Problem set 4. Due Feb 17 in class

February 14, 2011

- The Sun has has large scale magnetic field (of the order of 1 Gauss). Using the resistivity coefficient due to binary collisions derived in class, estimate resistive time scale of the Sun (time scale for resistive dissipation of magnetic field). Compare with e-e collision rate. Solar mass is $M_\odot = 2 \times 10^{33}$ g, Solar radius is $R_\odot = 7 \times 10^{10}$ cm. Hint: use Ohm's law and Poynting's theorem, neglect $E^2 \ll B^2$; in the Maxwell's equation $curl\mathbf{B} = (4\pi/c)j + \partial_t \mathbf{E}/c$, neglect $\partial_t \mathbf{E}/c$. To estimate average Solar temperature, use $p \sim \rho GM/R$.

$$\partial_t \left(B^2 + E^2\right)/(8\pi) = (\mathbf{B}\dot{\mathbf{B}} + \mathbf{E}\dot{\mathbf{E}})/(4\pi) = \nabla \cdot \mathbf{E} \times \mathbf{B}/(4\pi) - \mathbf{E} \cdot \mathbf{J} = \eta J^2$$

$E = \eta J$, and neglecting escape of magnetic field

$$\partial_t B^2/(8\pi) = \eta j^2$$

$$curl\mathbf{B} = (4\pi/c)j + \partial_t \mathbf{E}/c \approx (4\pi/c)j$$

$$\partial_t B^2/(8\pi) = \eta (c/4\pi)^2 (B/R)^2$$

$$\tau = \frac{(16\pi)^2}{\eta} \frac{R^2}{c^2}$$

$$\eta = \frac{m\nu_{coll}}{e^2 n} = \frac{e^2 \sqrt{m} \ln \Lambda}{T^{3/2}}$$

$$\tau = \frac{T^{3/2}}{e^2 \sqrt{m} \ln \Lambda} \frac{R^2}{c^2}$$

$$p \sim \rho GM/R = nT \to T = \frac{GMm_p}{R}$$

$$\tau = \frac{64\pi^3 G^{3/2} M^{3/2} {m_p}^{3/2} \sqrt{R}}{c^2 e^2 \sqrt{m_e}} = 10^{23} sec \qquad (1)$$

- Particle motion near neutral layer. Consider particle motion in magnetic field which reverse direction: $B_z = +B_0$ for $y > 0$ and $B_z = -B_0$ for $y < 0$. Find trajectory of a particle which starts at point $x_0 = 0$ $y_0 = V_0/\omega_B$, $V_{x,0} = V_0$, $V_{y,0} = 0$, where $\omega_B = eB_0/mc$.

- A laser at National Ignition Facility, or NIF, at Lawrence Livermore National Laboratory approaches a power of PetaWatts (10^{15} W) in a beam of cross-section radius $\approx$ 1 mm. Find the Lorentz factor of the quiver velocity of electrons in the laser electric field . Assume frequency of visible light. (Hint: $\gamma \approx a$).

$$
\begin{aligned}
&Power = \pi r^2 \frac{E^2}{8\pi} c \\
&a = \frac{eE}{m\omega c} = 2\sqrt{2}\frac{e\sqrt{Power}}{c^{3/2} m_e r\omega} = 2.8
\end{aligned}
\tag{2}
$$

For $\omega = 10^{14}$ rad/sec.