Problem set 1. Due Jan 25 in class

January 31, 2011

- At the Sun's photosphere, estimate ionization fraction, particle mean free path, $e-e$ and $i-i$ collision frequency, temperature equilibration times scale, electron plasma frequency, Debye radius, electron and ion cyclotron frequency and Larmor radii, electron skin depth and plasma parameter $T = 6000$ K, $n = 10^4$ cc, $B = 1$ G.

  Let the ionization fraction be $n_i/n_0 = \xi$. Then $n_e = n_i = \xi n_0$ and Saha equation becomes

$$\frac{\xi^2}{1-\xi} = e^{-\chi/(k_B T)}/(N\Lambda^3$$
$$\Lambda = \sqrt{\frac{2\pi\hbar^2}{m_e k_B T}} \tag{1}$$

  For the parameters given

$$\xi = 0.99998 \tag{2}$$

  Mean free path

$$l = \frac{1}{\sigma n}$$
$$\sigma \approx \pi e^4/(k_B T)^2 \ln \Lambda$$
$$l = 10^{-5}\ \text{cm}, \ \text{for} \ \ln\Lambda = 30 \tag{3}$$

  Collision times

$$\tau_{e-e} \sim \frac{\lambda}{v_e} = \frac{T^{3/2}\sqrt{m_e}}{\pi n e^4} = 3\times 10^{-13} sec^{-1}$$
$$\tau_{i-i} \sim \frac{\lambda}{v_i} = \tau_{e-e}\sqrt{m_i/m_e} = 10^{-11} sec^{-1} \tag{4}$$

  Equilibration time scale

$$\tau_{eq} = \frac{m_i}{m_e}\tau_{e-e} = 7\times 10^{-10} sec^{-1} \tag{5}$$

Electron plasma frequency

$$\omega_{p,e} = \sqrt{4\pi n e^2/m_e} = 5 \times 10^6 sec^{-1} \tag{6}$$

Electron cyclotron frequency

$$\omega_{B,e} = \frac{eB}{m_e c} = 1.8 \times 10^7 rad/sec \tag{7}$$

Ion cyclotron frequency

$$\omega_{B,i} = \frac{eB}{m_i c} = 10^4 rad/sec \tag{8}$$

Electron Debye radius

$$r_{D,e} = v_{T,e}/\omega_{p,e} = 5cm \tag{9}$$

Ion Debye

$$r_{D,i} = v_{T,i}/\omega_{p,i} = 5cm \tag{10}$$

Electron Larmor radius

$$r_{L,e} = v_{T,e}/\omega_{B,e} = 1.7cm \tag{11}$$

Ion Larmor radius

$$r_{L,i} = v_{T,i}/\omega_{B,i} = 74cm \tag{12}$$

Electron skin depth

$$\delta_e = c/\omega_p = 5 \times 10^3 cm \tag{13}$$

Plasma parameter

$$\Lambda = 4\pi n r_D^3 = 2 \times 10^7 \tag{14}$$

- Show that in non-relativistic plasma, $T \ll mc^2$, the electrostatic Coulomb force between particles is much larger than magnetic Lorentz force.

$$\begin{aligned} &F_c \sim eE,\ F_L = e\frac{v}{c}B \\ &E \sim en^{1/3}, B \sim e\frac{v}{c}n^{1/3} \\ &\frac{F_c}{F_L} \sim (c/v)^2 > 1 \end{aligned} \tag{15}$$

- A beam of electrons is accelerated to energy 10 keV and propagates through plasma of density 10^{15} cc. Estimate penetration depth due to binary collisions. Assume Coulomb logarithm $\Lambda = 30$. (Also assume that the beam is faster than electron and ion thermal motion).

$$\begin{aligned} &\sigma \sim (e^4/T^2)\ln\Lambda_c \\ &\lambda \sim 1/(n\sigma) = 1.6 \times 10^5 \text{cm} \end{aligned} \tag{16}$$