Problem set 7, Due Mar 24

March 24, 2011

- Find relation between helicity and magnetic energy in a constant-α force-free fields. Hint: derive an expression similar to curl$\mathbf{B} = \alpha\mathbf{B}$ for the vector potential $\mathbf{A}$.

  For constant-α force-free fields

$$\begin{aligned}
&\mathrm{curl}\,\mathbf{B} = curl curl\mathbf{A} = \alpha B = \alpha curl\mathbf{A} \\
&\mathrm{curl}\,\mathbf{A} = \alpha\mathbf{A} \\
&\mathbf{A}\cdot\mathbf{B} = \frac{1}{\alpha} curl\mathbf{A}\cdot\mathbf{B} = \frac{1}{\alpha}B^2
\end{aligned} \tag{1}$$

- Estimate magnetic Reynolds (Lundquist) number for the Sun. Solar mass is $M_\odot = 2\times10^{33}$ g, Solar radius is $R_\odot = 7\times10^{10}$ cm, magnetic field $B = 1$ G.

  Magnetic diffusivity κ

$$\partial_t\mathbf{B} = \nabla\times\mathbf{v}\times\mathbf{B} + \kappa\Delta\mathbf{B} \tag{2}$$

  Resistivity η

$$E = \eta j \tag{3}$$

  Conductivity σ

$$\sigma = \frac{1}{\eta} \tag{4}$$

  Magnetic diffusivity κ

$$\kappa = \frac{c^2}{4\pi\sigma} = \frac{c\eta}{4\pi} \tag{5}$$

  Magnetic Reynolds = Lundquist number

$$Lu = \frac{V_A L}{\kappa} \tag{6}$$

$$\eta = \frac{m_e\nu_{coll}}{e^2 n} = \frac{e^2\sqrt{m_e}\ln\Lambda}{T^{3/2}}$$

$$
\begin{aligned}
T &= \frac{GMm_p}{R} \\
V_A &= \frac{B}{\sqrt{4\pi\rho}} \\
\rho &= M/((4\pi/3)R^3)
\end{aligned}
\tag{7}
$$

$$
Lu \approx \frac{(Gm_p)^{3/2} M_\odot R_\odot}{e^2 c^2 \sqrt{m_e}} \approx 10^{10} \tag{8}
$$

- A plasma with the conductivity η is embedded in the magnetic field of the kind $B = B_0 \tanh(x/L)\hat{y}$ at t = 0. Find the magnetic field evolution if there is no plasma flows, $\mathbf{v} \equiv 0$.

$$
\begin{aligned}
\partial_t B &= \kappa \partial_x^2 B \\
B(x,t) &= \int dx_0 G(x - x_0, t) B(x_0) \\
G(x - x_0, t) &= \frac{1}{\sqrt{4\pi\kappa t}} e^{-(x-x_0)^2/(4\kappa t)}
\end{aligned}
\tag{9}
$$

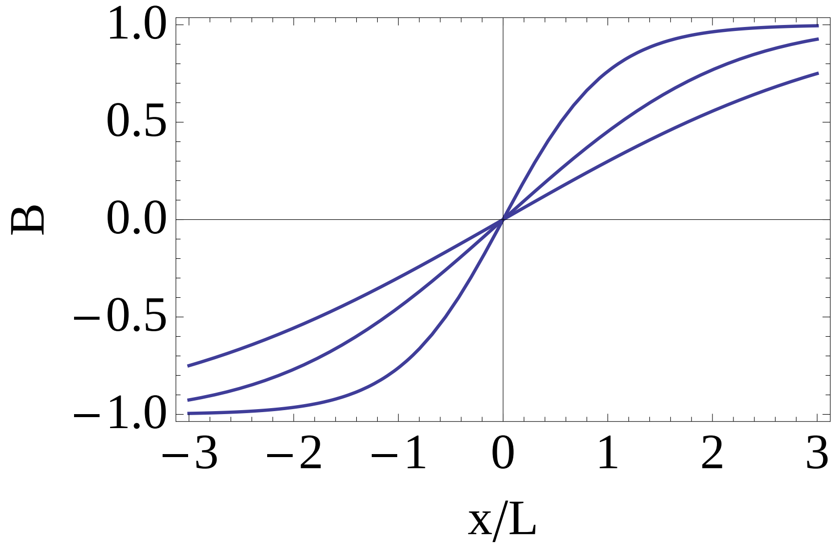

Figure 1: Evolution of the magnetic field .