Problem set 9, Due Apr 21

April 19, 2011

- Two identical beams with Maxwellian distribution (thermal velocity V_T) propagate in opposite directions with velocity $v = \pm v_0$. Find a condition when two stream/bump in tail instability is suppressed.

$$f = \exp\{-(v \pm v_0)^2/(2v_T^2)\}$$
$$\partial^2_{v^2} f(v=0) \to v_0 = V_T \tag{1}$$

- Plasma has a distribution function $f = \exp^{-v/v_T}/v_T$ (f is one-dimensional distribution). Find (i) thermal corrections to the real part of dispersion relation of Langmuir waves in the limit $v_{ph} = \omega/k \gg v_T$; (ii) damping rate $\Gamma(k)$ as a function of $z = kr_D$, where r_D is Debye radius.

$$< v^2 >= 2v_T^2, \omega^2 = \omega_p^2(1 + 6(kv_T/\omega_p)^2)$$
$$\epsilon_\parallel = 1 - \frac{\omega_p^2}{\omega^2}(1 + 3k^2 < v^2 > /\omega^2) - i\pi\frac{4\pi e^2}{m\omega k}\int vf'\delta(v - \omega/k)dv \to$$
$$(-i\pi)\frac{4\pi e^2}{m\omega k} vf'(v = \omega/k) = (\omega \to kv) = (-i\pi)\frac{\omega_p^2}{k^2} f' =$$
$$(-i\pi)\frac{\omega_p^2}{k^2}\frac{e^{-\omega/k/(v_T)}}{v_T^2} \tag{2}$$

- An ion beam is propagating along magnetic field B_0 with velocity v_0. Plasma has density ρ. What is a condition for a beam to emit an Alfven wave at anomalous cyclotron resonance $\omega - k_\parallel v_\parallel = -\omega_B/\gamma$ (note the minus sign on the rhs). What is the frequency of the emitted Alfven waves?

$$v_A k - k v_0 = -\omega_B$$
$$v_0 > v_A,\ k = \frac{\omega_B}{v_0 - v_A}$$
$$\omega = v_A k = v_A \frac{\omega_B}{v_0 - v_A} \tag{3}$$