Mach (V_{jet}/V_{cs})	$10 - 50$	> 1, the jet is supersonic	> 10 (for Ne and Ar)
β (P_{pl}/P_{magn})	$\gg 1$ near source $\ll 1$ at 10 AU		~ 0.35 (for Ne and Ar)
density contrast (n_{jet}/n_{amb})	> 1		$1 - 10$

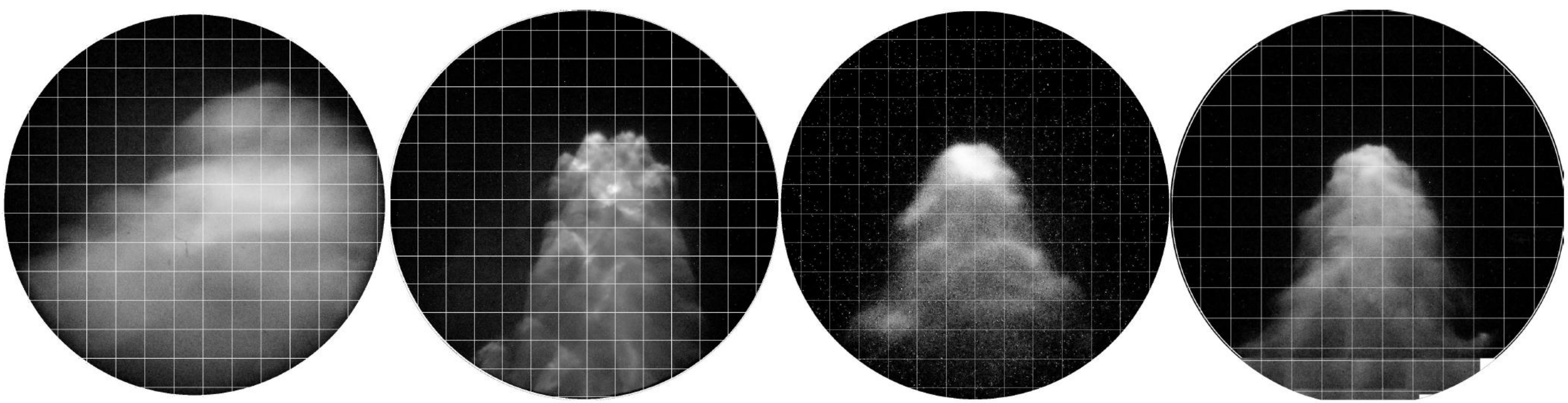
Laboratory experiment

Plasma focus facility – the very beginning



Laboratory experiment

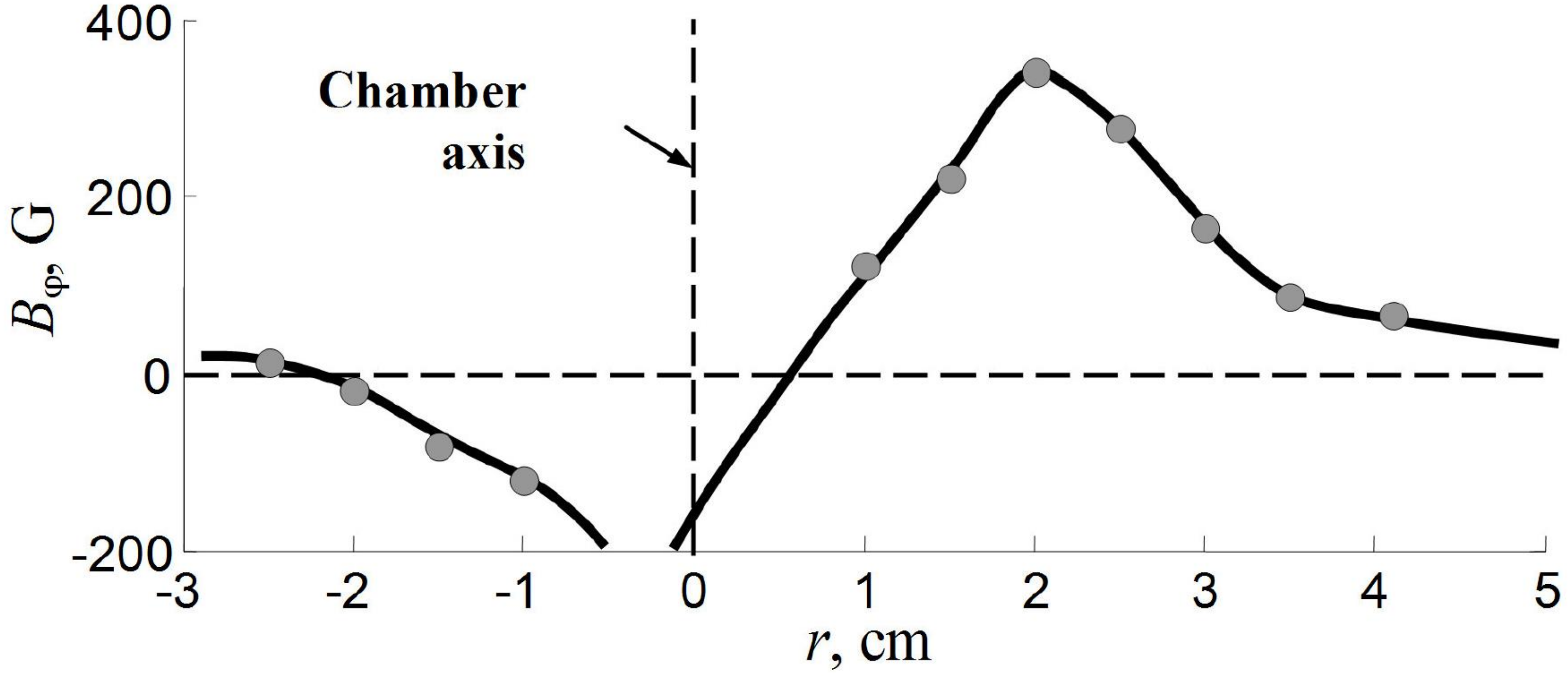
Plasma focus facility – rather stable shape, interaction with ambient gas



H_2 35 cm   Ne 35 cm   Ne 65 cm   Ar 95 cm

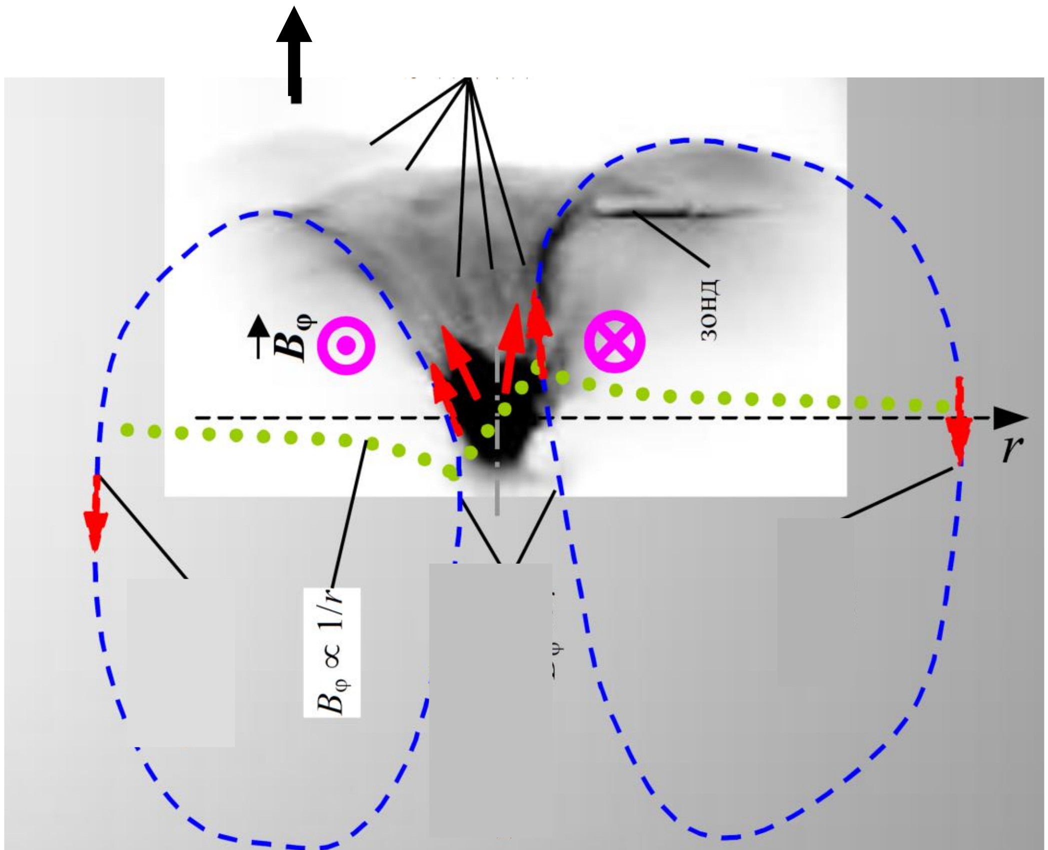
Laboratory experiment

Plasma focus facility – toroidal magnetic field



Laboratory experiment

Plasma focus facility – spheromak?



$$W_{\rm tot} \approx E_{\rm n}\eta_{\rm n}\Psi_0 \approx \Omega^{4/3}\dot{M}^{1/3}\Psi_0^{4/3}$$

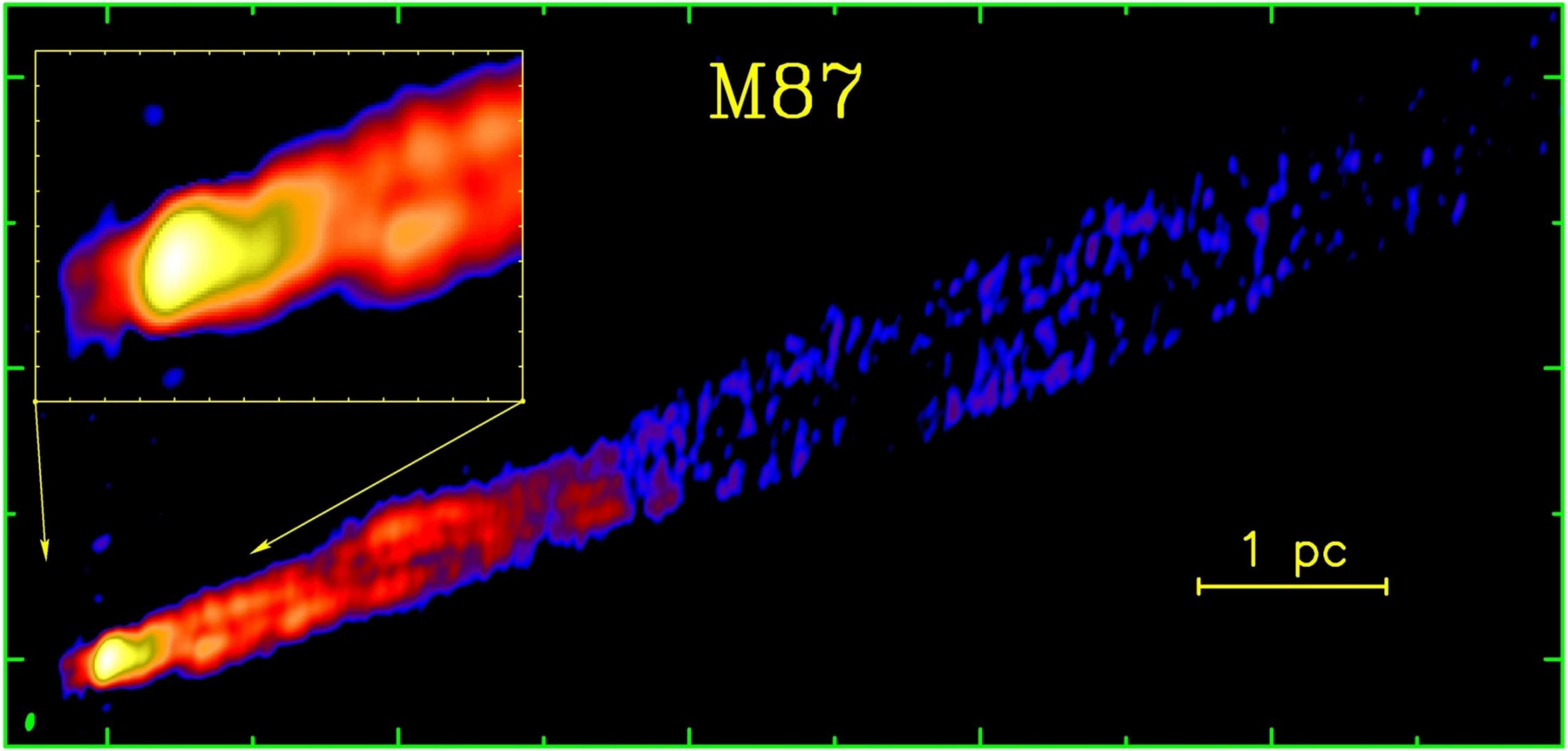
Theoretical result

Matching to ambient gas pressure –

how it affects the jet thickness?

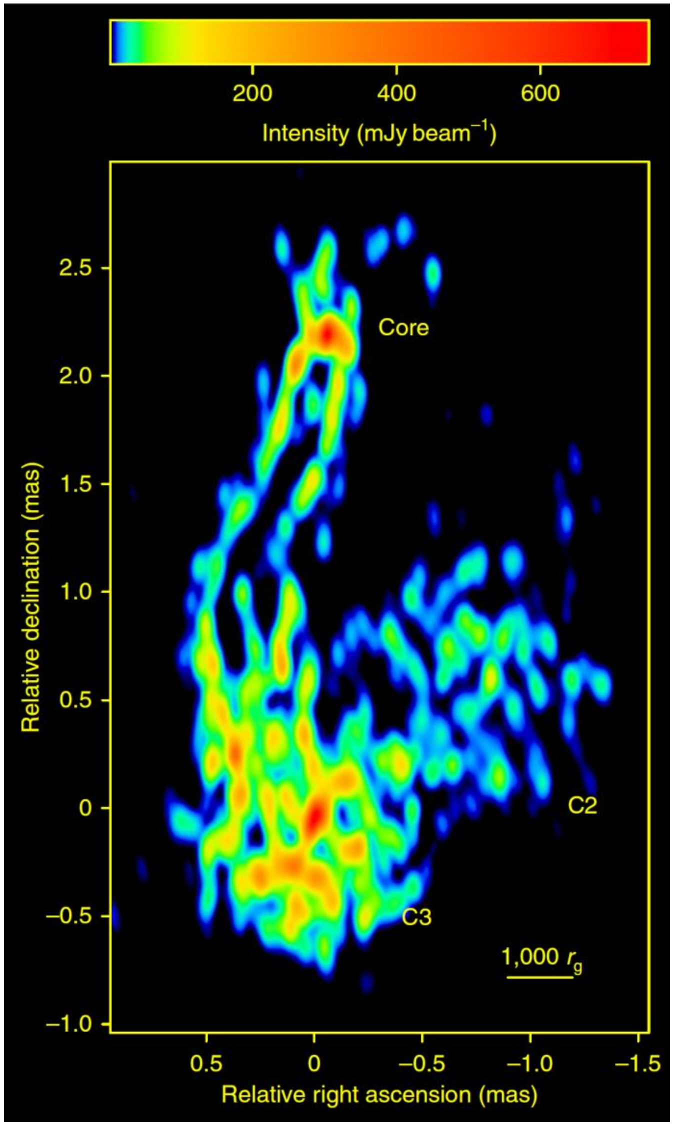
VLBA+VLA1, 15 GHz

The inner jet structure is clearly resolved, a short counter jet is detected



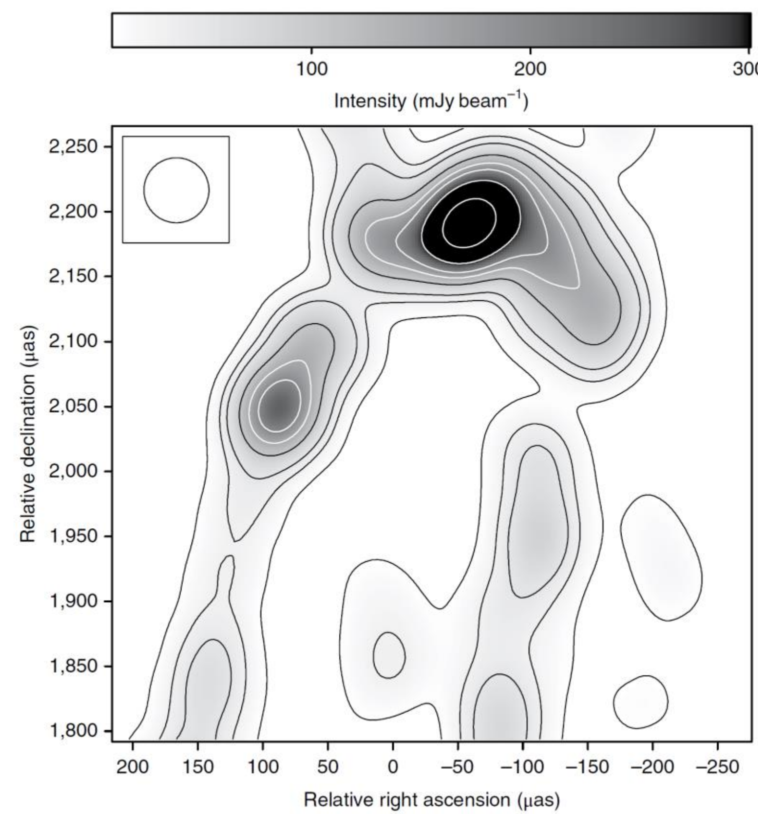
Y.Y.Kovalev et al, ApJ, **668**, L27 (2007)

nature astronomy
LETTERS
https://doi.org/10.1038/s41550-018-0431-2



A wide and collimated radio jet in 3C84 on the scale of a few hundred gravitational radii

G. Giovannini[1,2*], T. Savolainen[3,4,5*], M. Orienti[2], M. Nakamura[6], H. Nagai[7], M. Kino[8,9], M. Giroletti[2], K. Hada[9], G. Bruni[2,5,10], Y. Y. Kovalev[5,11,12], J. M. Anderson[13], F. D'Ammando[1,2], J. Hodgson[14], M. Honma[9], T. P. Krichbaum[5], S.-S. Lee[14,15], R. Lico[1,2], M. M. Lisakov[11], A. P. Lobanov[5], L. Petrov[12,16], B. W. Sohn[14,15,17], K. V. Sokolovsky[11,18,19], P. A. Voitsik[11], J. A. Zensus[5] and S. Tingay[20]



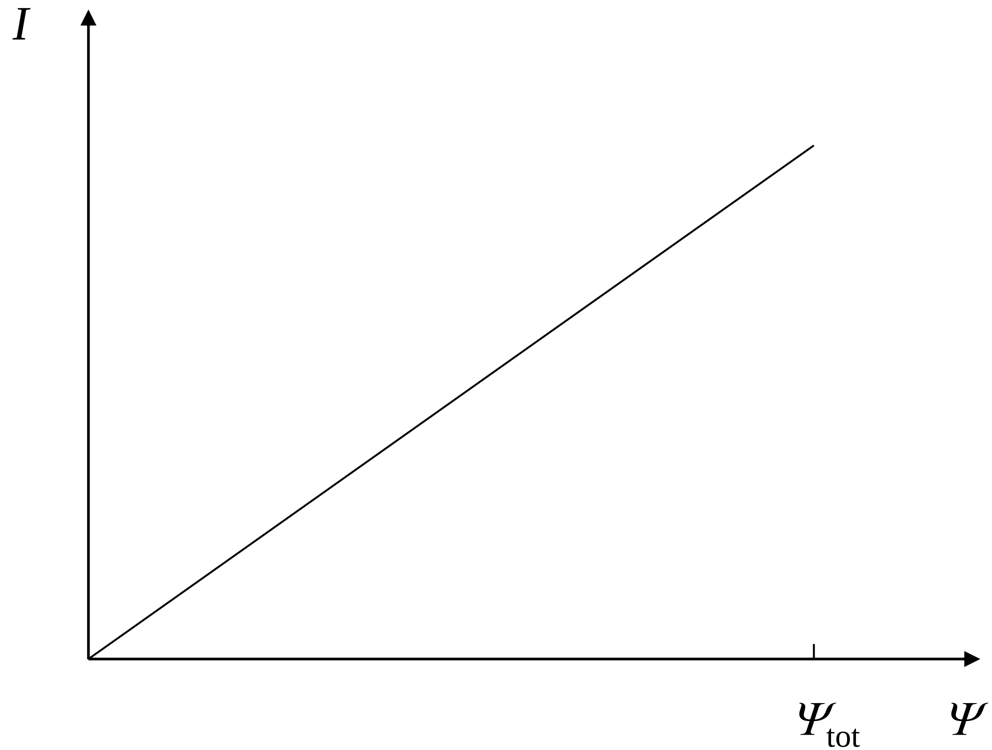
Matching to ambient gas pressure

Standard approach

$$\frac{B_\varphi^2}{8\pi} = P_{\rm ext}$$

$$B_\varphi = \frac{2I}{cr_\perp}$$

$$r_{\rm jet} = \left(\frac{I^2}{2\pi c^2 P_{\rm ext}}\right)^{1/2}$$



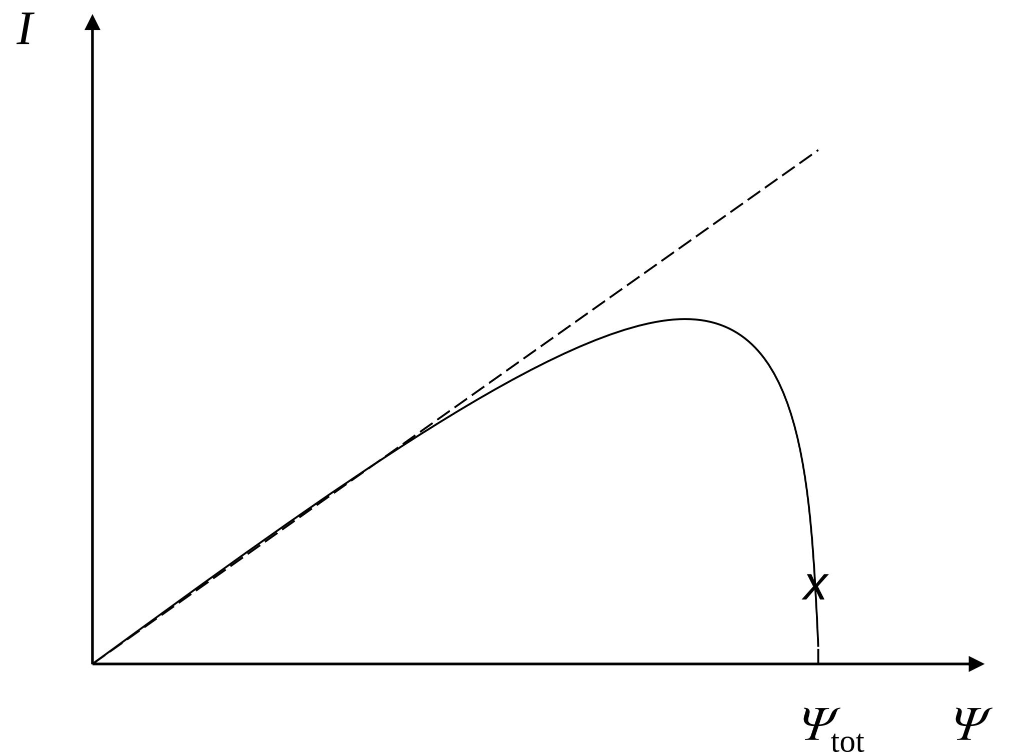
Matching to ambient gas pressure

More realistic

$$\frac{B_\varphi^2}{8\pi} = P_{\rm ext}$$

$$B_\varphi = \frac{2I}{cr_\perp}$$

$$r_{\rm jet} = \left(\frac{I^2}{2\pi c^2 P_{\rm ext}}\right)^{1/2}$$



Jets – theory

<u>Main parameters</u>

- Michel magnetization parameter (maximal <u>bulk</u> Lorentz-factor)

$$\sigma_{\rm M} = \frac{\Omega_0 e B_0 r_{\rm jet}^2}{4\lambda m_{\rm e} c^3}$$

μ now

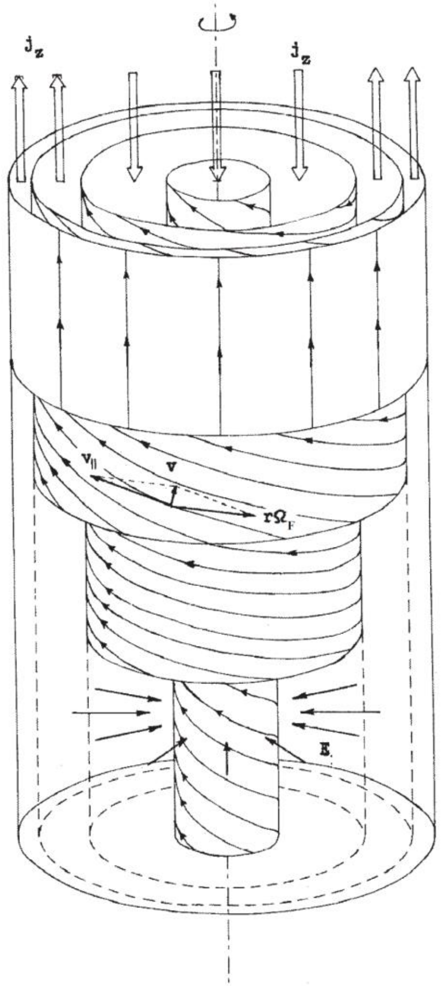
- Multiplicity parameter

$$\lambda = \frac{n^{\rm (lab)}}{n_{\rm GJ}}$$

$$\rho_{\rm GJ} = -\frac{\mathbf{\Omega}\cdot\mathbf{B}}{2\pi c}$$

- Total potential drop

$$\lambda\sigma_{\rm M} \sim \frac{eE_r r_{\rm jet}}{m_{\rm e}c^2}$$

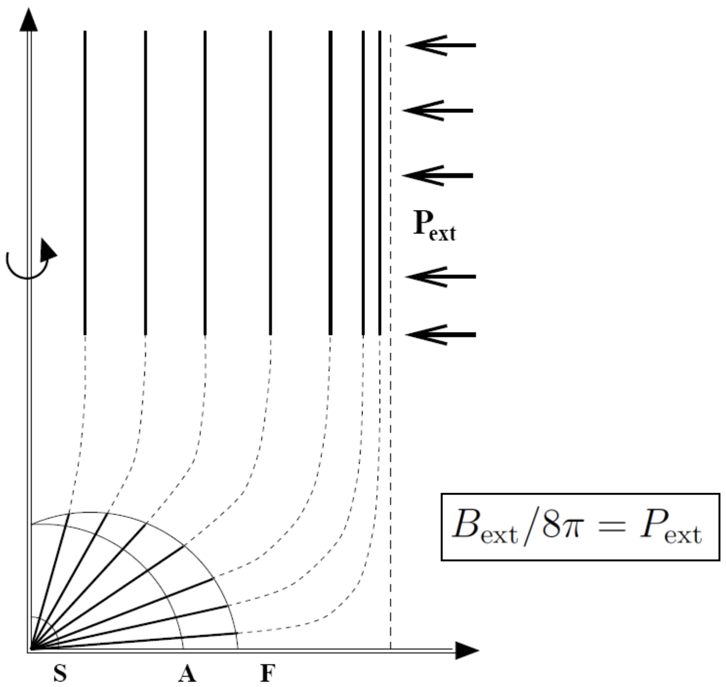


Jets – theory

It is necessary to include the external media into consideration. It is the ambient pressure that determines the jet transverse scale and particle energy.

1D approach for cylindrical jets

$$\begin{cases} \dfrac{\mathrm{d}\mathcal{M}^2}{\mathrm{d}r_\perp} = F_1(\mathcal{M}^2, \Psi, r_\perp) \\ \dfrac{\mathrm{d}\Psi}{\mathrm{d}r_\perp} = F_2(\mathcal{M}^2, \Psi, r_\perp) \end{cases}$$



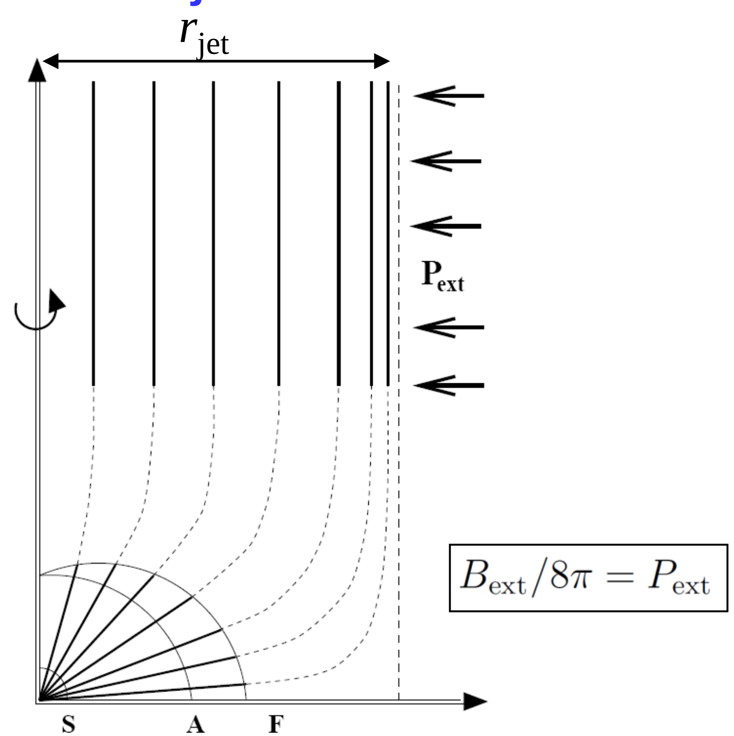
VB, L.M.Malyshkin. Astron. Lett., **26**, 208 (2000)
VB. Phys. Uspekhi, **40**, 659 (1997)

T.Lery, J.Heyvaerts, S.Appl, C.A.Norman. A&A, **347**, 1055 (1999)

Jets – theory

It is necessary to include the external media into consideration.
It is the ambient pressure that determines the jet transverse scale and particle energy.

$$r_{\rm jet} = \left(\frac{I^2}{2\pi c^2 P_{\rm ext}}\right)^{1/2}$$



VB, L.M.Malyshkin. Astron. Lett., **26**, 208 (2000)
VB. Phys. Uspekhi, **40**, 659 (1997)

T.Lery, J.Heyvaerts, S.Appl, C.A.Norman. A&A, **347**, 1055 (1999)

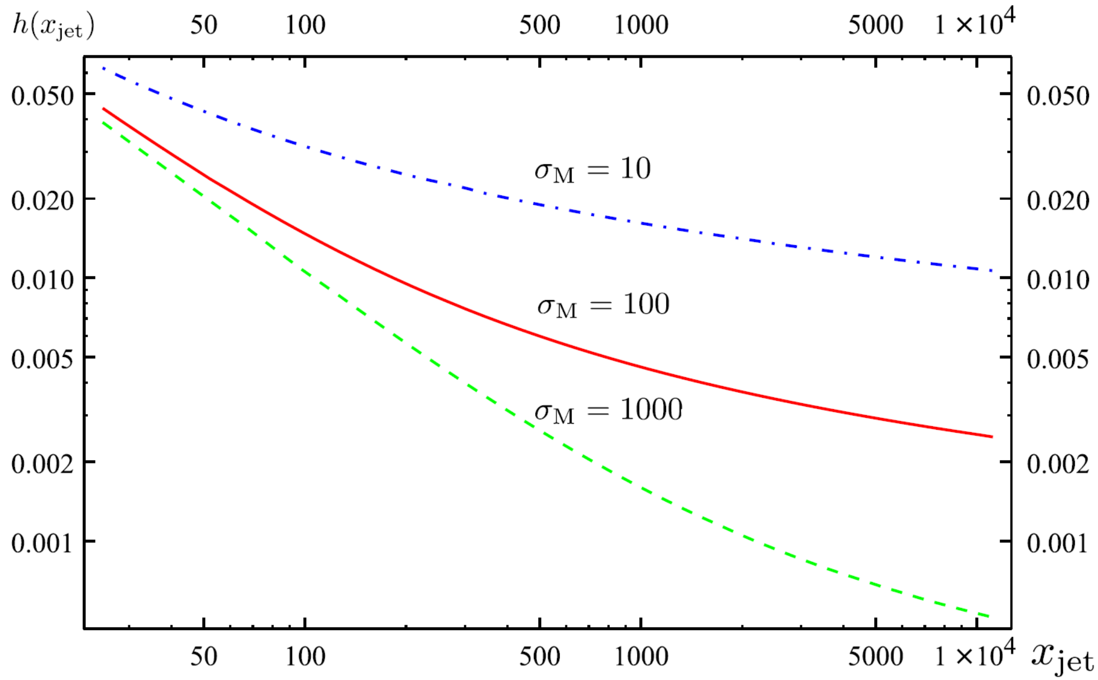
Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRAS **472,** 3971–3978 (2017)
doi:10.1093/mnras/s
Advance Access publication 2017 August 26

On the internal structure of relativistic jets collimated by ambient gas pressure

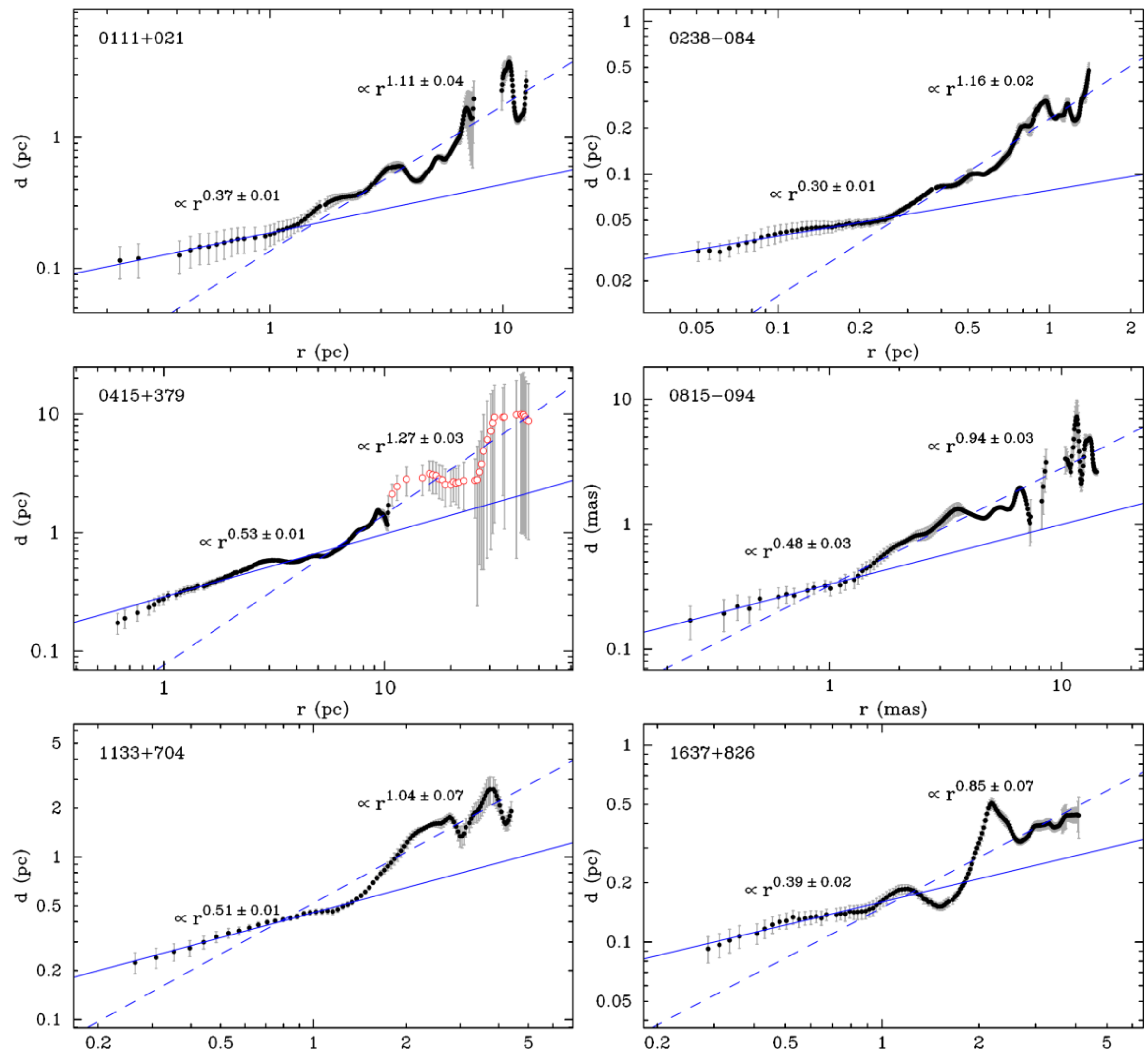
V. S. Beskin,[1,2]★ A. V. Chernoglazov,[1]★ A. M. Kiselev[1,2] and E. E. Nokhrina[1]
[1]Moscow Institute of Physics and Technology, Dolgoprudny, Institutsky per. 9, Moscow 141700, Russia
[2]P. N. Lebedev Physical Institute, Leninsky prosp. 53, Moscow 119991, Russia

$$x_{\rm jet} = \frac{1}{2(2\pi)^{1/2}} \frac{h(x_{\rm jet})}{k_{\rm I}} \frac{B_{\rm L}}{P_{\rm ext}^{1/2}}$$

$$x = \frac{\Omega_0 \, r}{c}$$

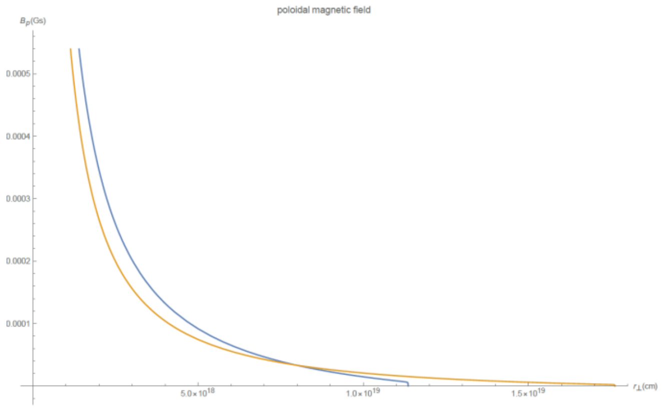


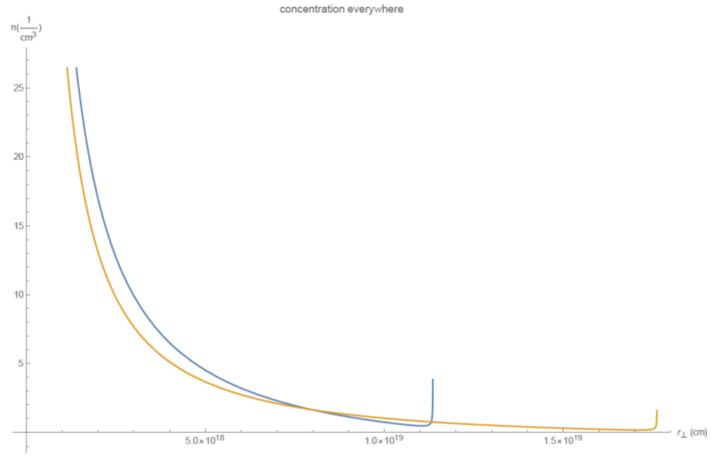
We can explain the break

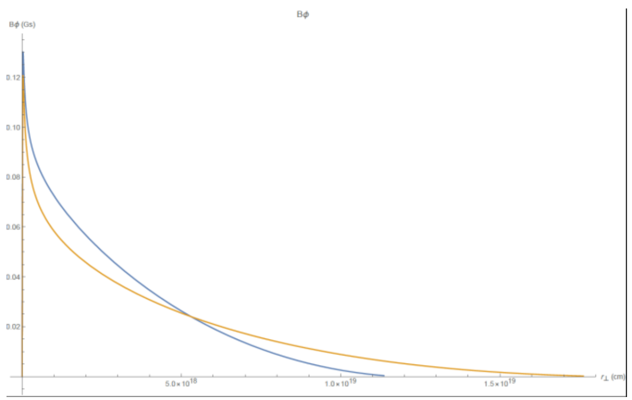


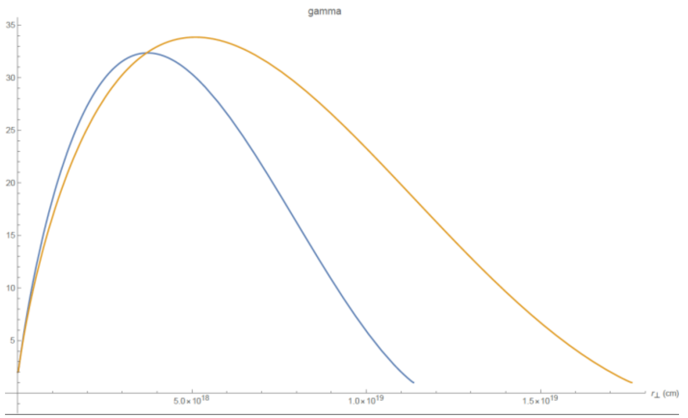
We can determine

- Internal structure



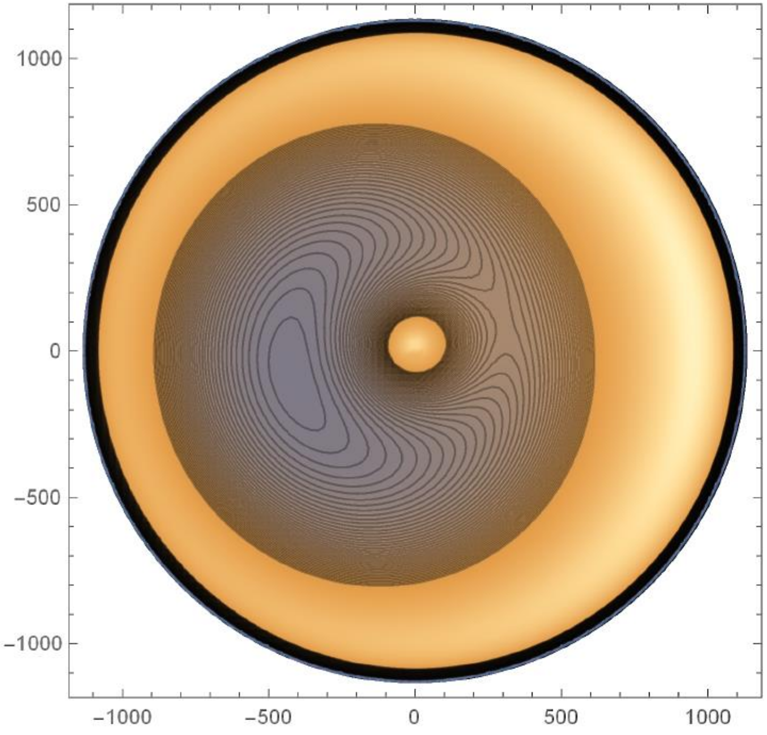




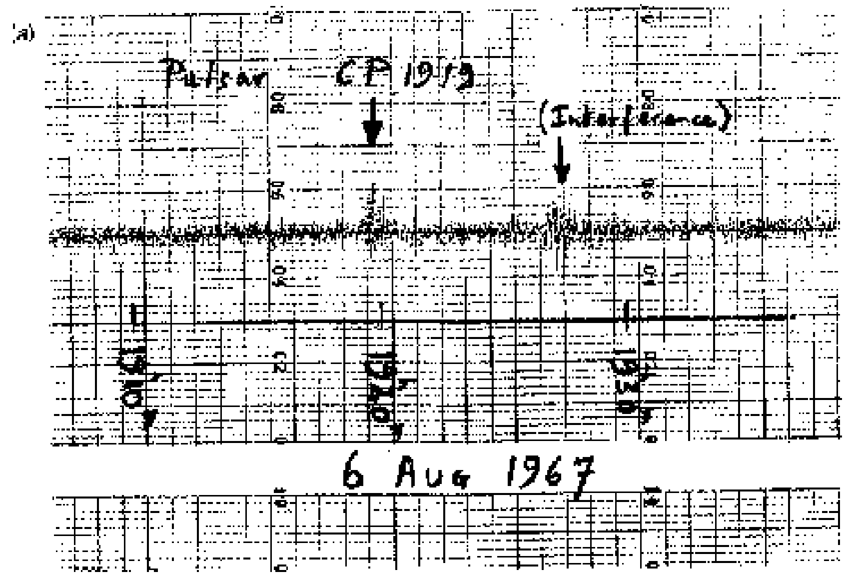


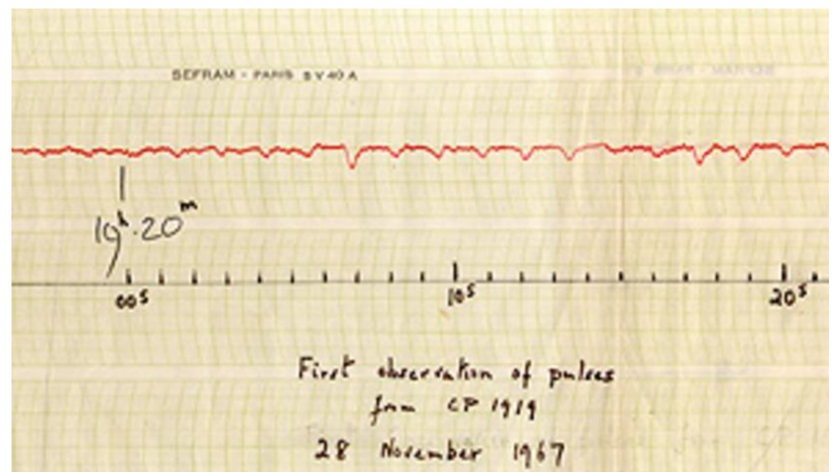
We can determine

- Doppler factor map



50 years!


Pulsar CP 1919
(Interference)
6 Aug 1967


SEFRAM - PARIS
19h 20m
00s
10s
20s
First observation of pulses
from CP 1919
28 November 1967

PSRs

The nature of the torque

- Our theory (BGI) – GJ current (and even smaller)

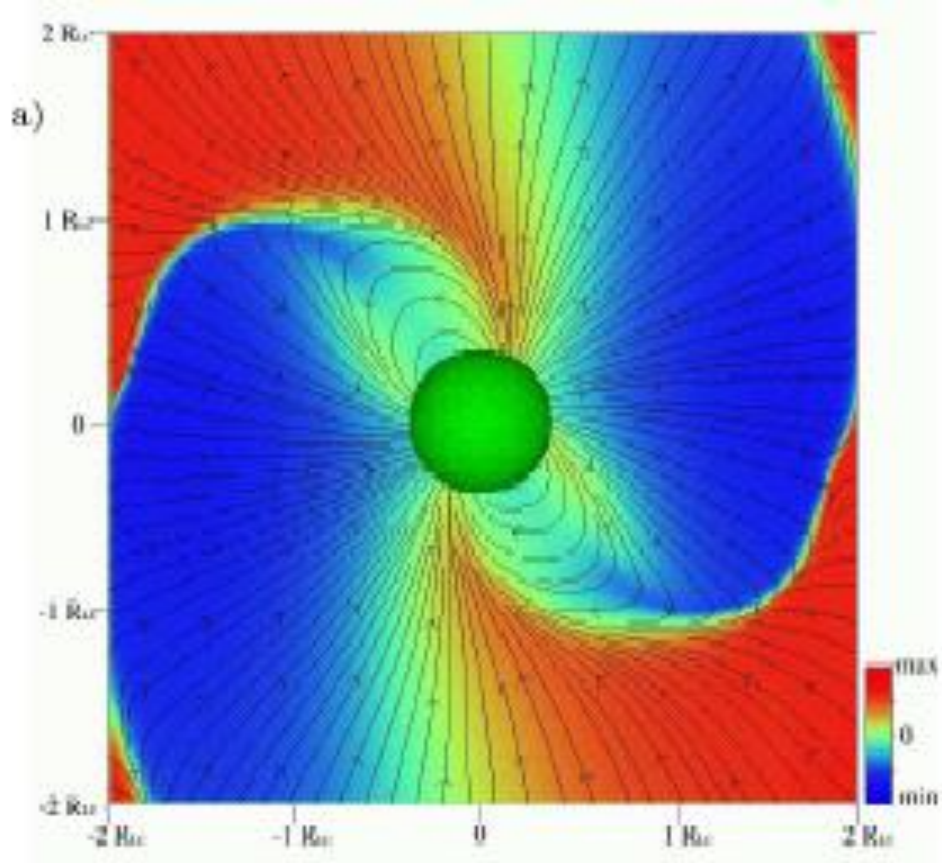
  no energy losses for zero current

  expression for current losses
- Standard approach – as large as necessary

How to explain?

$$W_{\rm tot}^{\rm (MHD)} \approx \frac{1}{4}\frac{B_0^2\Omega^4 R^6}{c^2}(1+\sin^2\chi)$$

What is the current system?



What is the current system?

Current losses

1. Direct current losses (BGI)

$$K_{\parallel} = -c_{\parallel}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s},$$

$$K_{\perp} = -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.$$

2. Mismatch
('second term')

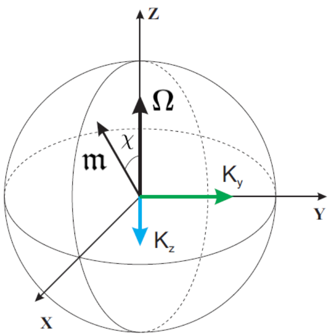
$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{B}\mathbf{n})\,{\rm d}o = \frac{R^3}{4\pi}\int\{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

3. Additional separatrix current

STEP #I

Vacuum magneto-dipole

Vacuum: magneto-dipole



$$\mathbf{K} = \frac{1}{c}\int \left[\mathbf{r} \times \left[\mathbf{J}_{\mathrm{s}} \times \mathbf{B}\right]\right] \mathrm{d}S$$

$$K_{z'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin^2\chi$$

$$K_{x'} = \frac{2}{3}\frac{\mathfrak{m}^2}{R^3}\left(\frac{\Omega R}{c}\right)^3 \sin\chi\cos\chi$$

Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\mathbf{\Omega}\times\mathbf{r}}{c}$$

$$W_{\rm tot} = \frac{c}{4\pi}\int(\beta_{\rm R}\mathbf{B})(\mathbf{B}{\rm d}\mathbf{S})$$

Vacuum (Deusch) 1 Ω Ω^3 1 1 $= \Omega^4$

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{B}\mathbf{n})\,{\rm d}o = \frac{R^3}{4\pi}\int\{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

$$\mathrm{H}_r = \mathrm{R}_1(a)\left\{\frac{a^3}{r^3}\cos\chi\cos\theta + \frac{h_1/\rho}{(h_1/\rho)_\alpha}\sin\chi\sin\theta\, e^{i\lambda}\right\}$$

Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\mathbf{\Omega}\times\mathbf{r}}{c}$$

Vacuum (Deusch) 1 Ω Ω^3 1 1 $= \Omega^4$

$$\mathrm{H}_r = \mathrm{R}_1(a)\left\{\frac{a^3}{r^3}\cos\chi\cos\theta + \frac{h_1/\rho}{(h_1/\rho)_\alpha}\sin\chi\sin\theta\, e^{i\lambda}\right\}$$

$$\mathrm{H}_\theta = \frac{1}{2}\mathrm{R}_1(a)\left\{\frac{a^3}{r^3}\cos\chi\sin\theta + \left[\left(\frac{\rho^2}{\rho h_2' + h_2}\right)_\alpha h_2 + \left(\frac{\rho}{h_1}\right)_\alpha\left(h_1' + \frac{h_1}{\rho}\right)\right]\sin\chi\cos\theta\, e^{i\lambda}\right\}$$

$$\mathrm{H}_\varphi = \frac{1}{2}\mathrm{R}_1(a)\left\{\left(\frac{\rho^2}{h_2' + h_2}\right)_\alpha h_2\cos 2\theta + \left(\frac{\rho}{h_1}\right)_\alpha\left(h_1' + \frac{h_1}{\rho}\right)\right\} i\sin\chi\, e^{i\lambda}$$

$$\mathrm{E}_r = \frac{1}{2}\omega\mu_0 a\,\mathrm{R}_1(a)\left\{-\frac{1}{2}\frac{a^4}{r^4}\cos\chi(3\cos 2\theta + 1) + 3\left(\frac{\rho}{\rho h_2' + h_2}\right)_\alpha \frac{h_2}{\rho}\sin\chi\sin 2\theta\, e^{i\lambda}\right\}$$

$$\mathrm{E}_\theta = \frac{1}{2}\omega\mu_0 a\,\mathrm{R}_1(a)\left\{-\frac{a^4}{r^4}\cos\chi\sin 2\theta + \left[\left(\frac{\rho h_2' + h_2}{\rho}\right)_\alpha \frac{\rho}{\rho h_2' + h_2}\cos 2\theta - \frac{h_1}{h_1(\alpha)}\right]\sin\chi\, e^{i\lambda}\right\}$$

$$\mathrm{E}_\varphi = \frac{1}{2}\omega\mu_0 a\,\mathrm{R}_1(a)\left\{\left(\frac{\rho}{\rho h_2' + h_2}\right)_\alpha \frac{\rho h_2' + h_2}{\rho} - \frac{h_1}{h_1(\alpha)}\right\} i\sin\chi\cos\theta\, e^{i\lambda}.$$

Landau-Lifshits, Field Theory

Orthogonal rotator

$$
\begin{aligned}
B_r^{\perp} &= \frac{|\mathfrak{m}|}{r^3}\sin\theta\,\mathrm{Re}\left(2-2i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\theta^{\perp} &= \frac{|\mathfrak{m}|}{r^3}\cos\theta\,\mathrm{Re}\left(-1+i\frac{\Omega r}{c}+\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
B_\varphi^{\perp} &= \frac{|\mathfrak{m}|}{r^3}\,\mathrm{Re}\left(-i-\frac{\Omega r}{c}+i\frac{\Omega^2 r^2}{c^2}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_r^{\perp} &= 0,\\
E_\theta^{\perp} &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\,\mathrm{Re}\left(-1+i\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right),\\
E_\varphi^{\perp} &= \frac{|\mathfrak{m}|\Omega}{r^2 c}\cos\theta\,\mathrm{Re}\left(-i-\frac{\Omega r}{c}\right)\exp\left(i\frac{\Omega r}{c}+i\varphi-i\Omega t\right).
\end{aligned}
$$

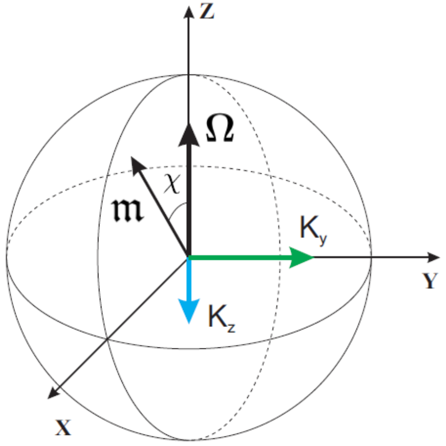
Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

$$W_{\rm tot} = \frac{c}{4\pi} \int (\beta_{\rm R} \mathbf{B})(\mathbf{B} d\mathbf{S})$$

Vacuum (Deusch)	1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (L&L) (2/3)	1	Ω	Ω^3	1	1	$= \Omega^4$



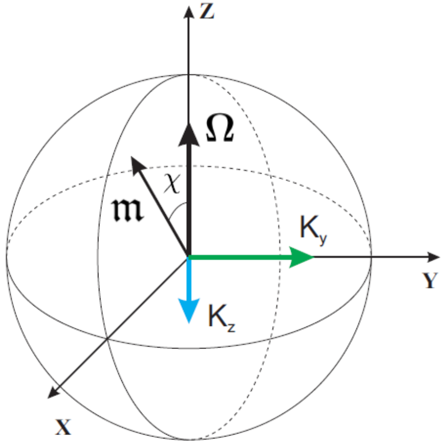
Vacuum: magneto-dipole

Energy losses

$$\beta_{\rm R} = \frac{\mathbf{\Omega} \times \mathbf{r}}{c}$$

$$W_{\rm tot} = \frac{c}{4\pi} \int (\beta_{\rm R} \mathbf{B})(\mathbf{B} d\mathbf{S})$$

Vacuum (Deusch)	1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (L&L) (2/3)	1	Ω	Ω^3	1	1	$= \Omega^4$
Vacuum (L&L) (1/3)	1	Ω	1	Ω^3	1	$= \Omega^4$



$$\mathbf{B}^{(3)} = -\frac{2}{3} \frac{\mathfrak{m}}{R^3} \left(\frac{\Omega R}{c}\right)^3 \mathbf{e}_{y'}$$

IMPORTANT CONCLUSION

Two terms can play role in energy losses

$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\rm s} \left(\mathbf{B}\mathbf{n} \right) \mathrm{d}o = \frac{R^3}{4\pi} \int \{ [\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n}) \} \, \mathrm{d}o$$

STEP #II

Pulsar magnetosphere

Force-free approximation

One can neglect energy of particles

$$\frac{1}{c}\mathbf{j} \times \mathbf{B} + \rho_{\rm e}\mathbf{E} = 0$$

Mestel equation (1973)

$$\nabla \times \tilde{\mathbf{B}} = \psi \mathbf{B}$$

$$\tilde{\mathbf{B}} = \left\{B_r\left(1 - \frac{\Omega^2 r^2}{c^2}\right), B_\theta, B_z\left(1 - \frac{\Omega^2 r^2}{c^2}\right)\right\}$$

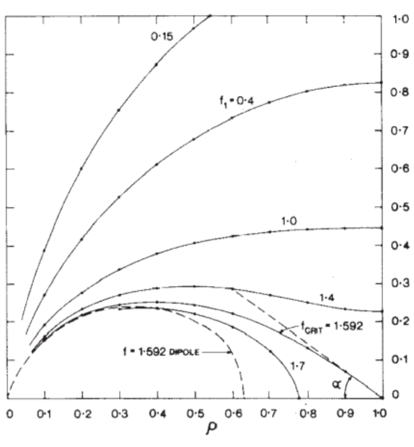
Pulsar equation

$$-\left(1 - \frac{\Omega_{\rm F}^2 \varpi^2}{c^2}\right)\nabla^2\Psi + \frac{2}{\varpi}\frac{\partial\Psi}{\partial\varpi} - \frac{16\pi^2}{c^2}I\frac{{\rm d}I}{{\rm d}\Psi} + \frac{\varpi^2}{c^2}\left(\nabla\Psi\right)^2\Omega_{\rm F}\frac{{\rm d}\Omega_{\rm F}}{{\rm d}\Psi} = 0$$

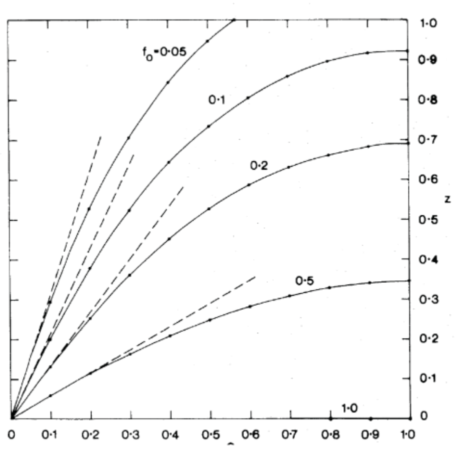
(Michel 1973, Mestel 1993, Scharlemann & Wagoner 1973,
Okamoto 1974, Mestel & Wang 1979)

First solutions

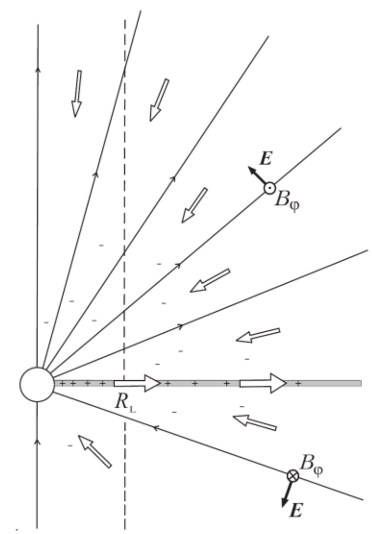
$$-\left(1-\frac{\Omega_{\mathrm{F}}^{2}\varpi^{2}}{c^{2}}\right)\nabla^{2}\Psi+\frac{2}{\varpi}\frac{\partial\Psi}{\partial\varpi}-\frac{16\pi^{2}}{c^{2}}I\frac{\mathrm{d}I}{\mathrm{d}\Psi}+\frac{\varpi^{2}}{c^{2}}\left(\nabla\Psi\right)^{2}\Omega_{\mathrm{F}}\frac{\mathrm{d}\Omega_{\mathrm{F}}}{\mathrm{d}\Psi}=0$$



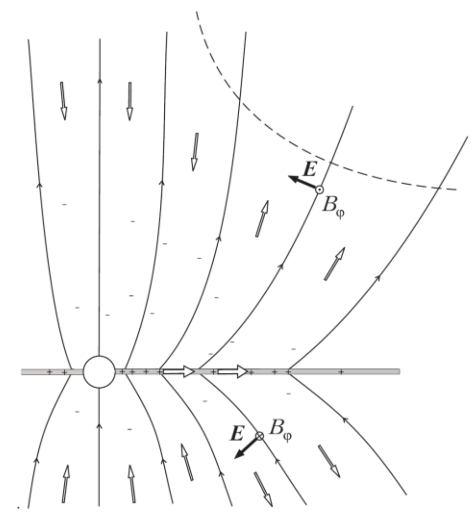
F. Michel (1973)



F. Michel (1973)



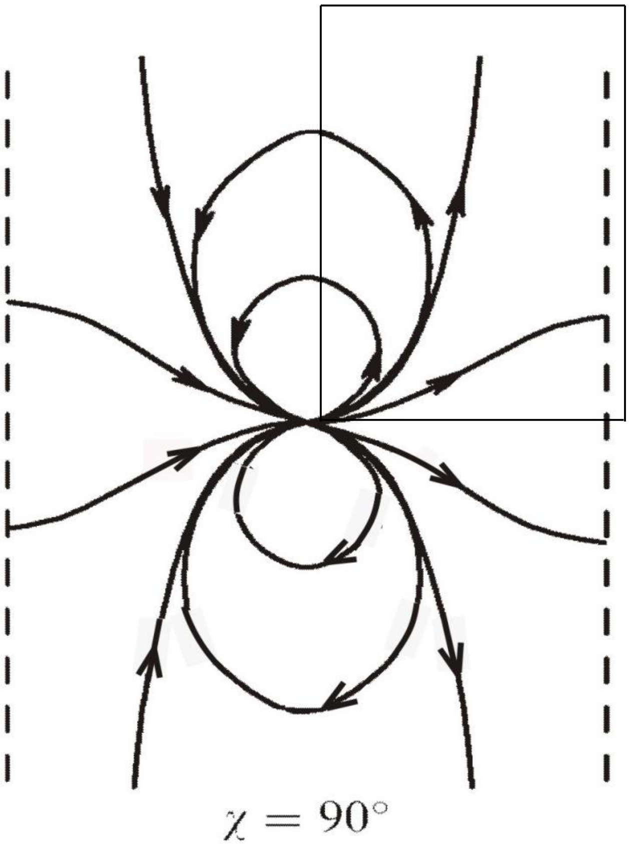
F. Michel (1973)



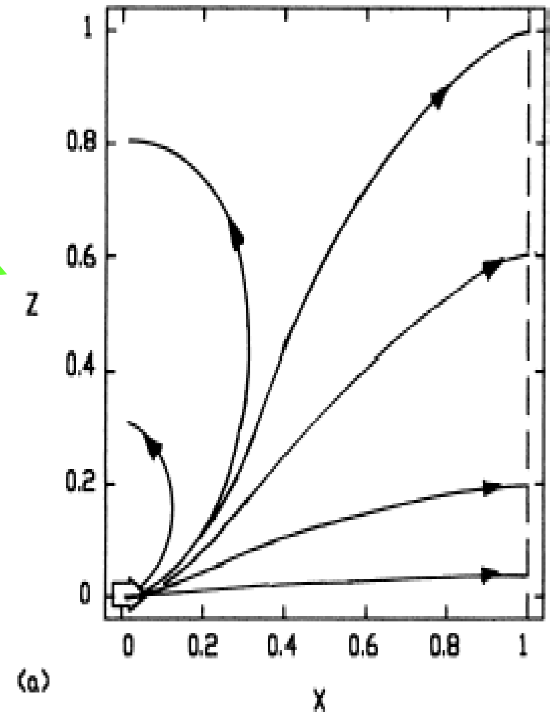
R.Blandford (1976)

Orthogonal Rotator – no currents

$$\nabla \times \tilde{\mathbf{B}} = \mathbf{0}$$



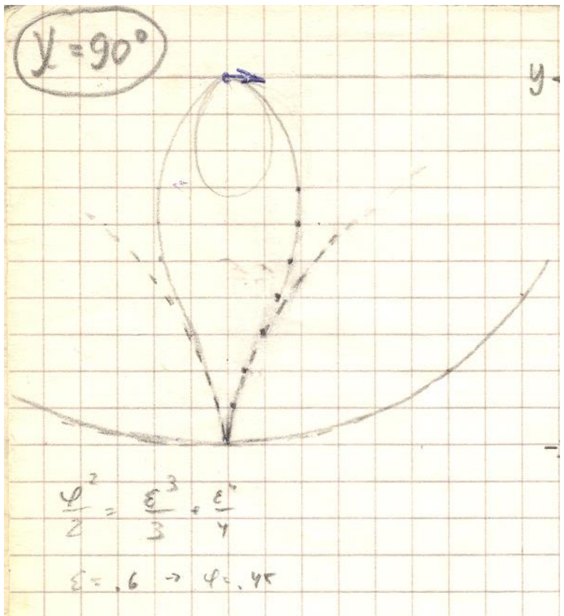
VB, A.V.Gurevich, Ya.N.Istomin,
Sov. Phys. JETP, **58**, 235 (1983)

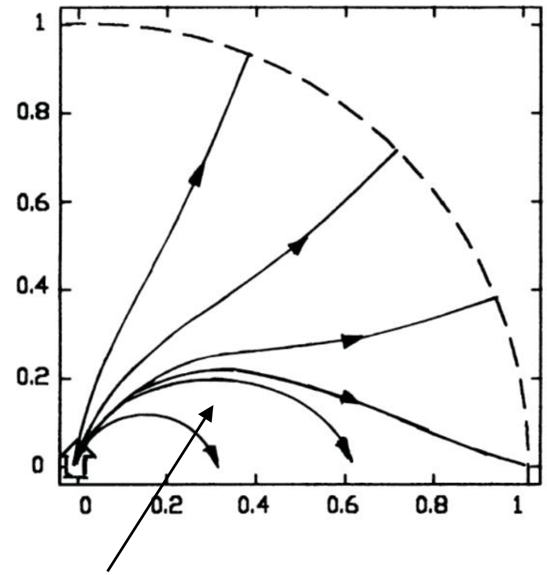


L.Mestel, P.Panagi, S.Shibata,
MNRAS, **309**, 388 (1999)

Orthogonal Rotator – no currents

VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)
L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)





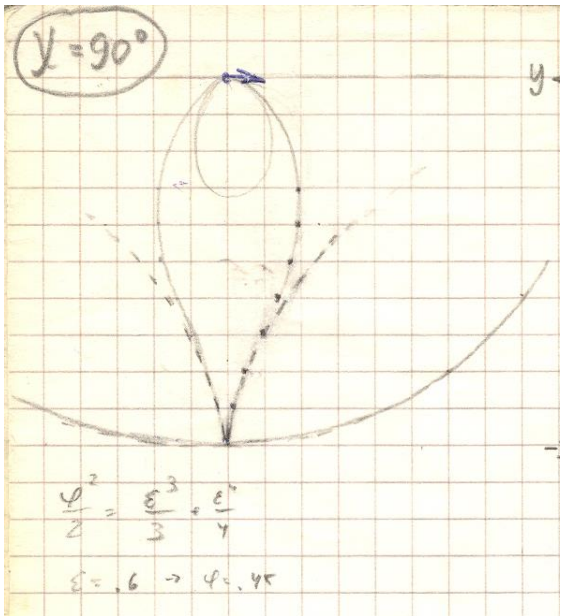
Equatorial plane

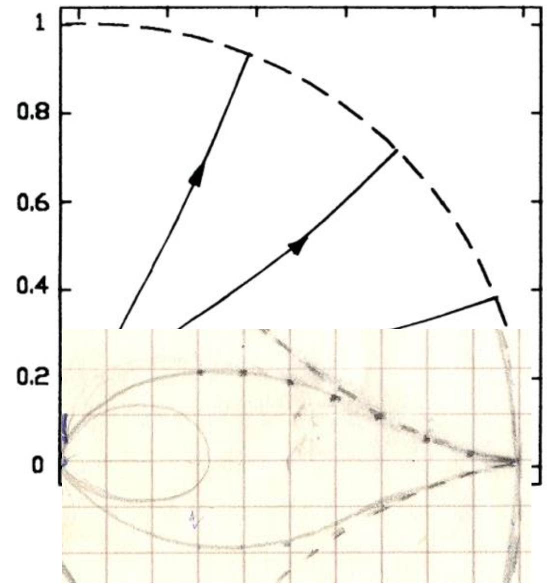
No energy flux through the light cylinder

$$B_\varphi \propto (1 - x_r^2)^2$$

Orthogonal Rotator – no currents

VB, A.V.Gurevich, Ya.N.Istomin, JETP, **58**, 235 (1983)
L.Mestel, P.Panagi, S.Shibata, MNRAS, **309**, 388 (1999)





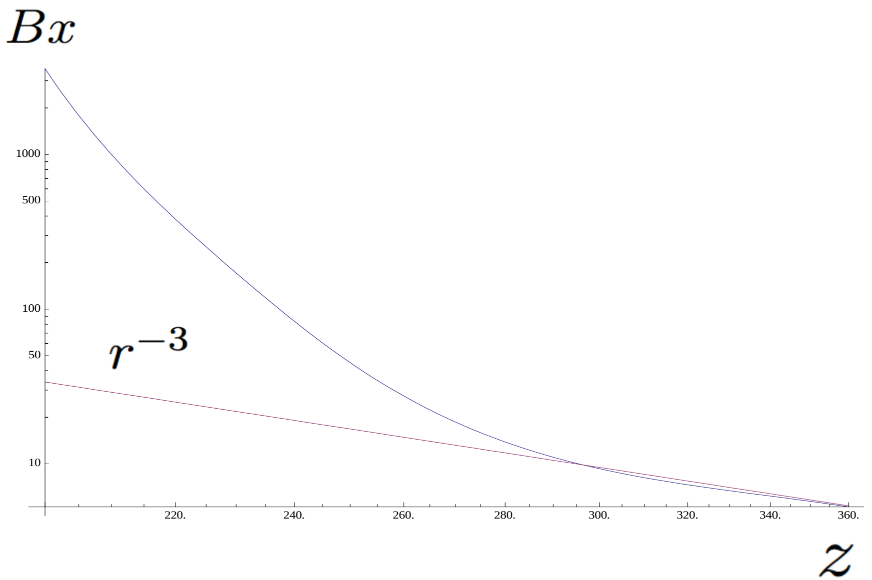
Equatorial plane

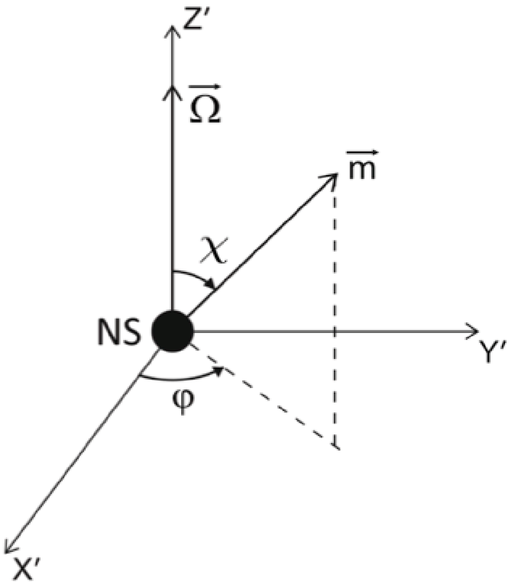
No energy flux through the light cylinder

$$B_\varphi \propto (1 - x_r^2)^2$$

Spitkovsky solution, $\chi = 60^o$

No magnetodipole radiation





In vacuum $B_x = \frac{\ddot{d}}{cr}$

VB, YA.N.Istomin, A.A.Philippov, Phys. Uspekhi, **56**, 164 (2013)

IMPORTANT CONCLUSION

No energy losses for zero longitudinal current

STEP #III

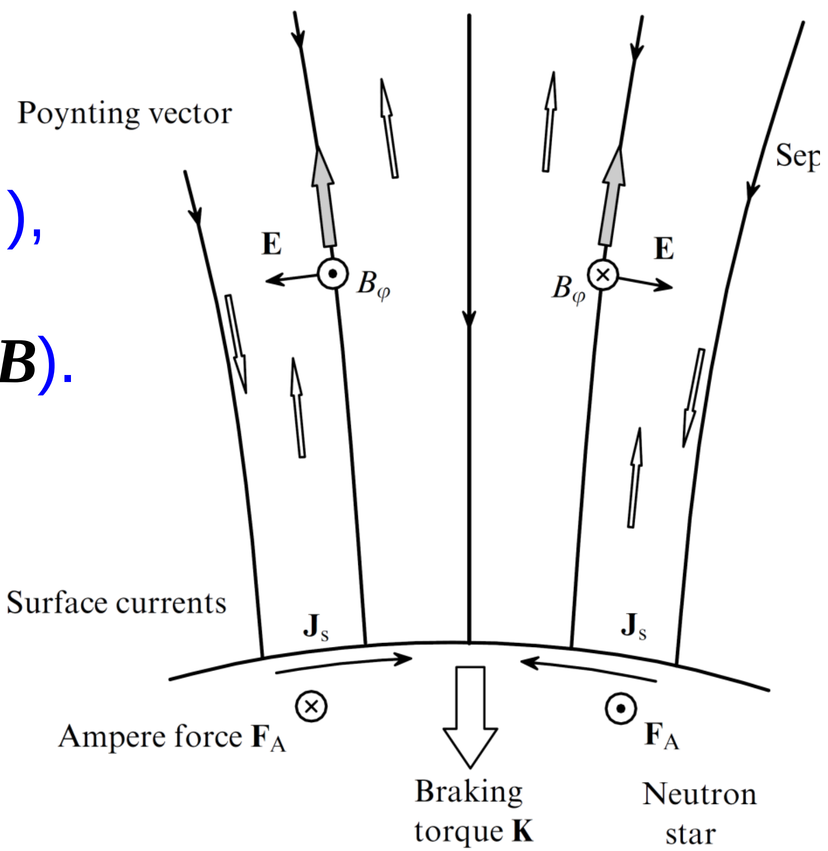
Current losses

Current losses

For current loss mechanism is necessary to have

- Plasma in the magnetosphere,
- regular poloidal magnetic field,
- rotation (inductive electric field ***E*,** EMF ***d**U*),
- longitudinal current I (toroidal magnetic field ***B***).

$$W_{\rm tot} = I\delta U$$



Current losses

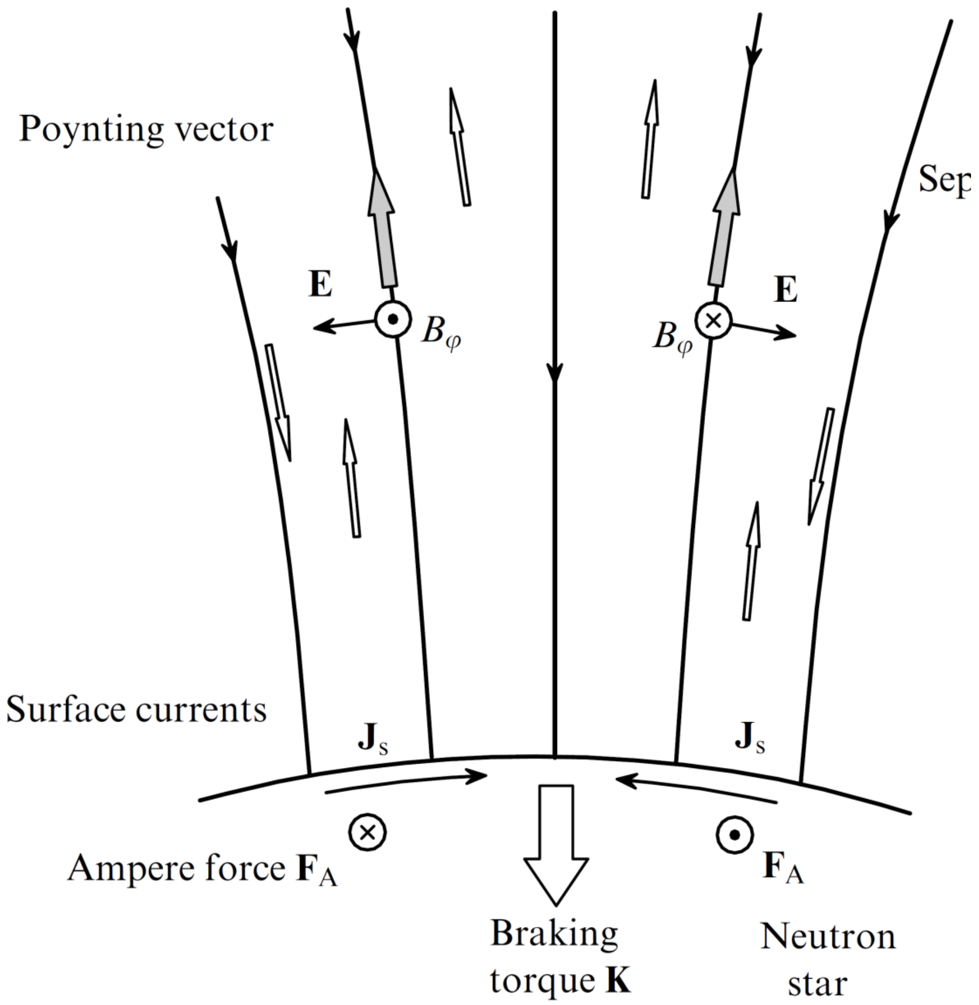
$$W_{\rm tot} = c_{\parallel} \frac{B_0^2 \Omega^4 R^6}{c^3} i_0$$

$$i_0 = j_{\parallel} / j_{\rm GJ}$$

$$W_{\rm tot}^{\rm (BGI)} \approx i_{\rm s}^{\rm A} \frac{B_0^2 \Omega^4 R^6}{c^2} \cos^2 \chi$$

$$W_{\rm tot}^{\rm (BGI)} \approx \frac{B_0^2 \Omega^4 R^6}{c^2} \cos^2 \chi$$

for GJ current



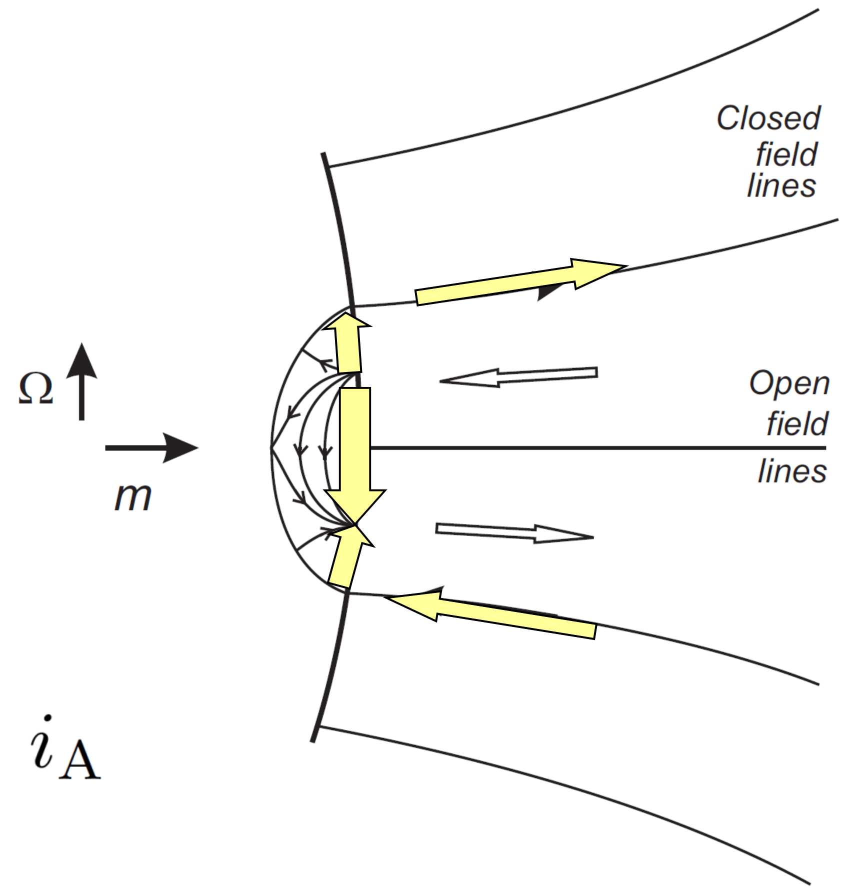
Orthogonal rotator

VB, A.V.Gurevich, Ya.N.Istomin JETP **58**, 235 (1983)

$$j_{\mathrm{GJ}} \approx \frac{\Omega B}{2\pi} \cos\theta$$

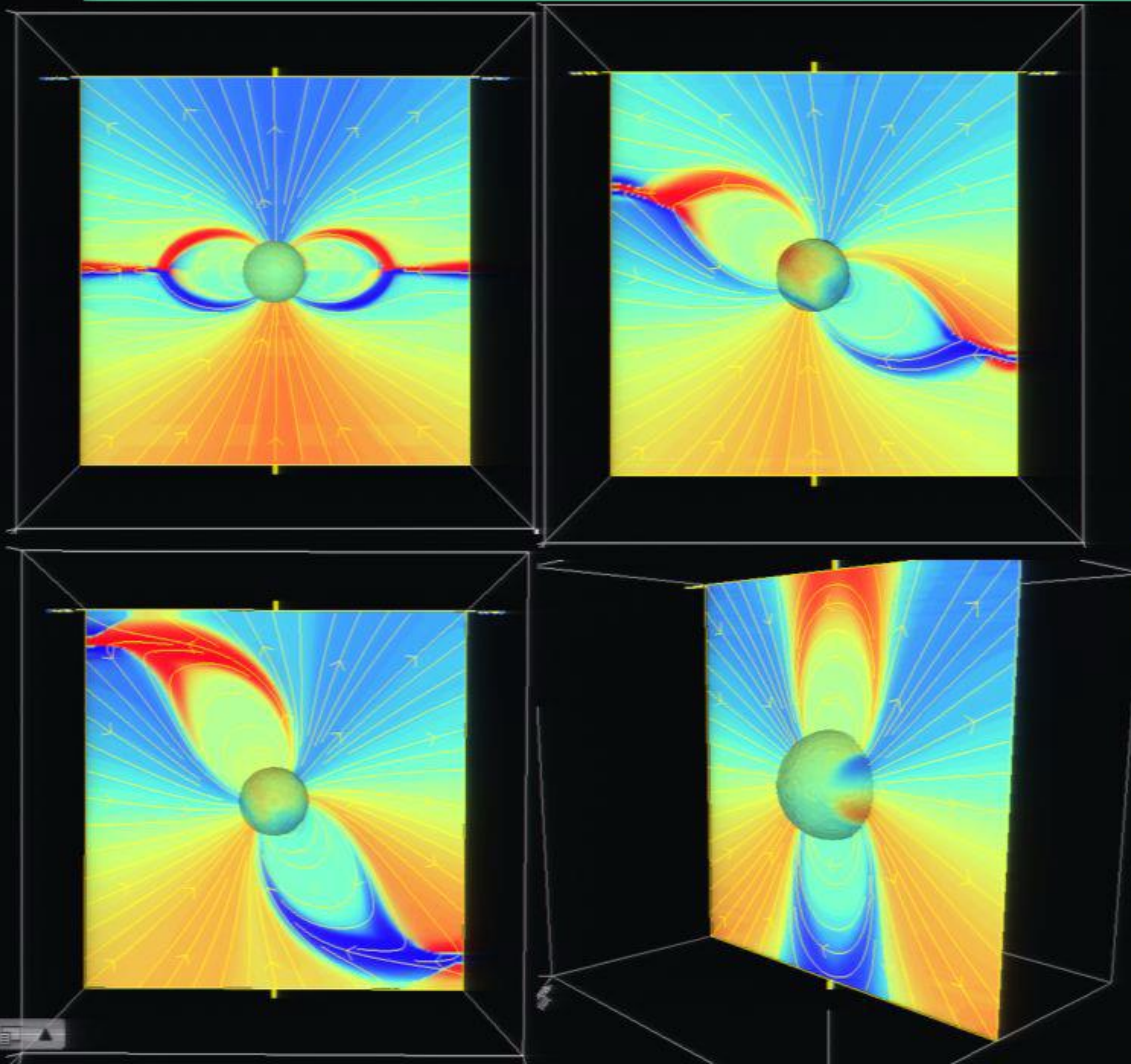
$$\mathbf{K} = \frac{1}{c} \int [\mathbf{r} \times [\mathbf{J}_{\mathrm{s}} \times \mathbf{B}]]\, \mathrm{d}S$$

$$W_{\mathrm{tot}} = c_{\perp} \frac{B_0^2 \Omega^4 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\mathrm{A}}$$



$\mathrm{i}_{\mathrm{A}} = j_{\parallel} / \rho_{\mathrm{GJ}} c$

Magnetospheric currents

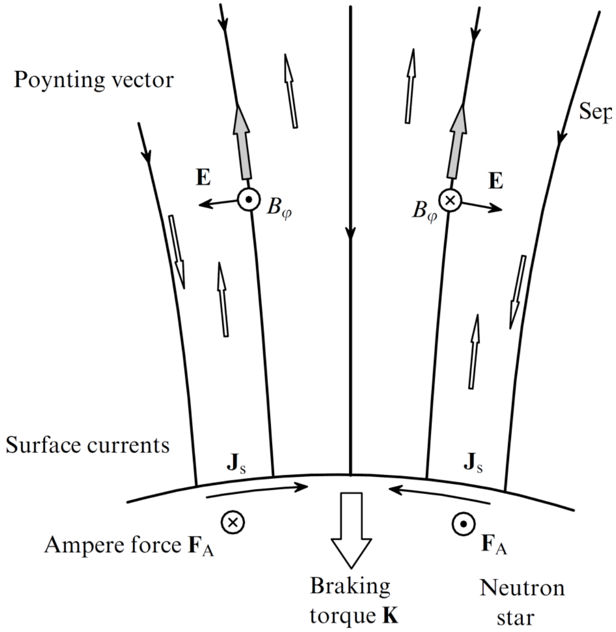


Oppositely flowing currents can occupy the same open flux tube. Does this have any obervational implications?

There is always a null-current field line in the open zone.

IMPORTANT REMARK

$$\boxed{W_{\mathrm{tot}} = I\delta U}$$



Direct current losses correspond to first term only

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_s\,(\mathbf{Bn})\,\mathrm{d}o = \frac{R^3}{4\pi}\int \{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,\mathrm{d}o$$

STEP #IV

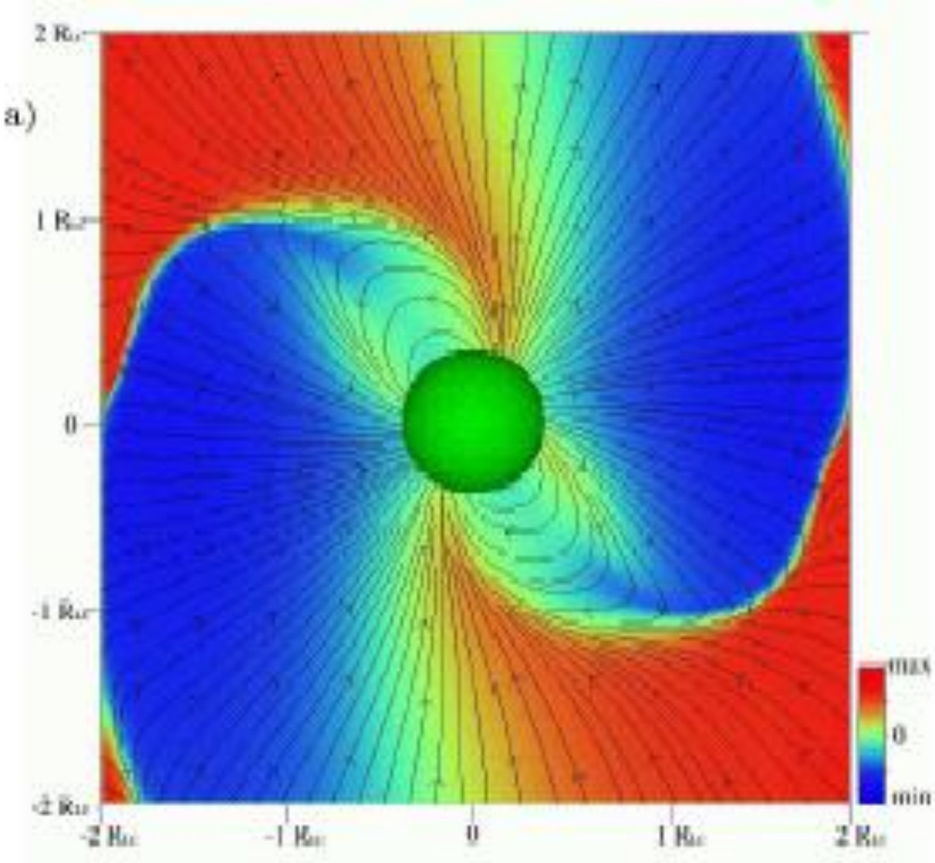
"Universal solution"

Inclined rotator

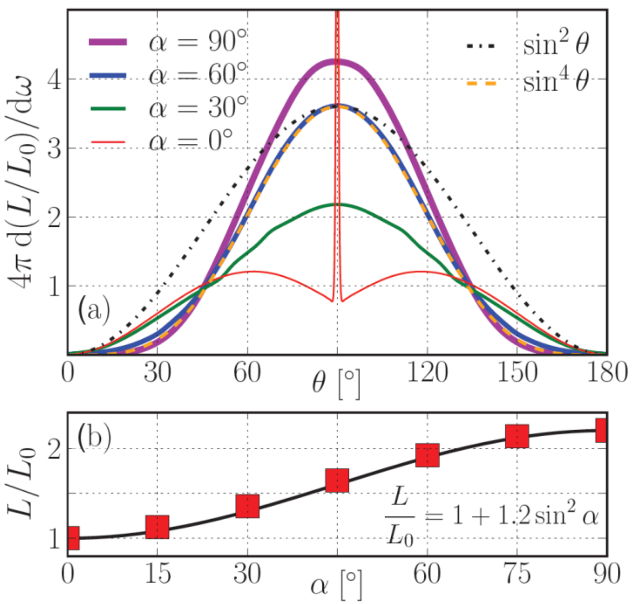
A.Spitkovsky, ApJ Lett., **648**, L51 (2006)

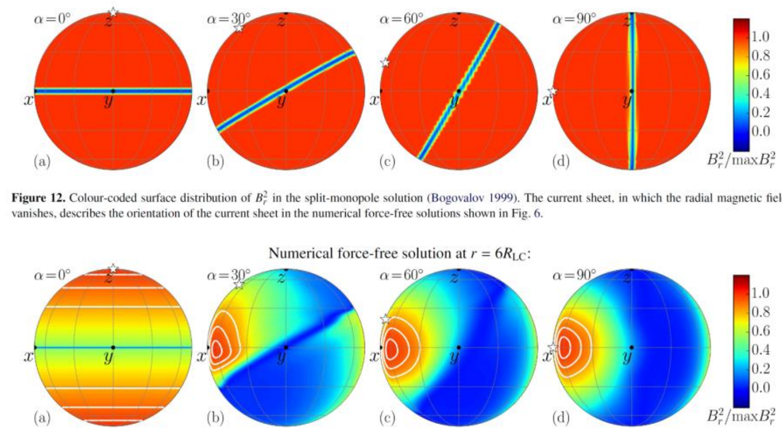
$$W_{\rm tot}^{\rm (MHD)} \approx \frac{1}{4}\frac{B_0^2\Omega^4 R^6}{c^2}(1+\sin^2\chi)$$

a)

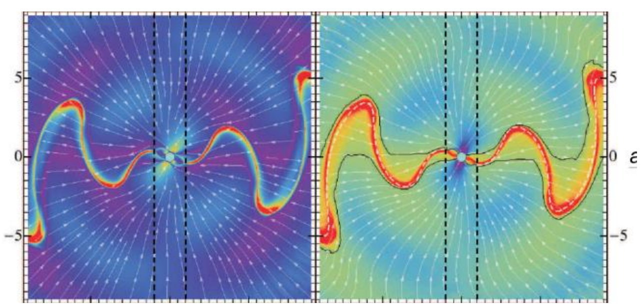


Inclined rotator – numerically





A.Tchekhovskoy, A.Philippov, A.Spitkovsky, MNRAS, **457**, 3384 (2016)



$$\langle B_r \rangle \sim \sin\theta$$
$$\langle E \rangle, \langle B_\varphi \rangle \sim \sin^2\theta$$

I.Contopoulos et al

$$W_{\rm tot}(\theta) = \sin^2\theta B_r^2(\theta)$$

Inclined rotator – MHD

- No monopole Michel-Bogovalov poloidal field
- Larger energy losses for orthogonal rotator

$$W_{\rm tot}^{\rm (MHD)} \approx \frac{1}{4}\frac{B_0^2\Omega^4 R^6}{c^2}(1+\sin^2\chi)$$

- Alignment: inclination angle evolves to 0 deg.

Problem 5.2. Show that the relation similar to (5.24) can be obtained for the conical solutions $\Psi = \Psi(\theta)$, but only at large distances $r \gg R_{\rm L}$ from the compact object. It has the form [Ingraham, 1973, Michel, 1974]

$$4\pi I(\theta) = \Omega_{\rm F}(\theta)\sin\theta\,\frac{{\rm d}\Psi}{{\rm d}\theta}. \qquad (5.25)$$

$E_\theta = B_\varphi$

S.Gralla, T.Jacobson, G.Menon, C.Dermer ($B_{\rm p} = 0$)

Wind – not a split-monopole

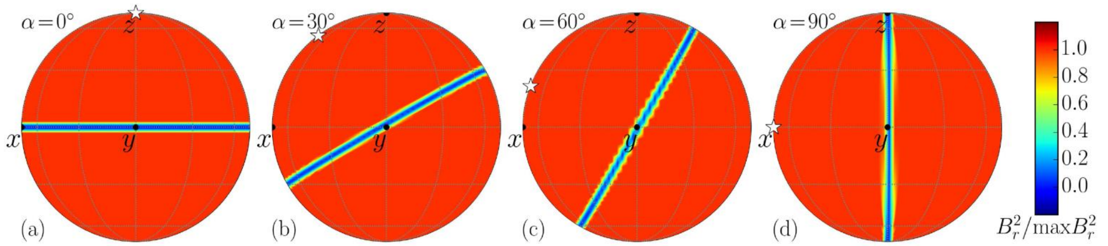
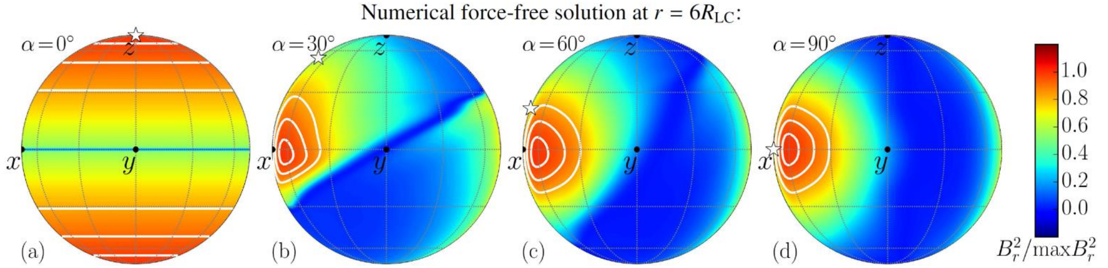


Figure 12. Colour-coded surface distribution of B_r^2 in the split-monopole solution (Bogovalov 1999). The current sheet, in which the radial magnetic field vanishes, describes the orientation of the current sheet in the numerical force-free solutions shown in Fig. 6.

$$W_{\rm tot}(\theta) = \sin^2\theta B_r^2(\theta)$$

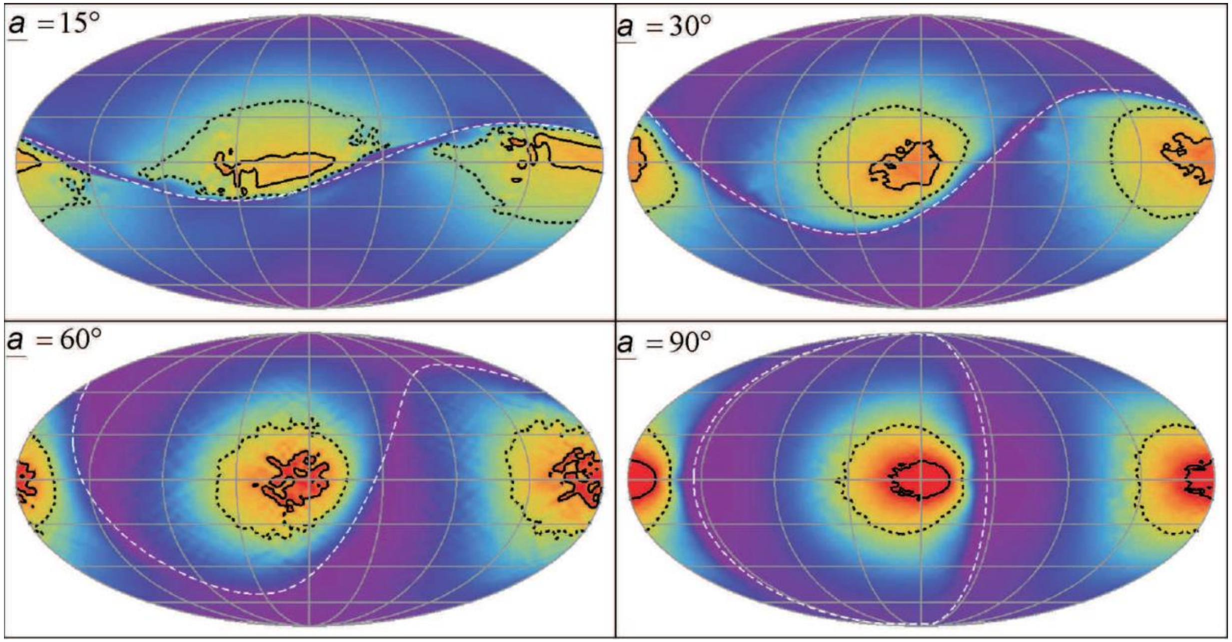


A.Tchekhovskoy, A.Philippov, A.Spitkovsky MNRAS, **457**, 3384 (2015)

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c).$$

Wind – not a split-monopole



C.Kalapotharakos, I.Contopoulos, D.Kazanas, MNRAS, **420**, 2793 (2012)

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c).$$

Asymtotic solution for orthogonal wind

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c).$$

Radial outflow

No current sheet

Asymtotic solution for orthogonal wind

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$
$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c).$$

Generalization

$$\psi(\theta, \varphi - \Omega t + \Omega r/c)$$

$$\begin{cases} B_r \approx B_\mathrm{L} \dfrac{R_\mathrm{L}^2}{r^2} \sin\theta \cos\left(\varphi - \Omega t + \dfrac{\Omega r}{c} + \varphi_0\right), \\ B_\theta \approx \dfrac{1}{r\sin\theta} \dfrac{\partial\psi}{\partial\varphi}, \\ B_\varphi \approx -B_\mathrm{L} \dfrac{\Omega R_\mathrm{L}^2}{cr} \sin^2\theta \cos\left(\varphi - \Omega t + \dfrac{\Omega r}{c} + \varphi_0\right) - \dfrac{1}{r} \dfrac{\partial\psi}{\partial\theta}, \\ E_r \approx 0, \\ E_\theta \approx -B_\mathrm{L} \dfrac{\Omega R_\mathrm{L}^2}{cr} \sin^2\theta \cos\left(\varphi - \Omega t + \dfrac{\Omega r}{c} + \varphi_0\right) - \dfrac{1}{r} \dfrac{\partial\psi}{\partial\theta}, \\ E_\varphi \approx -\dfrac{1}{r\sin\theta} \dfrac{\partial\psi}{\partial\varphi}. \end{cases}$$

What is the current system?

Current losses

Direct current losses?

$$
\begin{aligned}
K_{||} &= -c_{||}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s}, \\
K_{\perp} &= -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.
\end{aligned}
$$

Pulsar evolution: direst current losses?

$$
\begin{aligned}
I_{\rm r}\,\dot{\Omega} &= K_{||}\cos\chi + K_{\perp}\sin\chi, \\
I_{\rm r}\Omega\,\dot{\chi} &= K_{\perp}\cos\chi - K_{||}\sin\chi,
\end{aligned}
$$

$$
\begin{aligned}
K_{||} &= -c_{||}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s}, \\
K_{\perp} &= -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.
\end{aligned}
$$

$$
K_{\perp}^{A} \approx \left(\frac{\Omega R}{c}\right) K_{||}^{A}
$$

$$
i_{\rm s} \approx i_{\rm a} \approx 1
$$

VB, A.V.Gurevich, Ya.N.Istomin,
JETP **58**, 235 (1983)

Pulsar evolution: direst current losses?

$$I_r \dot{\Omega} = K_{\parallel} \cos\chi + K_{\perp} \sin\chi,$$
$$I_r \Omega \dot{\chi} = K_{\perp} \cos\chi - K_{\parallel} \sin\chi,$$

$$K_{\parallel} = -c_{\parallel} \frac{B_0^2 \Omega^3 R^6}{c^3} i_s,$$
$$K_{\perp} = -c_{\perp} \frac{B_0^2 \Omega^3 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_a.$$

$$K_{\perp}^A \approx \left(\frac{\Omega R}{c}\right) K_{\parallel}^A$$

$$i_s \approx i_a \approx 1$$

BGI

$$I_r \dot{\Omega} = K_{\parallel}^A + [K_{\perp}^A - K_{\parallel}^A] \sin^2\chi,$$
$$I_r \Omega \dot{\chi} = [K_{\perp}^A - K_{\parallel}^A] \sin\chi \cos\chi.$$

$$i_s = i_s^A \cos\chi,$$
$$i_a = i_a^A \sin\chi.$$

$$i_A \sim (\Omega R/c)^{-1}$$

Princeton (MHD)

Pulsar evolution: direst current losses?

$$I_{\rm r}\,\dot{\Omega} = K_{\parallel}\cos\chi + K_{\perp}\sin\chi,$$
$$I_{\rm r}\Omega\,\dot{\chi} = K_{\perp}\cos\chi - K_{\parallel}\sin\chi,$$

$$K_{\parallel} = -c_{\parallel}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s},$$
$$K_{\perp} = -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.$$

$$K_{\perp}^{A} \approx \left(\frac{\Omega R}{c}\right)K_{\parallel}^{A}$$

$$i_{\rm s} \approx i_{\rm a} \approx 1$$

BGI

$$I_{\rm r}\dot{\Omega} = K_{\parallel}^{A} + [K_{\perp}^{A} - K_{\parallel}^{A}]\sin^2\chi,$$
$$I_{\rm r}\Omega\dot{\chi} = [K_{\perp}^{A} - K_{\parallel}^{A}]\sin\chi\cos\chi.$$

$$i_{\rm s} = i_{\rm s}^{A}\cos\chi,$$
$$i_{\rm a} = i_{\rm a}^{A}\sin\chi.$$

$$i_{\rm A} \sim (\Omega R/c)^{-1}$$

?

Princeton (MHD)

How to write down the current

Drift approximation

$$\mathbf{j} = c\,\rho_{\rm e}\frac{[\mathbf{E}\times\mathbf{B}]}{B^2} + a\,\mathbf{B}$$

$$\boxed{\mathbf{j} = \rho_{\rm e}\,[\mathbf{\Omega}\times\mathbf{r}] + i_{\|}\,\mathbf{B}}$$

$$\boxed{\mathbf{j} = \frac{(\mathbf{B}\cdot\nabla\times\mathbf{B} - \mathbf{E}\cdot\nabla\times\mathbf{E})\mathbf{B} + (\nabla\cdot\mathbf{E})\mathbf{E}\times\mathbf{B}}{B^2}}$$

$$(\nabla i_{\|}\,\mathbf{B}) = 0$$

Mestel, BGI

Gruzinov

$$i_{\rm a} \sim \left(\frac{\Omega R}{c}\right)^{-1/2}$$

No point 1

$$B_r \approx B_0 \frac{R^2}{r^2} \sin\theta \cos(\varphi - \Omega t + \Omega r/c),$$

$$B_\varphi = E_\theta \approx -B_0 \frac{\Omega R^2}{cr} \sin^2\theta \cos(\varphi - \Omega t + \Omega r/c).$$

In the wind

$$i_\| = -3\,\frac{\Omega}{c}\cos\theta$$

Polar cap

$$i_{\rm a}^{\rm A} \approx f_*^{-1/2}\left(\frac{\Omega R}{c}\right)^{-1/2}$$

Current is too small!

What is the current system?

Current losses

1. Direct current losses

$$K_{\|} = -c_{\|}\frac{B_0^2\Omega^3R^6}{c^3}i_{\rm s},$$
$$K_{\perp} = -c_{\perp}\frac{B_0^2\Omega^3R^6}{c^3}\left(\frac{\Omega R}{c}\right)i_{\rm a}.$$

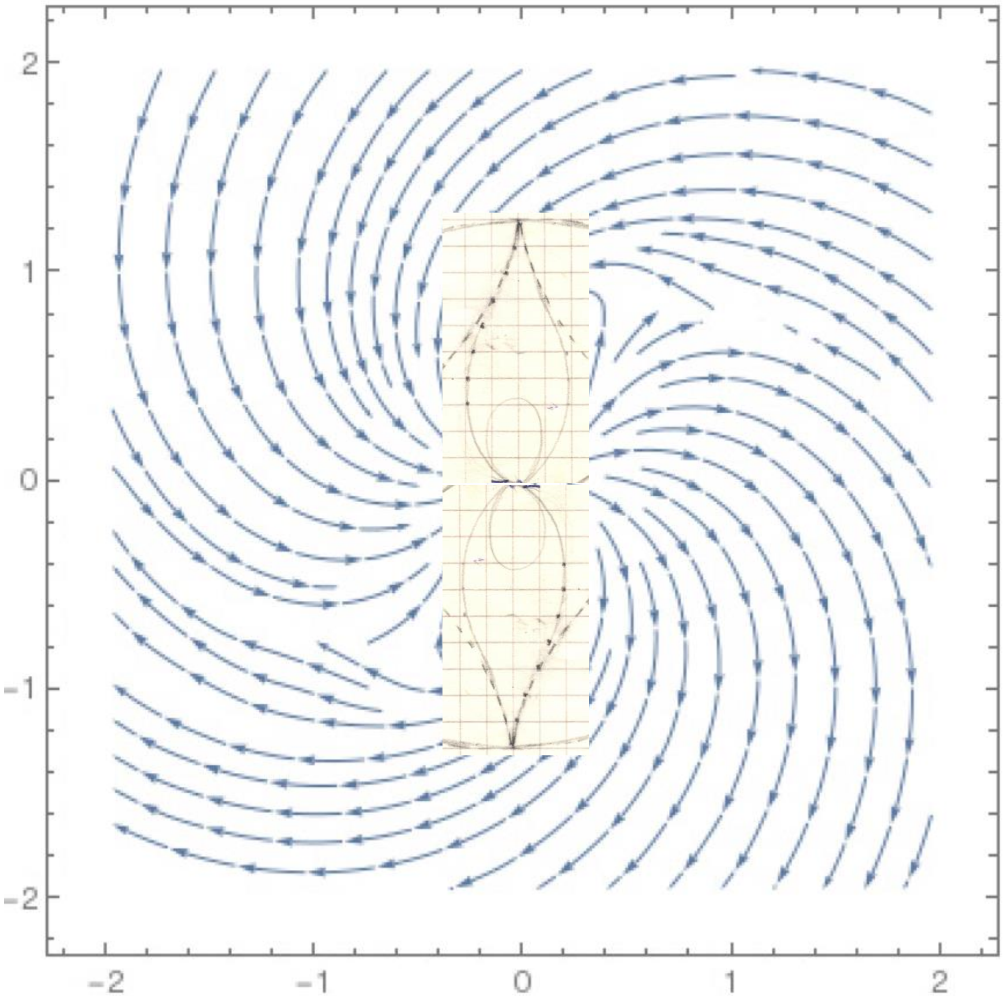
2. Mismatch
('second term')

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{B}\mathbf{n})\,{\rm d}o = \frac{R^3}{4\pi}\int\{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

3. Additional separatrix current

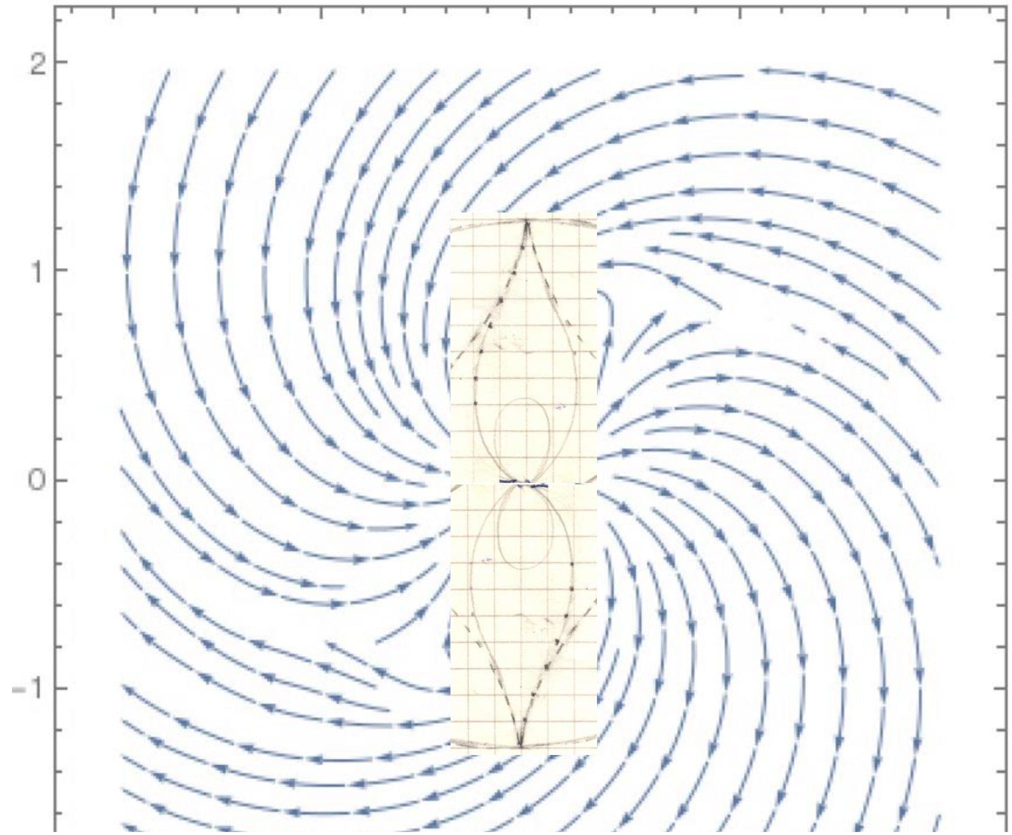
Point 2?

Mismatch



Point 2?

Mismatch



”Second term”

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\mathrm{s}}\left(\mathbf{B}\mathbf{n}\right)\mathrm{d}o = \frac{R^3}{4\pi}\int\{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,\mathrm{d}o$$

sarcastic maliciously smiling

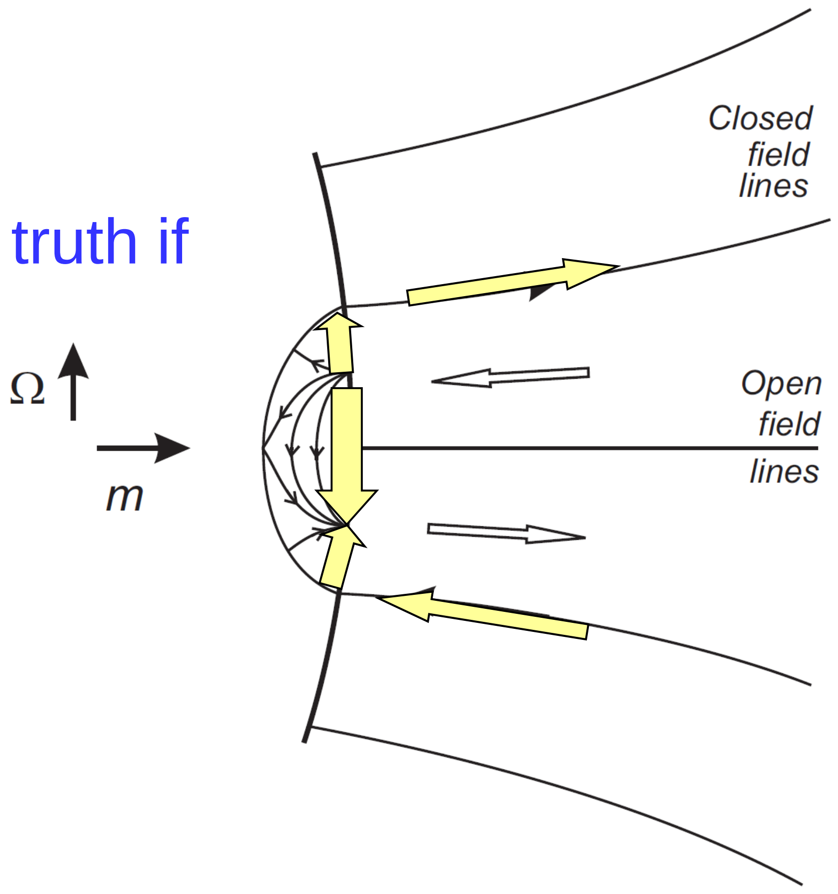
Point 3?

VB, E.E.Nokhrina. Astron. Letters, **30**, 685 (2004)

$$W_{\rm tot} = \frac{\Omega R^3}{c} \int J_\theta B_n {\rm d}o$$

Direct current losses are the only truth if

- No longitudinal currents in close magnetosphere (no additional current along the separatrix)



$$I_{\rm sep} = 3/4\, I_{\rm vol}$$

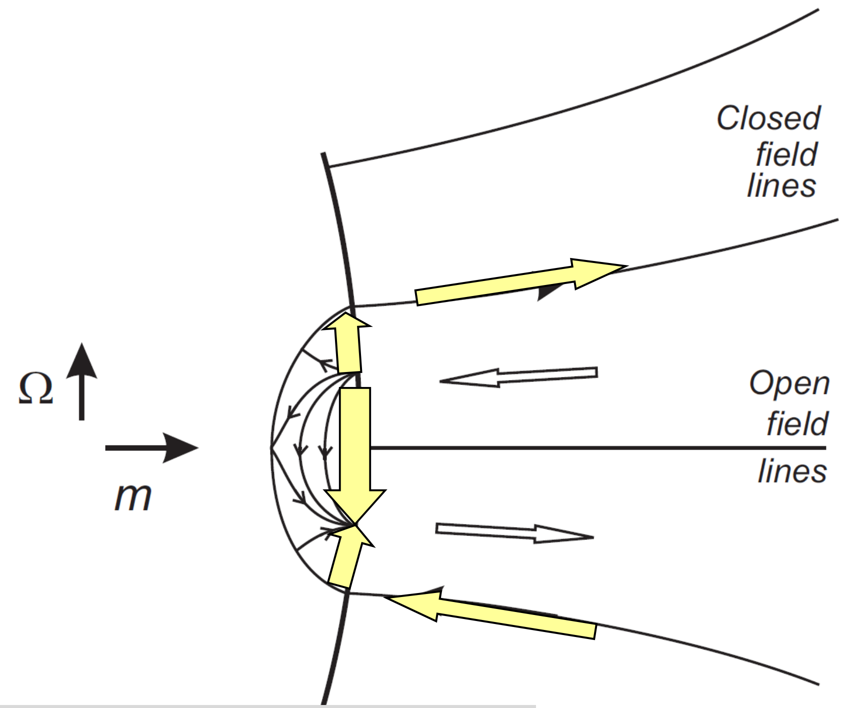
$$< J_\theta > = 0$$

$$< B_t > = 0$$

Point 3?

VB, MHD Flows in Compact Astrophysical Objects, Springer (2010)

$$W_{\mathrm{tot}} = \frac{\Omega R^3}{c} \int J_\theta B_n \mathrm{d}o$$



$$I_{\mathrm{sep}} = 3/4\; I_{\mathrm{vol}}$$

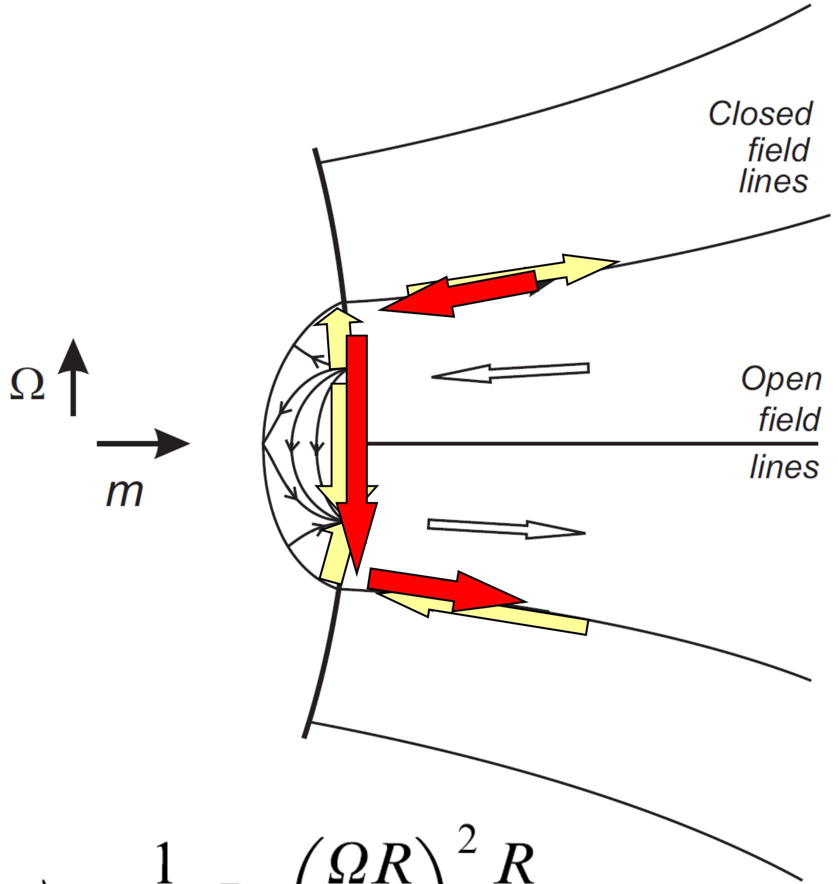
Problem 2.16. Show that in this case the total current I_{sep} flowing along the separatrix is 3/4 the total bulk current I_{bulk} flowing in the region of the open field lines:

$$\frac{I_{\mathrm{sep}}}{I_{\mathrm{bulk}}} = -\frac{3}{4}. \qquad (2.160)$$

Point 3?

$$W_{\text{tot}} = \frac{\Omega R^3}{c} \int J_\theta B_n \mathrm{d}o$$

Additional current along the separatrix.



$$I_{\text{sep}} < 3/4\ I_{\text{vol}}$$

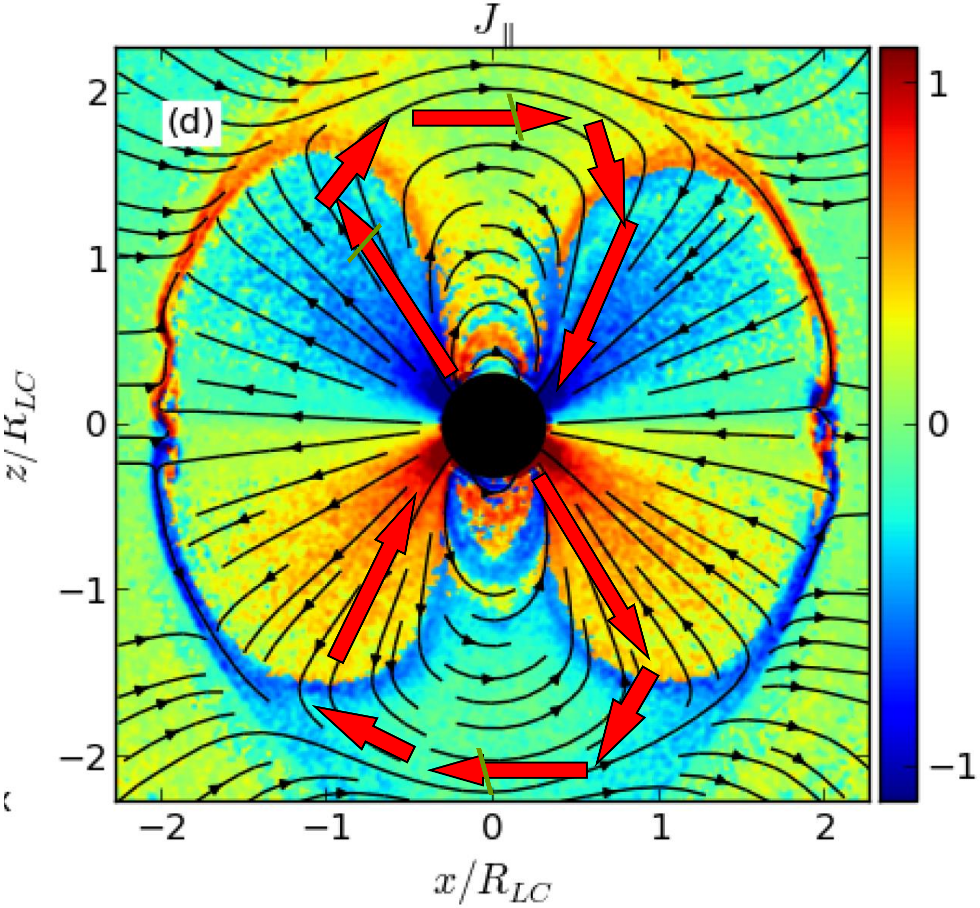
$$< J_\theta > \neq 0$$

$$\langle B_\varphi \rangle = \frac{1}{f_*} B_0 \left(\frac{\Omega R}{c} \right)^2 \frac{R}{r}$$

Point 3?

$$W_{\rm tot} = \frac{\Omega R^3}{c} \int J_\theta B_n {\rm d}o$$

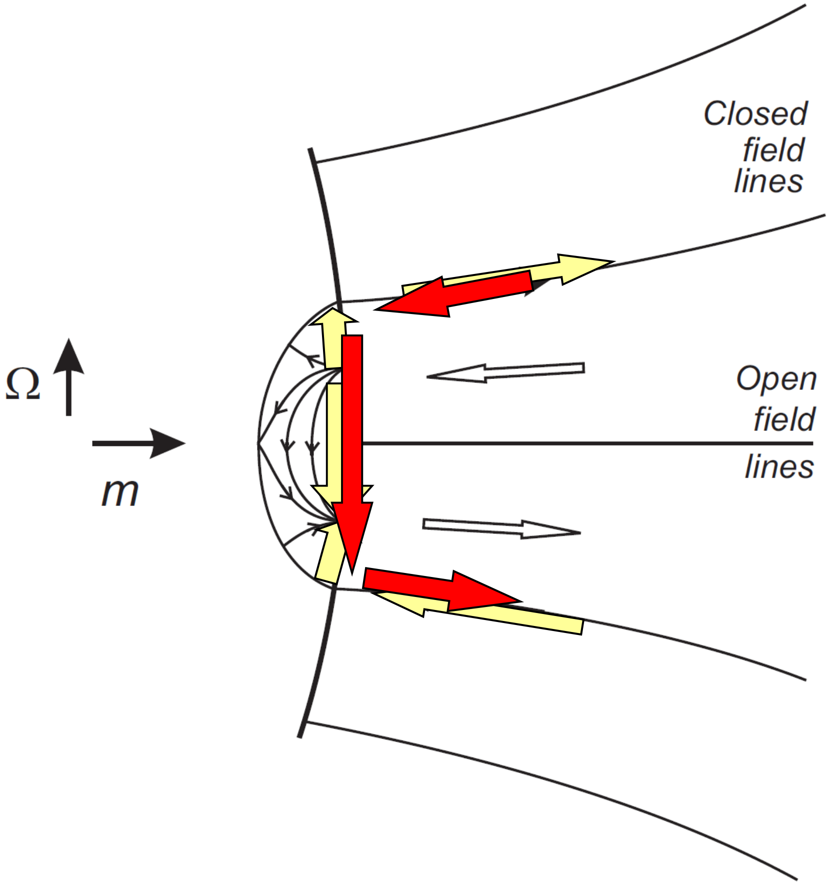
Direction corresponds
to energy losses.



Point 3?

X.-N. Bai, A.Spitkovsky ApJ, **715**, 1282 (2010)

$$I_{\mathrm{sep}} = 20\%\ I_{\mathrm{vol}}$$



$\mathrm{i}_{\mathrm{A}} = j_{\|} / \rho_{\mathrm{GJ}} c$

How to check?

Current losses

1. Direct current losses

$$K_{\parallel} = -c_{\parallel} \frac{B_0^2 \Omega^3 R^6}{c^3} i_{\rm s},$$
$$K_{\perp} = -c_{\perp} \frac{B_0^2 \Omega^3 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\rm a}.$$

2. Mismatch
('second term')

$$\mathbf{K} = \frac{R^3}{c}\int \mathbf{J}_{\rm s}\,(\mathbf{B}\mathbf{n})\,{\rm d}o = \frac{R^3}{4\pi}\int \{[\mathbf{n}\times\mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n}\times\mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\}\,{\rm d}o$$

3. Additional separatrix current

How to check?

Current losses

1. Direct current losses

$$K_{\parallel} = -c_{\parallel} \frac{B_0^2 \Omega^3 R^6}{c^3} i_{\rm s},$$
$$K_{\perp} = -c_{\perp} \frac{B_0^2 \Omega^3 R^6}{c^3} \left(\frac{\Omega R}{c}\right) i_{\rm a}.$$

2. Mismatch
('second term') ALL SURFACE WORKS

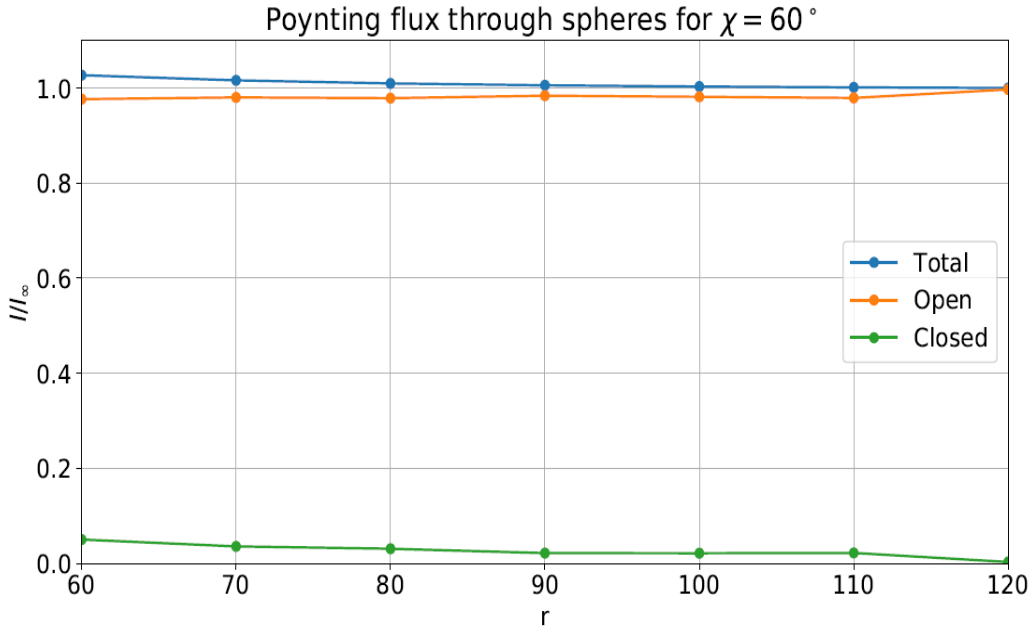
$$\mathbf{K} = \frac{R^3}{c} \int \mathbf{J}_{\rm s} \, (\mathbf{B}\mathbf{n}) \, {\rm d}o = \frac{R^3}{4\pi} \int \{[\mathbf{n} \times \mathbf{B}^{(3)}](\mathbf{B}^{(0)}\mathbf{n}) + [\mathbf{n} \times \mathbf{B}^{(0)}](\mathbf{B}^{(3)}\mathbf{n})\} \, {\rm d}o$$

3. Additional separatrix current

POLAR CAP ONLY

Direct check

VB, A.K.Galishnikova, E.M.Novoselov, A.A.Philippov, M.M.Rashkovetskyi JPhys: Conf. Series, **932**, 012012 (2017)

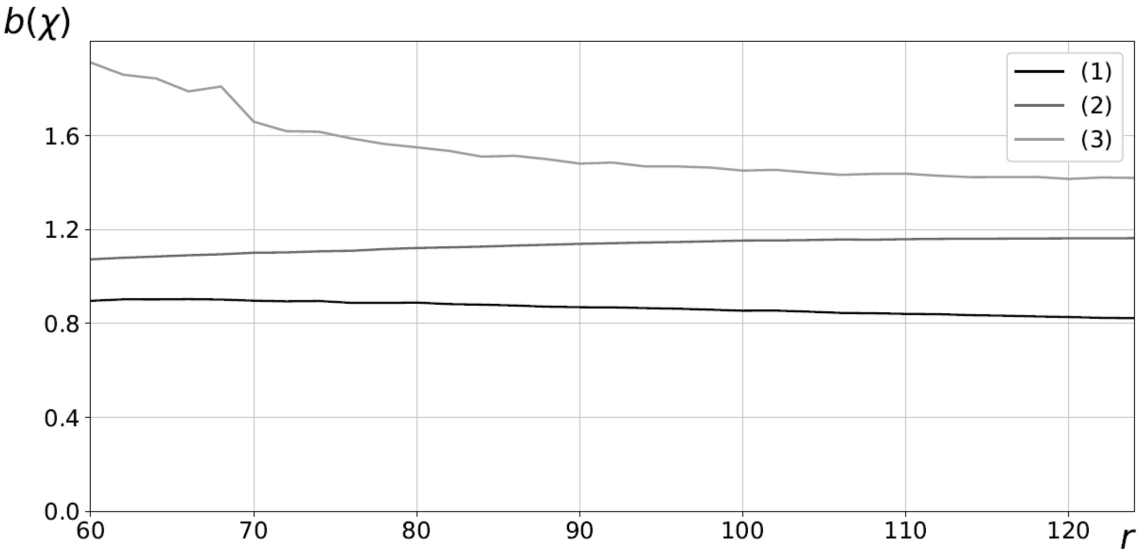


$R = 50$

$R_L = 500$

Direct check

M.M.Rashkovetskyi, VB, A.K.Galishnikova, E.M.Novoselov, A.A.Philippov (2018)



$$< B_\varphi >= b(\chi)\frac{1}{f_*} B_0 \left(\frac{\Omega R}{c}\right)^2 \frac{R}{r}$$

$$b(\chi) = \left[\frac{k_1 + k_2}{2} - \frac{f_*^{5/2}}{32}\left(\frac{\Omega R}{c}\right)^{1/2}\right] \sin\chi$$

$$W_{\rm tot}^{\rm MHD} \approx \frac{1}{4}\frac{B_0^2\Omega^4 R^6}{c^3}(k_1 + k_2 \sin^2\chi)$$

χ	30°	60°	90°
$b(\chi)$ (num)	0.8	1.2	1.4
$b(\chi)$ (anal)	0.6 ± 0.1	1.0 ± 0.1	1.2 ± 0.1

Direct check

M.M.Rashkovetskyi, VB, A.K.Galishnikova, E.M.Novoselov, A.A.Philippov (2018)

$$\frac{I_{\rm sep}}{I_{\rm vol}} = \frac{3}{4} - \frac{2}{f_*^{3/2}} \left(\frac{\Omega R}{c}\right)^{1/2}$$

$$I_{\rm sep} \sim 0.3\, I_{\rm vol}$$

X.-N. Bai, A.Spitkovsky ApJ **715**, 1282 (2010)

$$\boxed{I_{\rm sep} = 20\%\, I_{\rm vol}}$$

Fortunately for us

$$
\begin{aligned}
I_{\rm r}\dot{\Omega} &= K_{||}^{A} + [K_{\perp}^{A} - K_{||}^{A}]\sin^2\chi, \\
I_{\rm r}\Omega\dot{\chi} &= [K_{\perp}^{A} - K_{||}^{A}]\sin\chi\cos\chi.
\end{aligned}
$$

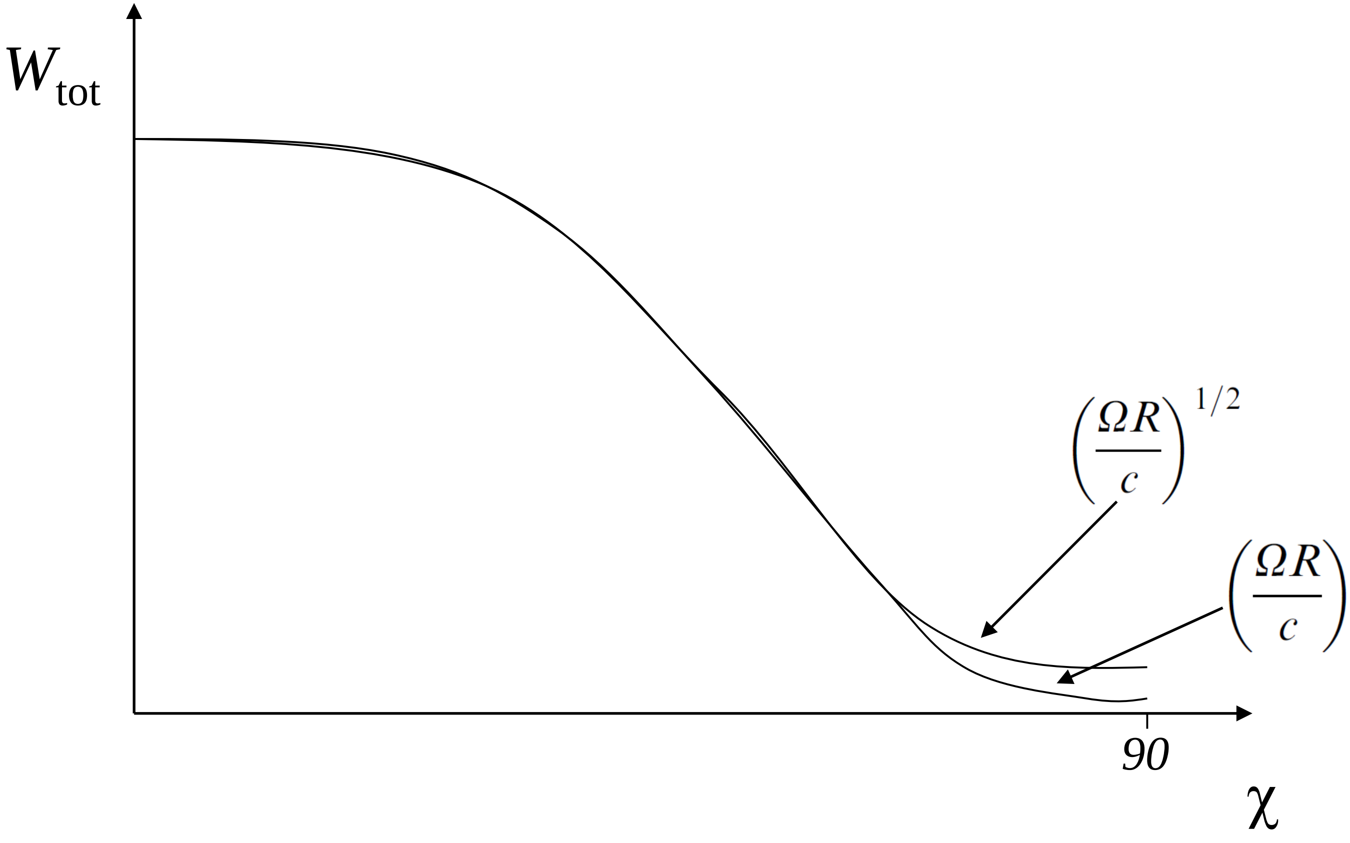
$$
K_{\perp}^{\rm mag} = -A\,\frac{B_0^2\Omega^3 R^6}{c^3}\,i_{\rm a}
$$

$$
\boxed{A \approx 2\left(\frac{\Omega R}{c}\right)^{1/2}}
$$

One can neglect additional losses for GJ current

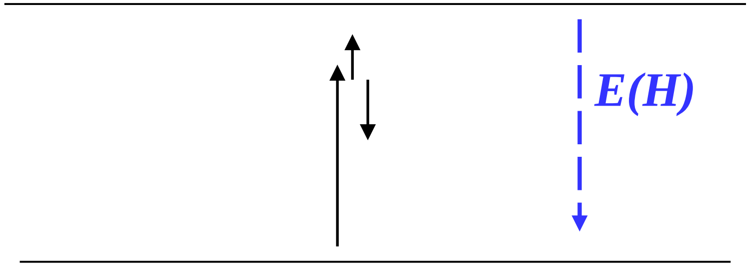
BGI correction

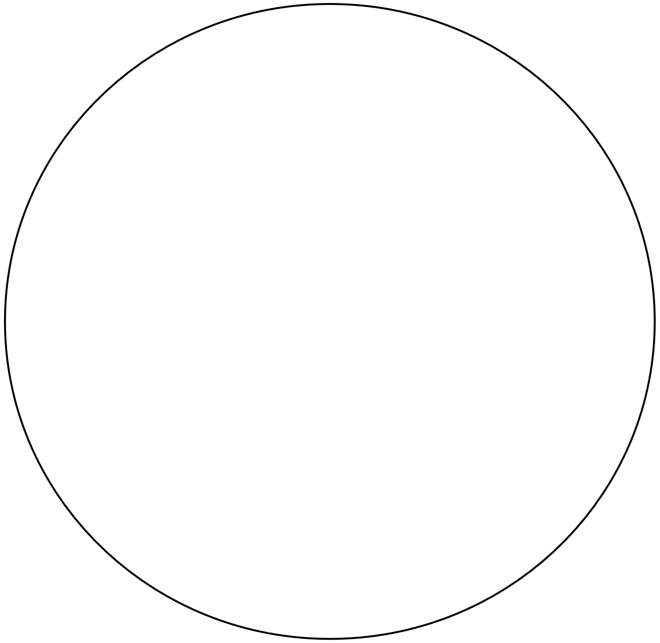
Some difference for orthogonal pulsars only



The last remark

Possible restriction of the longitudinal current

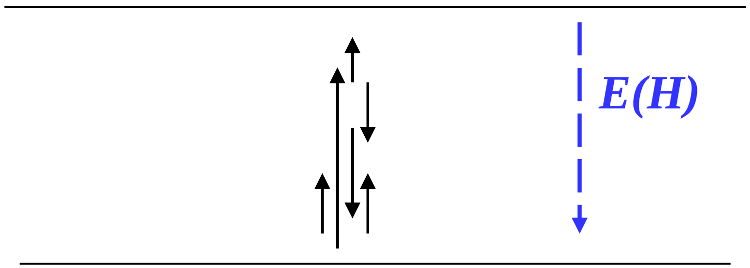




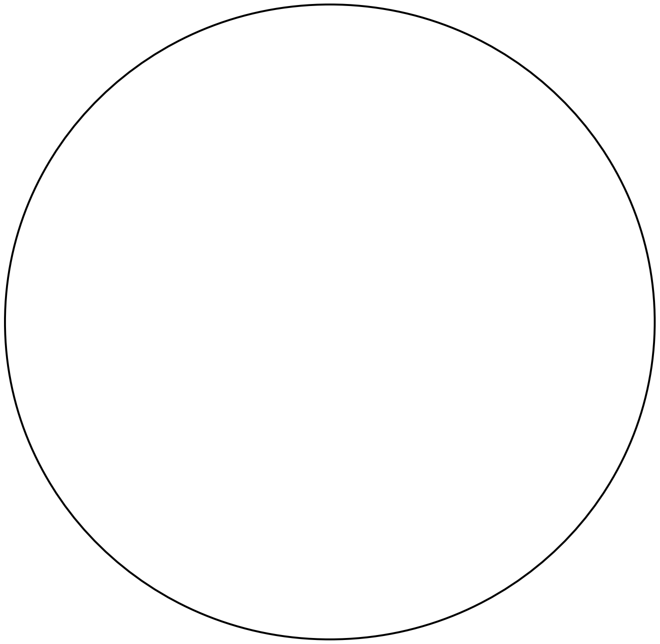
VB, Sov. Astron. **26**, 443 (1982)

The last remark

Possible restriction of the longitudinal current



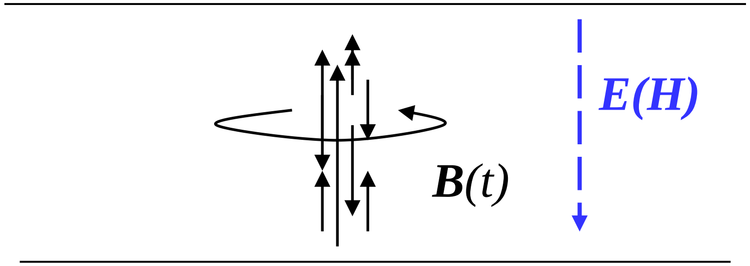
$$I = I_0 \exp\left(t/\tau\right)$$

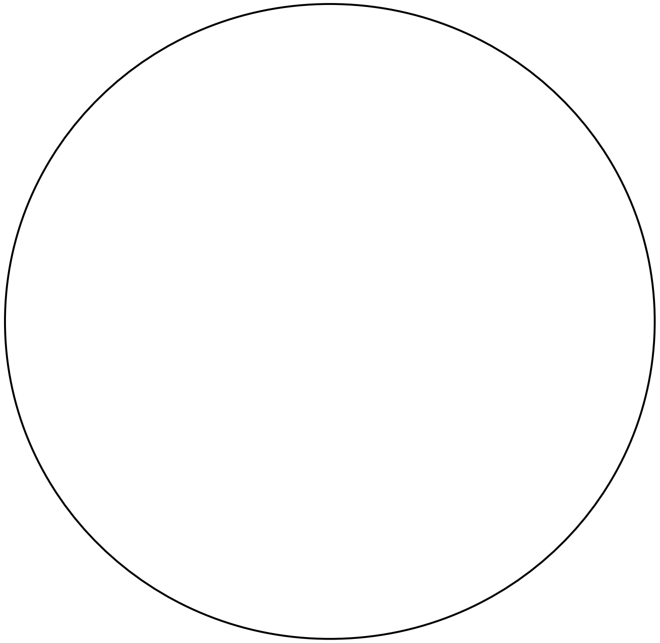


VB, Sov. Astron. **26**, 443 (1982)

The last remark

Possible restriction of the longitudinal current





$$I = I_0 \exp(t/\tau)$$

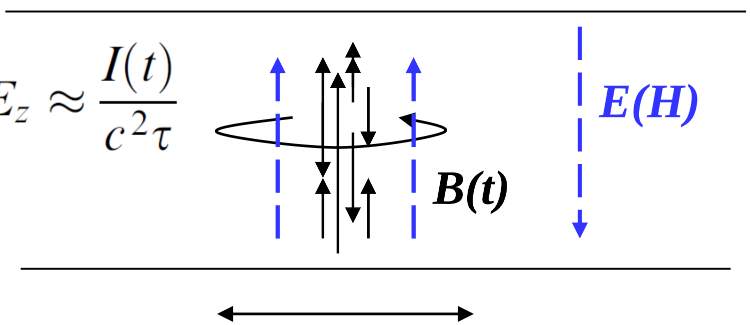
$$B_\varphi(t) = 2I(t)/(cr_\perp)$$

VB, Sov. Astron. **26**, 443 (1982)

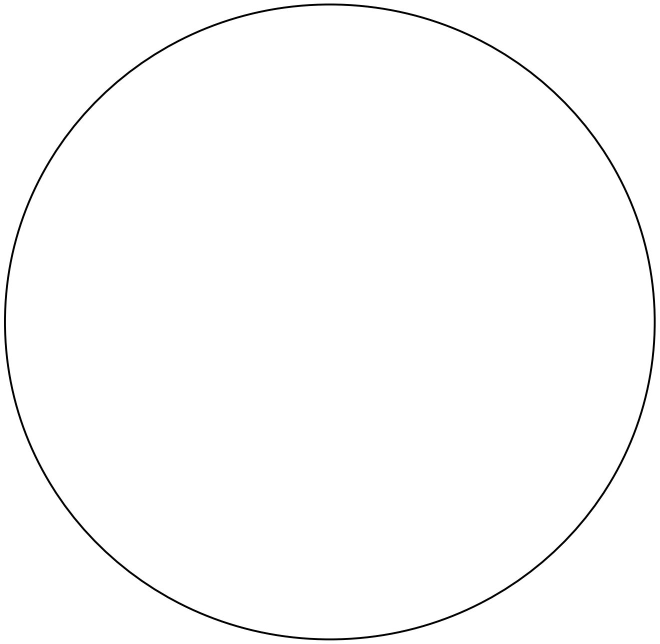
The last remark

Possible restriction of the longitudinal current

$$\delta E_z \approx \frac{I(t)}{c^2 \tau}$$



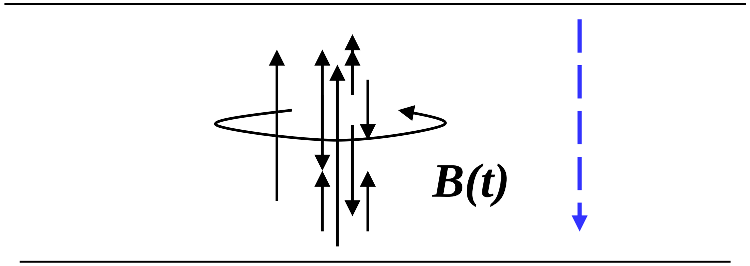
$$r_\perp \sim H$$



VB, Sov. Astron. **26**, 443 (1982)

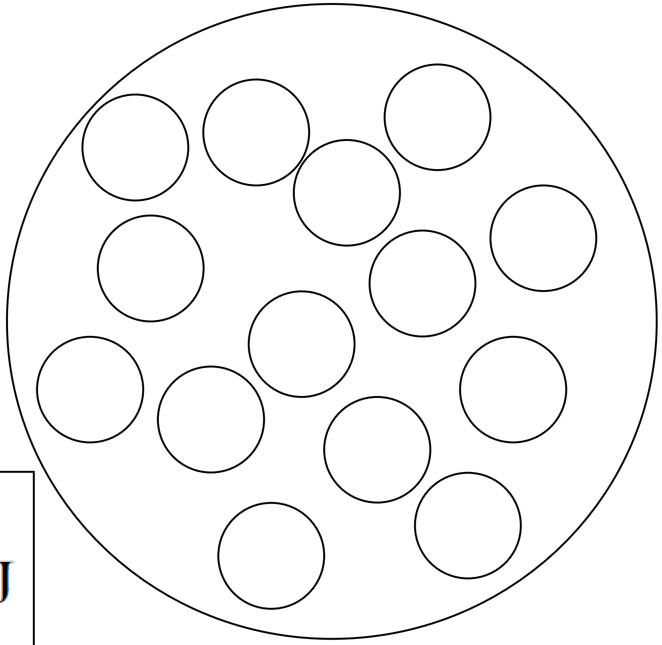
The last remark

Possible restriction of the longitudinal current



$$I_{\max} \approx \frac{c\tau}{H} c\rho_{\mathrm{GJ}} H^2$$

$$I_{\mathrm{tot}} \approx \frac{c\tau}{H} I_{\mathrm{GJ}}$$



$$N \approx R_0^2 / H^2$$

VB, Sov. Astron. **26**, 443 (1982)

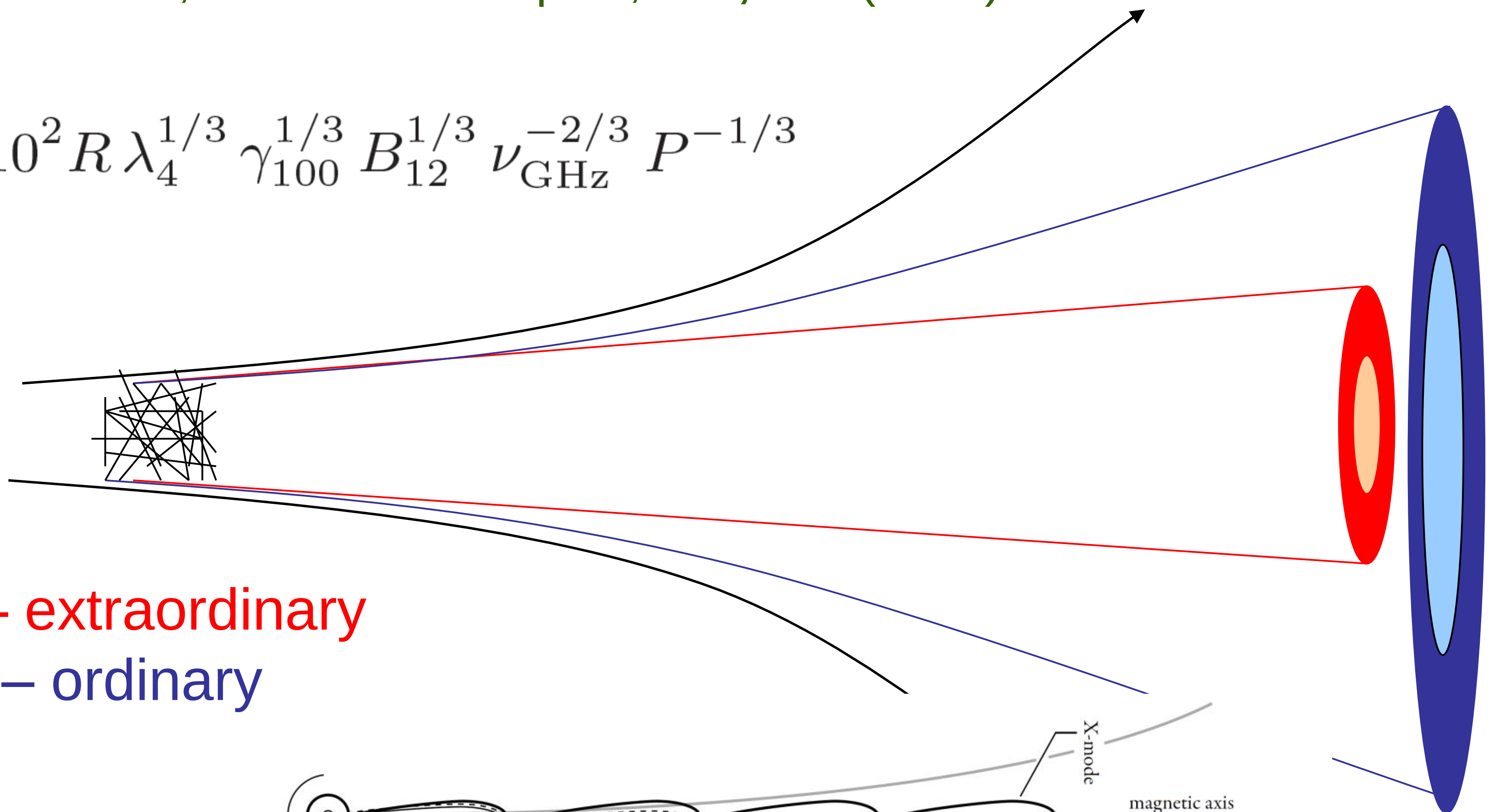
30 years!

VB, A.V.Gurevich, Ya.N.Istomin, Astrophys. Space Sci., **146,** 205 (1988)

Core & Conal

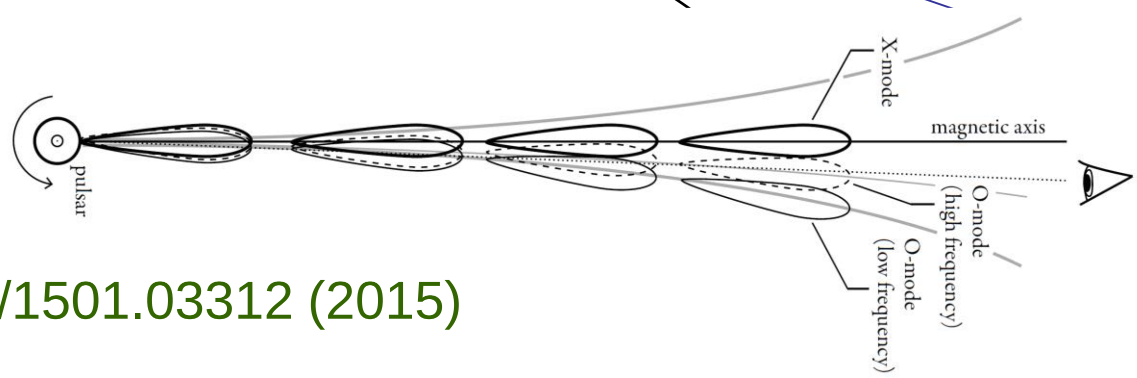
VB, A.V.Gurevich, Ya.N.Istomin. ApSS, **146,** 205 (1988)

$$r_{\mathrm{A}} \approx 10^2 R\,\lambda_4^{1/3}\,\gamma_{100}^{1/3}\,B_{12}^{1/3}\,\nu_{\mathrm{GHz}}^{-2/3}\,P^{-1/3}$$

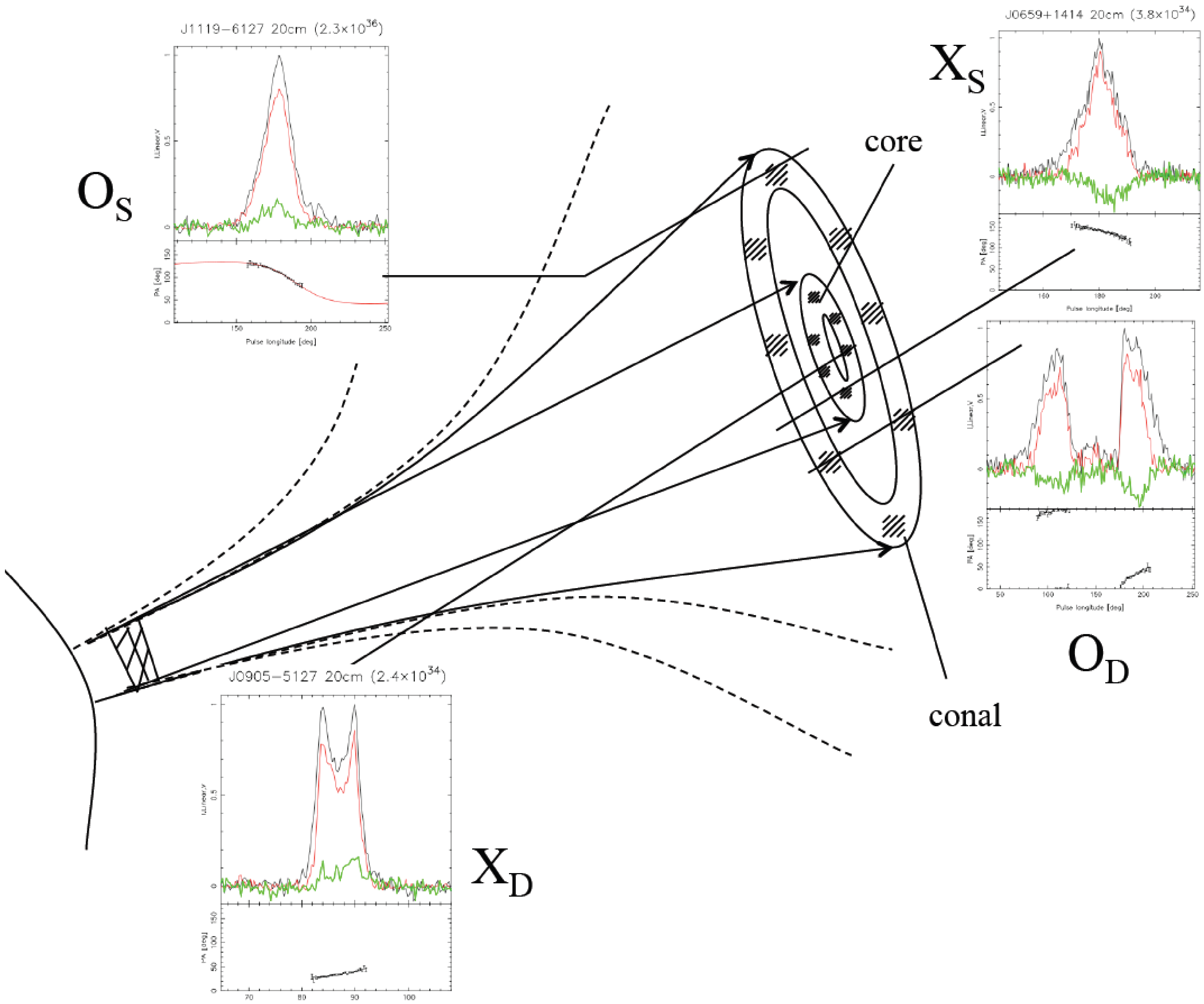


Core – extraordinary

Conal – ordinary



A.Noutsos et al. ArXiv/1501.03312 (2015)



J1119−6127 20cm (2.3×10^36)
O_S
Pulse longitude [deg]
J0659+1414 20cm (3.8×10^34)
X_S
core
O_D
conal
J0905−5127 20cm (2.4×10^34)
X_D

Core & Conal

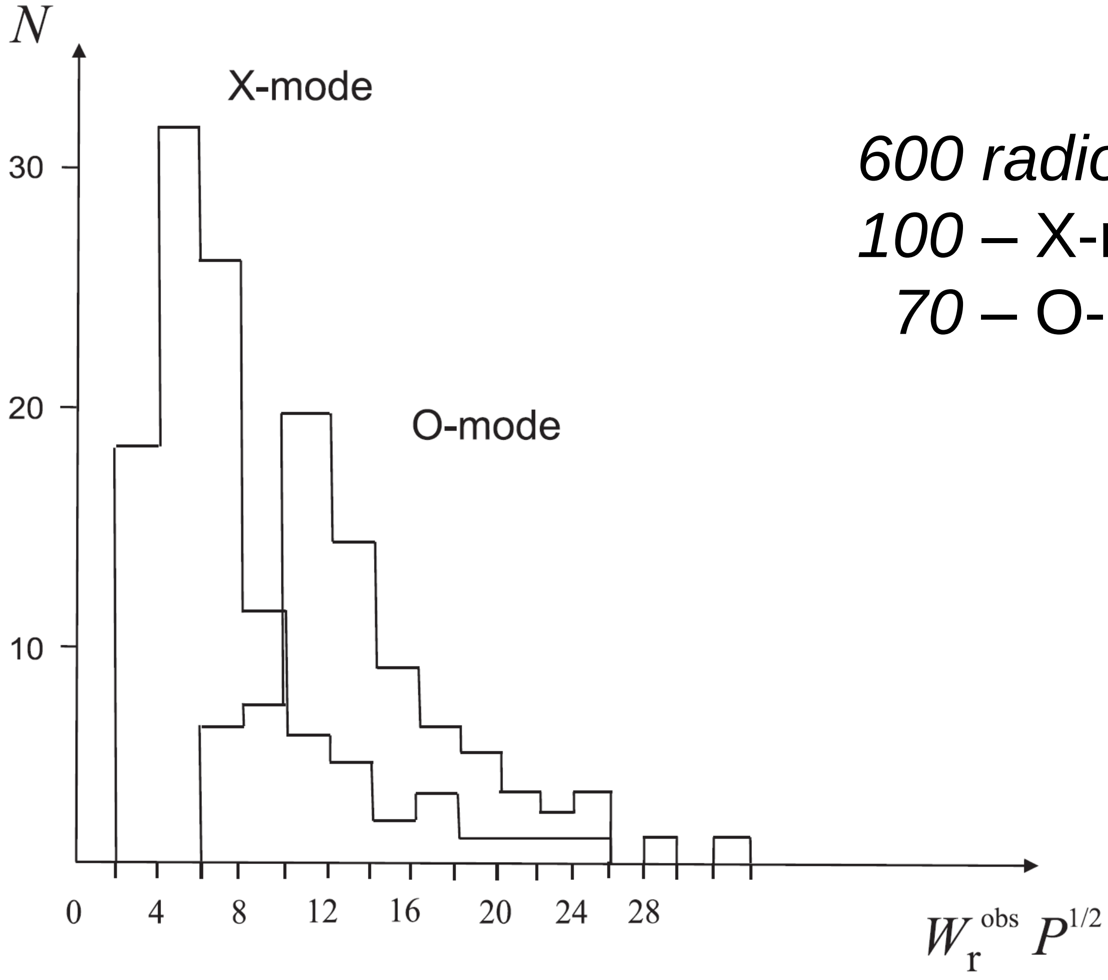
VB, A.V.Gurevich, Ya.N.Istomin. ApSS, **146,** 205 (1988)

$$W^{(1)}_{\boldsymbol{X}} \approx 3.6^\circ \left(\frac{P}{1\,s}\right)^{-3/4} \left(\frac{\nu}{1\mathrm{GHz}}\right)^{-1/2} \left(\frac{\lambda}{10^4}\right)^{1/8} \left(\frac{B}{10^{12}\mathrm{G}}\right)^{1/8} \left(\frac{\gamma}{100}\right)^{7/8},$$

$$W^{(2)}_{\boldsymbol{O}} \approx 7.8^\circ \left(\frac{P}{1\,s}\right)^{-0.43} \left(\frac{\nu}{1\mathrm{GHz}}\right)^{-0.14} \left(\frac{\lambda}{10^4}\right)^{0.07} \left(\frac{B}{10^{12}G}\right)^{0.07} \left(\frac{\gamma}{100}\right)^{-0.11},$$

$$W^{(2)} \approx 10^\circ \left(\frac{P}{1\,s}\right)^{-0.5} \left(\frac{\nu}{1\mathrm{GHz}}\right)^{-0.29} \left(\frac{\lambda}{10^4}\right)^{0.1} \left(\frac{B}{10^{12}\mathrm{G}}\right)^{0.1} \left(\frac{\gamma}{100}\right)^{-0.05}.$$

O- and X-modes



600 radio pulsars
100 – X-mode
70 – O-mode

S.Johnston, M.Kerr, MNRAS, **474**, 4629 (2018)

Conclusion

- Separatrix current can play important role
- Beskin, Gurevich & Istomin are still alive