Anomalous Transport by Magnetohydrodynamic Kelvin-Helmholtz Instabilities in the Solar Wind–Magnetosphere Interaction

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A magnetohydrodynamic simulation of Kelvin-Helmholtz instabilities in a compressible plasma has been performed for parallel \( (v_\parallel B_0) \) and transverse \( (v_\perp B_0) \) configurations, modeling high-latitude (or downstream flanks) and dayside low-latitude magnetospheric boundaries. In the parallel configuration, a super-Alfvénic and transonic shear flow (with \( 2 < M_s = v_0/c_s < 4 \) and \( M_\perp = V_0/c_s = 1 \), where \( V_0 \) is the total jump of the velocity across the velocity shear layer) leads to an oscillation of the velocity shear layer, which bends the initial uniform magnetic field. With a hyper-Alfvénic shear flow (\( M_\perp > 4 \)), the instability develops into a more turbulent state and the initial parallel shear flow develops into small eddies, which strongly twist, compress, and hence amplify the magnetic field by a dynamo action with an amplification factor \( M_\perp/2 \). In the nonlinear stage, however large the initial Alfvén Mach number \( M_A \) may be, the magnetic field, amplified and twisted by the hydromagnetic flow vortices, eventually reacts back upon the flow evolution, and the flow vortices cascade into smaller scale structures. In the transverse configuration, for a fast magnetosonic Mach number \( M_f = \sqrt{\gamma} \left( c_s^2 + v_\perp^2 \right) > 1 \), the instability leads to the formation of a fast shock discontinuity from an initially subfast shear flow. Anomalous tangential stress in the transverse configuration reaches \( 0.01 \rho v_\perp^2 \), and the energy flux across the boundary in the magnetospheric inertial frame reaches as much as \( 2\% \) of the magnetosheath flow kinetic energy flux, \( \rho v_\perp^2 \); this viscous tangential stress could account for the convection potential drop over the polar cap of 10–30 kV. The anomalous (eddy) viscosity \( \nu_{an} \) becomes \( \nu_{an} < 10^{-3} \Delta v_\perp \), where \( \Delta v_\perp \) is the thickness of the initial velocity shear layer, and becomes as large or even larger than the Bohm diffusion for conditions typical at the magnetospheric boundary, thereby satisfying the requirements of the “viscouslike” interaction hypothesis by Axford and Hines (1961). In the parallel configuration, the slow rarefactive wave (or Alfvén wave in the incompressible limit) is excited by the instability and contributes to a strong anomalous diffusion of momentum or a dissipation of vorticity by the magnetic viscosity (Maxwell stress); this in turn gives rise to momentum and energy fluxes and an anomalous viscosity 2-3 times larger than those brought about by the hydromagnetic Reynolds stress in the transverse configuration and hence provides a very efficient viscous interaction at the magnetospheric boundary.

1. INTRODUCTION

It has long been suggested that the solar wind plasma flow interacts or exchanges momentum and energy with the earth’s magnetospheric plasma in two basic ways: one is the “viscouslike” interaction hypothesis originally suggested by Axford and Hines [1961]; the other is the reconnection of the interplanetary magnetic field line with the earth’s dipole magnetic field on the frontside magnetospheric boundary (Dungey [1961], Petschek [1964], and Levy et al. [1964]; see also Vasyliunas [1975] and Sonnerup [1979]). Axford and Hines [1961] have suggested that a viscous interaction along the flanks of the magnetosphere can permit solar wind momentum to diffuse onto closed magnetospheric field lines. The resulting tailward convection flow would eventually be closed by an earthward return flow in the center of the tail. Later, this viscous interaction model was largely replaced by the Dungey-Petschek reconnection model of the open magnetosphere. In the last few years, however, considerable evidence has accumulated indicating that the viscous interaction is an important mechanism for momentum transfer, which could explain an observed residual convection potential drop over the polar cap - 35 kV [Reiff et al., 1981]. Although such a viscous contribution appears to be small in comparison with a reconnection induced convection potential drop, the determination of the nature of the “viscouslike” interaction and its contribution to magnetospheric convection by its tangential stress is important for understanding the solar wind–magnetosphere interaction. Several different mechanisms have been invoked in order to explain the nature of the viscouslike interaction [e.g., Piddington, 1960; Axford, 1964; Eviatar and Wolf, 1968; Coleman, 1970]; however, no definite answer has been given regarding just how much transport of momentum and energy across the magnetospheric boundary is realized by those processes. The Kelvin-Helmholtz (K-H) turbulence driven by the velocity shear at the magnetospheric boundary has long been suggested as a possible viscous interaction at the magnetospheric boundary; such a suggestion is based on an analogy with hydrodynamic eddy transport, in which the eddy turbulence excited in the boundary layer is known to greatly enhance the transport coefficient [e.g., Lamb, 1945]. But again no definite answer has been given regarding how much transport is realized by the Kelvin-Helmholtz instabilities.

The purpose of the present paper is to investigate by means of a magnetohydrodynamic (MHD) simulation the nonlinear dynamics involved in the hydromagnetic Kelvin-Helmholtz instabilities and nonlinear transport of momentum and energy by the instabilities for two basic configurations: we model the magnetospheric boundaries of high latitudes (or downstream flanks) and dayside low latitudes, the former with the flow velocity parallel to the magnetic field \( (v_\parallel B_0) \) and the latter transverse \( (v_\perp B_0) \). A preliminary result of our nonlinear study has been given previously [Miura, 1982]. This paper purses in more detail the nonlinear consequences of the in-
stabilities for several different Alfvén and sound Mach numbers in both configurations. We emphasize in particular the anomalous transport of momentum and energy by Kelvin-Helmholtz instabilities across the magnetospheric boundary, which is crucial in evaluating contributions of Kelvin-Helmholtz instabilities to magnetospheric convection. It is important to emphasize here that the hydrodynamic case leads to momentum, energy transport, and anomalous viscosity by the hydrodynamic Reynolds stress associated with K-H driven vortices [Miura and Sato, 1978a, b], whereas the hydrodynamic case allows basically two transfer mechanisms, hydromagnetic Maxwell (magnetic) stress, and hydrodynamic Reynolds stress. Therefore the transport process by the hydromagnetic Kelvin-Helmholtz instability, particularly the role of the magnetic field in the transport process, has yet to be clarified.

The Kelvin-Helmholtz instability has long been considered a possible instability at the magnetospheric boundary [e.g., Dungey, 1955; Parker, 1958], and a lot of theoretical efforts have been devoted to a linear analysis of the instability at the magnetospheric boundary; see, for example, Miura and Pritchett [1982] for references to and a summary of these linear works; however, the nonlinear evolution and the dynamical consequences of this basic flow instability in the MHD fluid have not yet been adequately considered in a context of the solar wind–magnetosphere interaction. It has also been recognized that the Kelvin-Helmholtz instability appears to be excited by strong shear flows at the magnetospheric boundary [e.g., Aubry et al., 1971; Lepping and Burlaga, 1979; Southwood, 1979; Williams, 1980; Hones et al., 1981; Scopke et al., 1981].

The principal conclusions obtained from the present self-consistent nonlinear treatment of the instability by an MHD simulation are (1) the anomalous tangential stress at the magnetospheric boundary caused by the instability reaches as much as or more than 1.0% of the magnetosheath flow momentum flux adjacent to the boundary and gives a contribution to the convection potential drop over the polar cap of 10–30 kV, and (2) the anomalous viscosity caused by the instability becomes comparable to the Bohm diffusion and satisfies the requirement in Axford and Hines' [1961] hypothesis for a viscouslike interaction. Specifically, it is shown that the slow magneto sonic wave (rarefactive wave) in the configuration $B_0 \cdot v_0 \neq 0$ contributes greatly to the diffusion of plasma momentum or the dissipation of vorticity by magnetic stress and thus allows a strong tangential drag at the magnetospheric boundary off the subsonic region, which is 2–3 times larger than that in the transverse configuration by the Reynolds stress. The simulation results also allow us to discuss quantitatively the observed characteristics of boundary oscillation and motion caused by the Kelvin-Helmholtz instability in terms of a total velocity jump across the boundary $V_0$ and scale length (thickness) $\Delta$ of the velocity shear layer and thereby to provide an explanation of the highly dynamic motion of the magnetospheric boundary [e.g., Aubry et al., 1971; Russell and Elphic, 1979; Lepping and Burlaga, 1979; Scopke et al., 1981].

An outline of this paper is as follows. In section 2 we give the basic equations and models used for the simulation. In section 3 we review the principal results of the instability's linear eigenmode analysis [Miura and Pritchett, 1982], which is the basis of the present nonlinear simulation. In section 4 we discuss anomalous transport by the instability, which forms the basis for a physical understanding of the numerical results in section 5. The numerical results are presented and discussed in section 5. Section 6 contains a summary of results and a discussion of their implications in the solar wind–magnetosphere interaction.

2. Basic Equations and Model

The conservative form of the ideal MHD equations which describe the Kelvin-Helmholtz instability are

$$\begin{align}
\frac{\partial \rho}{\partial t} &= - \nabla \cdot (\rho \mathbf{v}) \quad (1) \\
\left( \frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{v} \right) \mathbf{v} &= - \nabla \cdot \Pi \quad (2) \\
\frac{\partial \Pi}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}) \quad (3) \\
\frac{\partial \mathbf{E}}{\partial t} &= - \nabla \cdot \left[ (\rho + B^2/2\mu_0) \mathbf{v} - (1/\mu_0) (\mathbf{B} \cdot \mathbf{v}) \mathbf{B} \right] \quad (4)
\end{align}$$

Where $\rho$, $\mathbf{v}$, $\mathbf{B}$, and $p$ are the plasma mass density, bulk velocity of the plasma, magnetic field, and plasma pressure, and $\epsilon$ is the energy density defined by

$$\epsilon = \frac{1}{2} \rho v^2 + \frac{1}{2} \mu_0 B^2 + p/(\gamma - 1) \quad (5)$$

Calculations are performed in the $x - y$ plane where the initial velocity $v_x$ has a shear profile in the $x$ direction. We impose boundary conditions such that there is no mass flow ($v_x = 0$) across the boundaries at $x = \pm x_s$ and all quantities are periodic in the $y$ direction. It then follows from (1)–(4) that $B_x$ and derivatives with respect to $x$ of the remaining quantities ($p$, $v_y$, $v_z$, $B_y$, $B_z$, $p$) must vanish at the boundaries ($x = \pm x_s$). This boundary condition means that the flow kinetic energy flux and the Poynting flux across the boundaries vanish. Therefore there is no inflow or outflow of energy across the boundaries.

We show in Figure 1 the two basic configurations of the instability studied in this simulation. We use the simplest MHD equilibrium in which only the shear of the flow velocity $v_x$ is retained, the other equilibrium quantities ($B_0$, $p_0$, $\rho_0$) being uniform. For the magnetic field, however, we assume that the magnetic field is either parallel to the flow ((1) parallel configuration) or perpendicular to the flow ((2) transverse configuration). The parallel configuration models the magnetospheric boundary at high latitudes near the noon–midnight meridian, at downstream flanks, and at cusps, where there is a large magnetic field component parallel to the shear flow. The transverse configuration models the low latitude dayside magnetospheric boundary, where the magnetic field is almost perpendicular to the shear flow. By using these simple configurations, we can elucidate the basic dynamics of the instability involved in the above two basic configurations. For the velocity profile we assume a hyperbolic-tangent form,

$$v_0(x) = -(V_0/2) \tanh (x/\alpha) \quad (6)$$

characterized by a total velocity jump, $V_0$, and velocity shear scale length, $\alpha$. For the present calculation we have placed the boundaries at $x = \pm 10\alpha$, which is far enough from the shear region to make boundary effects negligible. The two-step Lax-Wendroff method [Richtmyer and Morton, 1967] with an artificial viscosity term [Lapidus, 1967] is used to solve equations (1)–(4); we employ a mesh system with a (100, 100) mesh. Physical parameters used in the calculation are the Alfvén Mach number $M_A = V_0/c_A$ and sound Mach number $M_s = V_0/c_s$, where $v_A = B_0/(\mu_0\rho_0)^{1/2}$ and $c_s = (\gamma p_0/\rho_0)^{1/2}$. We use time $t$ normalized by $\tau = 2\alpha/V_0$.  

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The fastest growing mode is characterized by a total velocity jump \( V_0 \) and a scale length 2\( a \).

3. LINEAR DISPERSION

Linear stability of the MHD Kelvin-Helmholtz instability for the two basic configurations in a compressible plasma has been studied in detail by Miura and Pritchett [1982].

Shown in Figure 2 are normalized growth rates as a function of the normalized wave number \( k_\parallel a \) for parallel (right panel) and transverse (left panel) configurations for the mode satisfying \( k_\parallel V_0 \). The right panel shows how compressibility stabilizes the K-H mode with an increase in \( M_\parallel \) for the transverse case. Two points on the dispersion curves denoted by cross signs represent the fastest growing mode, which is used as an initial perturbation in the simulation run described in section 5. The right panel shows the stabilizing effect of the field line tension as a function of \( M_\parallel \) for the parallel case. Three points on the dispersion curves denoted by cross signs represent the fastest growing mode, which is used as an initial perturbation in the simulation run described in section 5. For both configurations, the fastest growing mode always appears at wave numbers satisfying \( 0.5 < k_\parallel a < 1.0 \). Consequently, the fastest growing mode has a wavelength of the order of \( 2\pi \times 2a \), which is characterized by the finite thickness of the velocity shear layer \( 2a \).

4. ANOMALOUS TRANSPORT BY INSTABILITY

We formulate in this section anomalous transport by MHD waves, which is important in understanding anomalous tangential stress (anomalous momentum transfer) and anomalous energy transport across the magnetospheric boundary by the Kelvin-Helmholtz instability. In the following numerical calculations, we use a frame moving with the flow with a velocity \( \frac{1}{2} V_0 \), where the velocity profile becomes antisymmetric.

Since momentum and energy depend on the frame we choose, and since we are interested in evaluating the transport process in the earth's magnetospheric inertial frame, we need to clarify in particular the relationship between momentum and energy fluxes in the original frame and in the magnetospheric inertial frame.

4.1. Anomalous Momentum Transport

If we take a spatial average of the \( y \) component of (2) in the \( y \) direction, we obtain for the two-dimensional case \( (\partial / \partial z = 0) \) using the periodicity of perturbations in the \( y \) direction

\[
\frac{\partial}{\partial t} \langle \rho v_y \rangle = -\frac{\partial}{\partial x} \left( \rho v_x v_y - \frac{B_y B_x}{\mu_0} \right)
\]

(7)

where the brackets denote the spatial average over one wave period. From (7) we find that the instability can exert an anomalous tangential stress \( \langle \rho v_x v_y - \frac{B_y B_x}{\mu_0} \rangle \) on plasmas, where the first term is the hydrodynamic Reynolds stress and the second term is the hydromagnetic Maxwell stress. In order to calculate the change of momentum flux in a rectangular volume extending from \( x = 0 \) to \( x = \infty \) and surrounded by a unit surface at \( x = 0 \), we integrate equation (7) from \( x = \infty \) to \( x = 0 \) to obtain

\[
\frac{\partial}{\partial t} \int_0^\infty \langle \rho v_y \rangle \, dx = -\langle \rho v_x v_y - \frac{B_y B_x}{\mu_0} \rangle |_{x=0}^{x=\infty}
\]

(8)

This indicates that across the surface at \( x = 0 \) there is a net transfer of momentum in the \( y \) direction by the instability, which is expressed by the anomalous tangential stress at \( x = 0 \).

Let us denote the magnetospheric inertial frame by primed coordinates. In order to transform from the original frame used in the numerical calculations to the magnetospheric inertial frame, we need a Galilean transformation

\[
x' = x, \quad y' = y + \left( V_0 / 2 \right) t \quad z' = z \quad t' = t
\]

(9)

In the magnetospheric inertial frame, the momentum transport equation (8) becomes

\[
\frac{\partial}{\partial t'} \int_0^\infty \langle \rho v_y' \rangle \, dx' = -\langle \rho v_x v_y' - \frac{B_y B_x}{\mu_0} \rangle |_{x'=0}^{x'=\infty}
\]

where the magnetic field is invariant under the Galilean transformation. If we substitute the relation \( v_y' = v_y + V_0 / 2 \) into (10), we obtain

\[
\frac{\partial}{\partial t'} \int_0^\infty \langle \rho v_y' \rangle \, dx' = -\langle \rho v_x v_y - \frac{B_y B_x}{\mu_0} \rangle |_{x=0}^{x=\infty}
\]

(11)

Therefore, in the magnetospheric inertial frame, the additional momentum flux \( -V_0 / 2 \langle \rho v_y \rangle |_{x=0}^{x=\infty} \) arises due to the compressibility of the plasma.

In evaluating the momentum transport process across the magnetospheric boundary, an important quantity is the viscosity coefficient [Axford and Hines, 1961; Eardley and Lightman, 1975]. According to the definition of eddy viscosity in ordinary hydrodynamics [e.g., Lamb, 1945], anomalous viscosity in MHD fluid can be defined as

\[
v_{\text{ano}} = \rho_0 \langle B_x B_y / \mu_0 - \rho v_x v_y (d (v_y) / dx) \rangle^{-1}
\]

(12)

The part of \( v_{\text{ano}} \) due to the Reynolds stress, \( \rho v_x v_y \), is the eddy viscosity in ordinary hydrodynamics, and the part due to the Maxwell (magnetic) stress, \( B_x B_y / \mu_0 \), is anomalous magnetic viscosity [e.g., Eardley and Lightman, 1975]. As is obvious from the last term of the right-hand side of (11), compressible
turbulence can also give rise to a viscous stress in the magnetospheric inertial frame. Therefore, in the magnetospheric inertial frame anomalous viscosity should be defined by

\[ \nu_{\text{anom}} = \rho_0^{-1} \langle B_x B_y / \mu_0 - \rho v_x v_y - \frac{1}{2} V_0 \rho v_x^2 \rangle \]

(13)

4.2. Anomalous Energy Transport

The energy flux, \( Q \), produced by MHD waves is generally expressed by

\[ Q = (e + p) v + 1/\mu_0 \mathbf{E} \times \mathbf{B} \]

(14)

where the first term is the sum of enthalpy flux and flow kinetic energy flux and the second term is the electromagnetic Poynting flux. We integrate equation (4) from \( x = \infty \) to \( x = 0 \) and take the spatial average over one wave period in the \( y \) direction to obtain

\[ \frac{\partial}{\partial t} \int_0^x \langle \epsilon \rangle \, dx = \langle Q_x \rangle_{x=0} \]

(15)

In the original frame of reference, where the velocity is antisymmetric, the net energy transfer across \( x = 0 \) is obviously zero owing to the symmetry of the system. Therefore we have

\[ \langle Q_x \rangle_{x=0} = 0 \]

(16)

in the original inertial frame.

In the magnetospheric inertial frame, the averaged energy transport equation becomes

\[ \frac{\partial}{\partial t} \int_0^x \langle \epsilon' \rangle \, dx' = \langle Q_x' \rangle_{x=0} \]

(17)

where \( \langle Q_x' \rangle_{x=0} \) is expressed as

\[ \langle Q_x' \rangle_{x=0} = \langle Q_x \rangle_{x=0} + V_0 \langle pv_x v_y - (1/\mu_0) B_x B_y + \frac{1}{2} \rho V_0^2 \rangle \]

(18)

By using (18), (19) is reduced simply to

\[ \frac{\partial}{\partial t} \int_0^x \langle \epsilon' \rangle \, dx' = V_0 \langle pv_x v_y - (1/\mu_0) B_x B_y + \frac{1}{2} \rho V_0^2 \rangle \]

(19)

Thus in the magnetospheric inertial frame, the energy flux is simply equal to the tangential stress in the original frame multiplied by the total velocity jump \( V_0 \) across the magnetospheric boundary. Therefore an evaluation of the tangential stress caused by the instability also enables us to calculate the net energy flux across \( x = 0 \).

5. Simulation Results

We present in this section simulation results obtained for parallel and transverse configurations. For the parallel configuration, simulations are performed for three different values of \( M_a \) and for a fixed \( M_f \) in order to clarify the dependence of instability consequences on the Alfvén Mach number \( M_a \) or plasma \( \beta \) \((\beta \sim M_a^2 / M_s^2)\). For the transverse configuration, simulations are performed for two different values of \( M_f \) (fast
magnetosonic Mach number) in order to illustrate the effect of compressibility on the instability. Finally, anomalous transport of momentum and energy by the instability is evaluated for both configurations, and the importance of the hydrodynamic Maxwell stress (magnetic stress) in anomalous transport is elucidated. For each simulation run, the system length $L_y$ is set equal to the wavelength of the fastest growing mode, which is marked by cross signs in both panels of Figure 2.

5.1. Parallel Configuration

First, we show results of a simulation run performed for the parallel configuration with parameters $M_s = 1.0$, $M_A = 2.5$. The right panel in Figure 2 shows that the growth rate peaks at $2\alpha k_y = 0.5$, and it is given by $\gamma = 0.05 \sqrt{V_o/2a}$. Therefore in the simulation we have used the system length $L_y = 5a\lambda$, which is equal to the wavelength $\lambda$ of the fastest growing mode.

Solid and dashed curves in Figure 3 marked by the parallel symbol show the time evolution of normalized peak amplitudes $v_{\text{max}}/V_o$ and $B_{\text{max}}/B_0$, where $B_0 = B_0$ and $V_o = V_0$. In the early stages, the amplitudes grow linearly with the predicted linear growth rate. At $T/\tau = 90$ the instability saturates, $v_{\text{max}}/V_o$ reaches 30%, and the magnetic field perturbation (transverse component) becomes comparable to the initial background magnetic field intensity.

Figure 4 shows flow vectors (left panels) and magnetic field vectors (right panels) at $T/\tau = 40$ and 80. Since we are using the comoving frame where the velocity changes from $\frac{1}{2}V_o$ to $-\frac{1}{2}V_o$, the time sequence in Figure 4 may be regarded in the actual magnetospheric inertial frame as the spatial evolution from upstream to downstream along the magnetospheric boundary over a distance $l = 80\tau \times \frac{1}{2}V_o \approx 5\lambda$, where $\lambda$ is the wavelength of the fastest growing mode. In the early phase ($T/\tau = 40$) the sheared plasma flow undulates slightly with the development of the instability (top left panel). Since the magnetic field is frozen into the plasma, the above undulation of the plasma flow leads to a slight bending of the magnetic field (as seen in the top right panel). In the saturation stage at $T/\tau = 80$, a large oscillation of the velocity shear layer occurs with a large twin eddylike circulation pattern imbedded near $y/a \approx 20$. By this time, the magnetic field is more strongly bent than at $T/\tau = 40$, and the magnetic field is also slightly compressed and intensified along the velocity shear layer owing to a finite compressibility (bottom right panel). It is obvious from the bottom right panel and equation (7) that at this stage the finite Maxwell stress is imposed on the plasma in the $y$-direction by the bending ($B_\perp \neq 0$) of field lines. We have found
that the total magnetic field energy in the whole calculation domain increased by 8.7% of the initial total magnetic field energy due to the compression and bending of the magnetic field; therefore the instability constitutes a dynamo process, whereby the magnetic field is amplified. As we will see later, the pressure is reduced where the magnetic field is intensified (top panel of Figure 7), and this region constitutes a standing slow rarefaction layer, since the plasma flows toward this region with a super-slow magnetosonic speed.

We should emphasize here that in this low \( M_A \) case a strong twisting of the magnetic field line by the instability is prevented by the large tension of the magnetic field lines. For much larger \( M_A \) or larger plasma \( \beta \) (\( \beta \approx M_A^2/M_s^2 \)), the tension of the magnetic field line becomes smaller relative to the inertial term of the equation of motion, and hence the instability is expected to lead to a stronger bending or twisting of the magnetic field line by a differential rotation associated with the development of vortices [Miura, 1982]. In order to demonstrate this strong dependence of the bending of the magnetic field line on the Alfvén Mach number \( M_A \), we show in Figures 5 and 6 flow vectors and magnetic field vectors for two different Alfvén Mach numbers. Figure 5 shows flow vectors (left panels) and magnetic field vectors (right panels) at \( T/\tau = 20 \) and 40 for a hyper-Alfvénic case with \( M_A = 5.0 \) and \( M_s = 1.0 \) (plasma \( \beta = 30 \)). In this case the shear flow is extremely disturbed by the saturation stage at \( T/\tau = 40 \), and a pair of eddies is formed inside a large vortex at \( 7a < y < 15.7a \). In the center of the large vortex, the flow is almost stagnant. A stagnation region also appears in between the larger vortices, and the plasma flow toward this stagnation region induced by vortices is forced to diverge along a layer formed tangent to the vortices. The initially uniform magnetic field is slightly sheared and compressed inside the velocity shear layer at \( T/\tau = 20 \), and eventually at \( T/\tau = 40 \), a strong compression of the magnetic field occurs at \( 0 < y < 7a \) along the layer formed tangent to the vortices. On the other hand, at \( 7a < y < 15a \) the magnetic field line is stretched and twisted strongly as a consequence of the wrapping up of the field lines by the differential rotation associated with each of the twin vortices. In this case, the total magnetic energy increased by \( \sim 26\% \) of the initial total magnetic energy at the expense of the initial flow kinetic energy. Figure 6 shows flow vectors and magnetic field vectors for a case with a much larger Alfvén mach number (\( M_s = 1.0, M_A = 10.0 \)). This case has a plasma \( \beta \) equal to 75. In the early phase at \( T/\tau = 25 \), a large vortex cell is formed (upper left panel). From the good agreement between the vortex pattern at this time and that in the transverse case (the bottom left panel in Figure 12), we can deduce that the mag-
magnetic field tension does not greatly affect the development of the flow by this time; this is because the initial magnetic field is too weak to affect the plasma flow in the present hyper-Alfvénic case with \( M_a \gg 1 \) and \( \beta \gg 10 \). On the other hand, the magnetic field is affected strongly by the flow, and it is sheared and compressed strongly by the converging plasma flow induced by the vortices (upper right panel). In the later stage at \( T/\tau = 45 \), the magnetic field flux is strongly compressed by the incoming plasma flow along a thin layer formed tangent to the vortices, and the magnetic flux is wrapped up several times at \( 7a < y < 15a \) (lower right panel) due to the differential rotation associated with the vortex motion. Within the compressed magnetic flux tube, the initial magnetic field is amplified six times and the plasma inside is squeezed out of the magnetic flux tube, so that the plasma \( \beta \) inside the compressed magnetic flux tube decreases considerably. In such a stage, however, the magnetic force term becomes large enough to affect the plasma flow locally or the magnetic energy density becomes comparable to the flow kinetic energy density; consequently, it is seen in the bottom left panel that the original vortex at \( T/\tau = 25 \) is about to split into smaller eddies, since the magnetic field can now react back upon the flow evolution under the influence of the strong Maxwell stress. We see in the bottom left panel that the shear layer or the layer connecting the almost stagnant region has an undulation with much smaller scale sizes than is seen in the top left panel. Therefore, although the flow is still ordered, the large \( M_a \) case definitely leads to a more "turbulent" flow in the sense that the large scale flow perturbation has excited much smaller scale perturbations. In the magnetospheric inertial frame, such a flow perturbation would appear as a highly dynamic motion of the boundary layer (velocity shear layer) propagating downstream along the magnetospheric boundary.

Note in the lower panels that this hydromagnetic "turbulence" of flow and magnetic field excited by the Kelvin-Helmholtz instability, in particular, the wrapped region of the magnetic field lines, dominates a region much wider than the initial velocity shear scale length. A reason for the striking difference of the evolution of the instability, in particular, the evolution of the magnetic field among the above three cases, seems to be the fact that in the above three cases the maximum transverse flow velocities, or the flux transfer velocities associated with the vortices, vary greatly from sub-Alfvénic to super-Alfvénic with respect to the initial Alfvén velocity \( v_A \). In the first case the peak amplitude of \( v_x \) is \( \sim 0.3V_0 = 0.75v_A \) (see Figure 3); therefore strong piling up of the magnetic field by
the incoming plasma flow should be rather weak, since the piled magnetic flux is easily transported away from the compressed region by an MHD wave.

Shown in Figure 7 are three-dimensional views of the pressure distributions at saturation stages of the instability for the above three different values of $M_A$. A substantial depletion of the plasma pressure is seen for all cases along the compressed flux tube formed tangent to the vortices. This is because the flux tube is compressed on both sides by the incoming plasma flow induced by the vortex motion and the plasma inside is squeezed out of the flux tube. In the final stage, the maximum depletion of the plasma pressure reaches $\sim 20\%$ of the initial uniform pressure for $M_A = 2.5$. As $M_A$ or the plasma $\beta$ increases, the slow rarefaction layer becomes much narrower and deeper. In all the panels, the position of the slow rarefaction layer corresponds to that of the strong compression layer of the magnetic flux tube seen in the bottom right panels of Figures 4–6. The region corresponding to the strong curling of the field line, on the other hand, is seen to be nearly incompressible and to have a strong Alfvénic component.
In order to demonstrate the formation of a plasma depletion layer by a strong compression of the magnetic flux tube, we show in Figure 8 profiles of $B_x$, $B_y$, $\rho$, $p$, in the $x$ direction at the saturation stage of the parallel configuration for $M_A = 5.0, M_s = 1.0$. It is seen that plasma pressure and density are reduced where the magnetic field is strongly compressed. Both the $x$ and $y$ components of the magnetic field have solitary structures at $x \sim 0$ as a result of the strong compression caused by the incoming plasma flow induced by vortices, and the magnetic field is amplified to about 3 times the initial value. Inside this rarefaction layer, the plasma $\beta$ decreases considerably from the initial $\beta = 30$ to $\beta = 1.06$. Such a configuration of the slow rarefaction region, namely, the increased magnetic field and the decreased pressure or density, is similar to the plasma depletion layer formulated by Zwan and Wolf [1976] at the subsolar magnetosheath which is compressed by the shocked solar wind. In their model, it is shown that the depletion layer thickness is proportional to $M_A^{-2}$. This strong dependence of the thickness of the slow rarefaction layer $d_0$ on the Alfvén Mach number has also been seen for the present case and is summarized in Figure 9. This figure shows the normalized thickness of the slow rarefaction layer as a function of $M_A$. Here, we define the thickness of the slow rarefaction layer $d_0$ as the full width at half maximum of the magnetic field energy within the slow rarefaction layer. At $M_A = 2.5 \sim 5$, the thickness of the rarefaction layer decreases strongly with the increase of $M_A$, and this dependence is well represented by $M_A^{-2}$ dependence, the same dependence as the depletion layer formulated by Zwan and Wolf [1976].

We summarize in Figure 10 the plasma dynamics involved in the parallel configuration for the case of $M_A = 5.0$, which has led to a strong compression and twisting of magnetic field lines and the formation of a slow rarefaction layer. Illustrated also in this figure is an amplification of the magnetic field at the site of small eddies by the dynamo action, $E \cdot J = v \cdot (J \times B) < 0$, where $E$ and $J$ are the electric field and current induced by eddy motion and field line twisting. Note that in the present two-dimensional configuration the electric field $E = -v \times B$, which is in the $z$ direction, is essentially the inductive field, since $\partial B/\partial z = 0$. At the site of the slow rarefaction layer, the magnetic field is also amplified by slow rarefaction due to the accumulation of magnetic field lines by flux transfer associated with frozen-in vortex motion. Both of these processes, i.e., twisting and compression of magnetic field lines, contribute to a dynamo action, whereby the flow kinetic energy is converted into magnetic energy. This dynamo process is caused by a deceleration of the flow by the $J \times B$ magnetic force; that is the magnetic field gains its energy from the vortex flow by $v \cdot (J \times B) = 0$. In order to show that the total magnetic energy in the whole calculation domain is increased by the dynamo action, we start from the Poynting theorem for ideal MHD plasma, i.e., without the displacement current,

$$\frac{1}{2} \mu_0 (\partial B/\partial t)^2 = -(1/\mu_0) \nabla \cdot (E \times B) - v \cdot (J \times B)$$  \hspace{1cm} (20)

If we integrate this over the volume $V$ formed by the calculation domain and a unit length in the $z$ direction, we obtain

$$E = c_0 (1/\mu_0) \nabla \times (J \times B) = 0$$

**Fig. 8.** Profiles in the $x$ direction of $B_x$, $B_y$, $\rho$, $p$ normalized by $B_0$, $\rho_0$, $p_0$ at $T/r = 40$ for the parallel configuration with $M_s = 1.0, M_A = 5.0$.

**Fig. 9.** Normalized thickness of slow rarefaction layer $d_0/a$ as a function of the Alfvén Mach number.

**Fig. 10.** Summary of the plasma dynamics involved in the parallel configuration.
slow rarefaction layer (= B_0), may be calculated as follows: Figure 10 shows that the slow rarefaction layer is formed as a consequence of the compression of the magnetic flux tube by the incoming plasma flow induced by vortices. The pressure balance between the inside and outside of the slow rarefaction layer becomes

$$P_{\text{out}} + \rho_{\text{out}} v_{\text{out}}^2 + \frac{B_{\text{out}}^2}{2\mu_0} \approx P_0 + \frac{B_0^2}{2\mu_0}$$

(23)

where $v_{\text{out}}$ is the velocity of the incoming plasma flow, which is almost normal to the slow rarefaction layer. From simulation results, we found that $P_{\text{out}} - P_0 \ll \rho_{\text{out}} v_{\text{out}}^2$. Therefore the above pressure balance condition gives simply

$$P_{\text{out}} v_{\text{out}}^2 \approx \frac{B_0^2}{2\mu_0}$$

(24)

This relation means that the dynamic pressure due to the incoming plasma flow is nearly balanced by the magnetic pressure inside the rarefaction layer. If we use the empirical fact obtained from simulation results (see Figure 3), that

$$v_{\text{out}} \sim \frac{V_0}{3}$$

(25)

(24) can be rewritten as

$$\rho_{\text{out}}(V_0)^2 \approx \frac{B_0^2}{2\mu_0}$$

(26)

Thus we can conclude that the amplification factor of the magnetic field is

$$\frac{B_f}{B_0} = \left[\frac{B_f^2/2\mu_0}{B_0^2/2\mu_0}\right]^{1/2} \approx \frac{\rho_0(V_0/3)^2}{B_0^2/2\mu_0} \approx \frac{1}{2} M_A$$

(27)

We show in Table 1 the amplification factors, obtained from simulations, of the magnetic field inside the slow rarefaction layer for three different values of $M_A$ in the saturation stage. Although the above calculations are based on a simple argument, the calculated amplification factor agrees very well with the simulation results, suggesting that the simple dynamo relation (24) based on slow magnetosonic rarefaction is well satisfied in the actual simulation results. Summarized also in Table 1 are the value of the initial plasma beta and that of the plasma beta within the slow rarefaction layer at the saturation stage for three different values of $M_A$. Although the initial plasma beta is much larger than unity for all cases, the plasma beta at their saturation stages becomes 1–2 for all cases, owing to the squeezing process. As we have noted in Figure 4, because of this large decrease of the plasma beta within the rarefaction layer, $V_p$ and $J \times B$ forces eventually become almost comparable, and hence the flow becomes strongly affected by the magnetic field, even though the initial (seed) magnetic field satisfying $\beta \gg 1$ was too weak to affect the plasma motion.

**TABLE 1.** The Plasma $\beta$ at the Initial Stage, the Plasma $\beta$ at the Saturation Stage of the Instability, the Amplification Factor of the Magnetic Field, and the Relative Magnetic Energy Increase, in the Parallel Configuration for Three Different Alfvén Mach Numbers

<table>
<thead>
<tr>
<th>$M_A$</th>
<th>Initial $\beta$</th>
<th>$\beta$ at Saturation Stage</th>
<th>$B_f/B_0$</th>
<th>$\frac{W_p - W_f}{W_f}(t = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>7.5</td>
<td>1.6</td>
<td>1.9</td>
<td>8.7%</td>
</tr>
<tr>
<td>5.0</td>
<td>30.0</td>
<td>1.7</td>
<td>3.6</td>
<td>26%</td>
</tr>
<tr>
<td>10.0</td>
<td>120.0</td>
<td>2.3</td>
<td>6.0</td>
<td>140%</td>
</tr>
</tbody>
</table>
5.2. Transverse Configuration

Solid and dashed curves in Figure 3 marked by the perpendicular sign show the time evolution of normalized peak amplitudes $v_{\text{rms}}/V_0$, $\delta B_{\text{rms}}/B_N$ for the transverse configuration for $M_s = M_A = 2.0$ (M_s = 1.44). Both amplitudes grow linearly with the predicted linear growth rate $\gamma = 0.09 \times 2a/V_0$ (see Figure 2). In the saturation stage, both normalized amplitudes reach $\sim 25\%$. In this case, the total magnetic field energy increased only slightly, by 1.6% of the initial total magnetic field energy, by the fast magnetosonic compression.

We show in Figure 12 simulation results of the transverse configuration with its time evolution shown in Figure 3. The system length $L_x$ in this case is equal to 17.9$a$, which is equal to the wavelength of the fastest growing mode $\lambda$. Left panels show flow patterns at $T/\tau = 25, 55$. The initially laminar sheared plasma flow is disturbed slightly at $T/\tau = 25$, and it develops into a flat vortex cell at $T/\tau = 55$. Again, this time evolution may be regarded in the magnetosonic inertial frame as a spatial evolution along the magnetospheric boundary from upstream to downstream over a distance of $55 \times \frac{1}{2}V_0 \sim 3\lambda$. The transverse vortex size at $T/\tau = 55$ becomes much larger than the initial thickness of the velocity shear layer ($2a$), and therefore a large-scale mixing of plasma is accomplished by a vortex motion. It is seen that the initial laminar shear flow is accelerated and decelerated periodically in the $y$ direction, since the perturbed vortex motion periodically changes the direction of rotation. Notice that the ultimate energy for this acceleration is provided by the inertial force $\rho \partial (\partial u_y/\partial x)u_x$ due to the velocity shear. An interesting consequence of the instability found for this case is the formation of a pair of fast shock structures aligned side by side across the velocity shear layer, even though the initial maximum flow

---

Fig. 12. (left) Flow velocity at $T/\tau = 25$ and 55 and (right) three-dimensional plots of the pressure distribution at $T/\tau = 40, 45, 50, \text{and } 55$ for the transverse configuration ($M_s = M_A = 2.0$).

---

Fig. 13. Summary of the plasma dynamics involved in the transverse configuration.
speed is less than the magnetosonic speed $V_f (v_{\text{max}} = V_0/2 = 0.71V_f)$. The right panels in Figure 12 show a three-dimensional view of the pressure distribution. It is seen that at $T/\tau = 50$ and 55, the pressure gradient presents a clear-cut discontinuity, which appears at the strongly accelerated flow region in the bottom left panel. The physical picture leading to this fast shock formation from the initially subsonic flow is summarized in Figure 13. Initially, the plasma was uniform and the maximum flow speed was below the magnetosonic speed. As the instability grows, however, the vortices are excited and the flow is accelerated and decelerated periodically in the $y$ direction by the perturbed vortex motion. Therefore the decelerated flow is overtaken by the accelerated flow causing the pressure gradient to steepen more and more with time. Eventually, the accelerated flow speed exceeds the local magnetosonic speed, and a fast shock discontinuity is formed. In order to resolve the fast shock structure, we show in Figure 14 profiles in the $y$ direction of pressure $p$, density $\rho$, and temperature $T$ normalized by their initial values $p_0$, $\rho_0$, $T_0$, and profiles of $v_y$ and the magnetosonic speed $(c_s^2 + v_y^2)^{1/2}$ normalized by $V_0$ at $x = 3.0a$ in the saturation stage ($T = 55$). At $y \approx 11a$, those quantities present clear-cut discontinuities, across which the flow speed changes from superfast($M_{\text{free}} = 1.09$) in the upstream side to subfast ($M_{\text{free}} = 0.7$) in the downstream side, consistent with the shock condition [Landau and Lifshitz, 1959], where $M_{\text{free}}$ is the real fast magnetosonic Mach number defined using the phase velocity, i.e., $M_{\text{free}} = (\delta V_0)/(c_s^2 + v_y^2)^{1/2}$. Notice that in the present ideal MHD scheme the dissipation mechanism necessary for the formation of the fast shock discontinuity is provided by the artificial viscosity introduced following Lapidus [1967].

It is obvious in Figure 14 that the initial uniform flow velocity $v_y$ is perturbed and steepened to form a fast shock discontinuity where this velocity exceeds the local magnetosonic speed. The maximum perturbation of $v_y$ reaches about 20% of $V_0$, which is almost comparable to the maximum velocity in the $x$ direction of the vortex motion. Therefore, if we assume $(\Delta v_y)_{\text{max}} \sim v_{\text{max}}$ in the saturation stage, the condition for the fast shock formation can be written simply as

$$\frac{1}{2}V_0 + (\Delta v_y)_{\text{max}} \sim \frac{1}{2}V_0 + v_{\text{max}} > (c_s^2 + v_y^2)^{1/2}$$

If we use the fact that $v_{\text{max}} \leq 0.25v_0$, we obtain from the above equation

$$M_f > 1.3$$

as a rough condition for the fast shock formation.

Therefore for a fast magnetosonic Mach number less than some critical value, say 1.3, the acceleration of the initial flow is not expected to be strong enough to form the fast shock discontinuity. In order to verify this speculation, we have performed a simulation run for a less compressible case $M_f = 0.5$, $M_s = 3.33$, and $M_f \approx 1.0$. Figure 15 shows flow patterns at $T/\tau = 25, 45, 65$, and 85 for this case. Here the growth rate is larger than the previous case shown in Figure 12, and a clear-cut vortex pattern is formed in a relatively early period of the evolution (at $T/\tau = 45$). In contrast to the previous highly compressible case, however, the acceleration of the flow is not so strong, and the instability results only in the change of the flow configuration without any strong compression.

5.3. Anomalous Transport

We show in Figure 16 anomalous stresses normalized by $\rho_0V_0^2$ (upper panel) and velocity shear profiles (lower panel) for the two basic configurations; the time evolutions for these cases are shown in Figure 3 and the velocity shear profiles in this figure are those at their saturation stages. For the transverse configuration ($B_0 \perp V_0$), assuming two-dimensionality where $\partial / \partial z = 0$ (i.e., the field line is not allowed to bend), the Maxwell stress vanishes and only the Reynolds stress at $T/\tau = 30$ is plotted (dotted-dashed curve). The anomalous Reynolds stress peaks at $x = 0$, and the peak stress becomes $0.006 \rho_0V_0^2$, which is 0.6% of the flow momentum flux far from the shear layer. This anomalous momentum transport by the Reynolds stress leads to a finite diffusion of momentum shown as a relaxation of the velocity shear profile from dashed curves to dotted-dashed curve in the lower panel. In the parallel configuration, the Maxwell stress (solid curve) at $T/\tau = 50$ is much larger than the Reynolds stress (dashed curve), and the Maxwell stress reaches $\approx 0.5\%$ of the flow momentum flux far from the shear layer. Note that the anomalous Maxwell stress is strongly confined within the region of the velocity shear where the magnetic field line is bent most strongly; this causes a very strong relaxation and widening of the initial velocity shear (solid curve in the lower panel), which in turn leads to a dynamo amplification of the magnetic field. Since the net transferred momentum is proportional to the area between the initial velocity shear profile and the velocity shear profile at the saturation stage (shown by the hatched area for the transverse case), it is seen from the lower panel that the momentum transport in the parallel configuration is about 2–3 times as large as that in the transverse configuration. This is a very important finding, as it means that the hydromagnetic Maxwell stress is more efficient than the hydrodynamic Reynolds stress in the momentum transport. Since the net momentum transport is given by the time integral of the tangential stress at $x = 0$ (see equations (10) and (11)), this large difference of the net momentum transport between the two configurations should have been caused by the difference of the tangential stress right before their saturation stages between the two configurations. In order to check this, we show in Table 2 the hydrodynamic Reynolds stress and the hydromagnetic Maxwell stress for a transverse case and for three different parallel cases right before their saturation stages. The Maxwell stress for the cases of $M_s = 5.0$ and 10.0 at their saturation stages reach $\sim 2 \times 10^{-2} \rho_0V_0^2$, which is about 2–3 times larger than the maximum attainable Reynolds stress in the transverse configuration. It is found from this
table that in spite of the large difference in values of $M_A$ and the large difference in the nonlinear stage in their flow and magnetic field configurations, the net momentum transport in the parallel configuration is not so different for different values of $M_A$ (see Table 2). This seemingly weak dependence of the Maxwell stress on $M_A$ might be a natural consequence of the dynamo action by the Kelvin-Helmholtz instability in the parallel configuration, in which the maximum attainable magnetic field is independent of $M_A$ and is proportional to $V_0$ (see (24)). It is interesting to note that the anomalous drag force per unit area, or the Reynolds stress $\varepsilon > 0.01 \rho_0 V_0^2$ obtained for the transverse configuration, is comparable to the drag force between a flowing fluid and a solid object obtained empirically in the hydrodynamic case with a large Reynolds number [e.g., Taylor, 1915; Lamb, 1945; Landau and Lifshitz, 1959]. This empirically obtained drag force has been found to be almost independent of molecular viscosity and is believed to be the anomalous momentum transport by small hydrodynamic eddies excited at the interface between the fluid and a solid object, although details of the transport mechanism have never been established. By using the definition of anomalous viscosity (12), we obtain for the parallel case ($M_s = 1.0, M_A = 2.5$) at $T/\tau = 50$, $\eta_{\text{anom}} = 2.6 \times 10^{-2} \alpha V_0$ at $x = 0$, which is mainly due to magnetic stress (magnetic visco-

Fig. 15. Flow velocity at $T/\tau = 25, 45, 65,$ and $85$ of the transverse configuration with $M_s = 1.0, M_A = 3.33$.

Fig. 16. Spatial averages of $v_x$ at $T = 0$ and in the growing phases (lower panel) for the parallel ($M_s = 1.0, M_A = 2.5$) and transverse ($M_s = M_A = 2.0$) configurations. Spatial averages of anomalous stresses for the two configurations (upper panel). The hatched area corresponds to the net momentum transport for the transverse case.
TABLE 2. The Hydrodynamic Reynolds Stress and Hydromagnetic Maxwell Stress for a Transverse Case, for Two Different Parallel Cases, and for an Oblique Case Right Before Their Saturation Stages

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Reynolds Stress $M_A$</th>
<th>Maxwell Stress $M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse</td>
<td>0.5 0.5</td>
<td>$1.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>Parallel</td>
<td>5.0 1.0</td>
<td>$2.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>Parallel</td>
<td>10.0 1.0</td>
<td>$-8.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>Oblique</td>
<td>2.5 1.0</td>
<td>$-3.9 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

cosity; Eardley and Lightman [1975]. For the transverse case ($M_s = M_A = 2.0$) we obtain $\nu_{\text{max}} = 1.2 \times 10^{-2} \ aV_0$ at $x = 0$, $T/r = 80$. Since $\langle \rho v_o \rangle = 0$ at $x = 0$ as we will see later (Figure 17), these anomalous viscosities at $x = 0$ are independent of the inertial frame (see (13)).

These results strongly suggest that the momentum diffusion process is an intrinsic and inevitable feature of the Kelvin-Helmholtz instabilities which in turn leads to the saturation of the instability. It is interesting to note that in the resistive MHD plasma the importance of the MHD wave, the Alfvén wave or slow magnetosonic wave in the compressible case, in increasing the diffusion rate of the magnetic field, has been recognized by Petscheck [1964] and Levy et al. [1964]. In the present case the slow rarefactive wave contributes to the strong diffusion of momentum or dissipation of vorticity as was seen in Figure 16 through the dynamo action. Such an increase in the dissipation rate of the vorticity by the hydromagnetic Kelvin-Helmholtz instability is reasonable from the following analogy with ordinary hydrodynamics: in ordinary hydrodynamics the kinematic molecular viscosity dissipates the flow kinetic energy at a rate proportional to the square of the inverse scale length of the eddy. In the present case, the flow kinetic energy is reduced and converted into magnetic energy, by the small-scale twisting and slow magnetosonic rarefaction, and this process gives rise to an anomalous (magnetic) viscosity in the sheared hydromagnetic fluid.

In order to clarify the energy transport process across the velocity shear layer, we show in Figure 17 spatial averages of different energy fluxes for the parallel configuration with $M_s = 1.0$, $M_A = 2.5$ (left panel) and for the transverse configuration with $M_s = M_A = 2.0$ (right panel) in the original inertial frame (i.e., the flow kinetic energy flux $1/2 \rho_0 v_o^2$, the Poynting flux $\rho_0 v_o v_x$, the Maxwell stress $\rho_0 v_o v_y$). Also shown in Figure 17 is the energy flux $\langle \rho_0 v_x v_y \rangle$, which appears in the magnetospheric inertial frame due to finite compressibility. In this figure, all positive fluxes point in the positive $x$ direction. As is expected from the choice of an antisymmetric velocity profile (6), all energy fluxes are antisymmetric in both configurations, and the net energy transport is zero in this frame of reference (see equation (16)). In the magnetospheric inertial frame, however, the energy flux is given by (18), namely, the tangential stress in the magnetospheric inertial frame multiplied by $v_0$. In the parallel configuration, we see from Figures 16 and 17 that $\langle B_x B_y v_0 \rangle > |\langle \rho_0 v_x v_y \rangle| > |\langle 1/2 \rho_0 v_o v_x \rangle|$. Therefore the net energy in this case is transferred mainly across the velocity shear layer by the Poynting flux. Notice that this Poynting flux carries along with it momentum as well as the energy. In the transverse configuration, we see from Figures 16 and 17 that $|\langle \rho_0 v_x v_y \rangle| > |\langle 1/2 \rho_0 v_o v_x \rangle|$. Therefore, the Reynolds stress due to the convection (vortex) cell is primarily responsible for the net energy transport, although the Poynting flux also gives a substantial contribution. For both cases, the peak stresses occur at $x = 0$ and become $\geq 10^{-2} \rho_0 v_o^2$ (Table 2). Consequently, the anomalous energy fluxes become $\geq 10^{-2} \rho_0 v_o^2$ for both configurations. This implies that the anomalous incident energy flux into the magnetosphere caused by the instability reaches $\geq 2\%$ of the initial flow kinetic energy flux far from the shear layer (magnetosheath).

Shown also in Figure 17 are spatial averages of Joule dissipation, i.e., the work done by the $\mathbf{J} \times \mathbf{B}$ force $\langle \mathbf{E} \cdot \mathbf{J} \rangle = \langle \mathbf{v} \cdot (\mathbf{J} \times \mathbf{B}) \rangle$. For both configurations, the averaged Joule dissipation is negative at $x = 0$. Accordingly, the $\mathbf{J} \times \mathbf{B}$ force decelerates the flow, and the plasma flow kinetic energy is converted into magnetic energy. This provides us with additional evidence that the Kelvin-Helmholtz instability constitutes a dynamo process [e.g., Batchelor, 1959; Moffatt, 1978] whereby the flow kinetic energy is converted into magnetic energy.

6. Summary and Discussion

We have investigated with a MHD simulation the magnetohydrodynamic Kelvin-Helmholtz instabilities for two basic configurations, parallel ($B_b || v_0$) and transverse ($B_b \perp v_0$). The principal results obtained for each configuration by the simulation may be summarized as follows.

6.1. Summary

6.1.1. Parallel configuration.
1. For $M_A = 2.5$ and $M_s = 1.0$, the instability leads to oscillation of the velocity shear (boundary) layer in association with a bending of the magnetic field line with a ratio of wavelength to amplitude of oscillation $> 10$.
2. For hyper-Alfvénic shear flow ($M_A > 4$, with $M_s = 1.0$), the magnetic field line is strongly compressed, twisted, and hence amplified (dynamo action) by the vortex motion induced by the instability, and the initial parallel shear flow develops into a highly irregular flow (turbulence) with perturbations of a much smaller scale size.
3. The plasma is squeezed out of the magnetic flux tube, which is compressed on both sides by the incoming plasma flow associated with the vortex, to form a slow rarefaction layer. This squeezing process becomes much stronger with an increase in $M_A$ or plasma $\beta$. As a consequence of the squeezing process, the magnetic field is intensified with an amplification factor $\sim M_A^2/2$ and the plasma $\beta$ inside the squeezed flux tube decreases considerably to the order of unity.
4. The anomalous Maxwell stress caused by the instability reaches $0.02 \rho_0 v_o^2$ and the anomalous energy flux in the magnetospheric inertial frame reaches $4\%$ of the flow kinetic energy density far from the shear layer. The anomalous magnetic viscosity due to the Maxwell stress reaches $2.6 \times 10^{-2} \ aV_0$, which is much larger than the hydrodynamic eddy viscosity. Therefore the MHD wave or slow rarefactive wave (Alfvén wave in the incompressible limit) contributes to a strong anomalous diffusion of momentum or dissipation of vorticity.
5. The initial velocity shear is diffused and the shear layer widened several times by the anomalous momentum transfer across the shear layer.

6.1.2. Transverse configuration.
1. The instability results in a vortex flow, which accelerates and decelerates the original flow.
2. For $M_e = M_A = 2.0$ ($M_e = 1.41$), a pressure gradient induced by the acceleration eventually develops into a fast shock discontinuity, whereby the flow kinetic energy is dissipated and converted into thermal energy.

3. The anomalous Reynolds stress caused by the instability reaches as much as $0.01 \rho_0 V_0^2$, the net energy flux across the velocity shear layer becomes $\sim 2.0\%$ of the flow kinetic energy flux far from the shear layer (magnetosheath), and the eddy viscosity becomes $1.2 \times 10^{-2} a V_0$.  

4. The initial velocity shear is relaxed and the width of the velocity shear layer is increased by a few times by the anomalous momentum transfer by the instability.

### 6.2. Small-Scale Characteristics of the Nonlinear Kelvin-Helmholtz Mode

One of the obvious consequences of the Kelvin-Helmholtz instability seen in Figures 4–6 relevant to previous observational results near the magnetospheric boundary is the large amplitude oscillation of the velocity shear (boundary) layer seen for example, in the bottom panel of Figure 4. The present simulation has enabled us to calculate the amplitude of the oscillation and the speed of the boundary oscillation in terms of the total velocity jump $V_o$ and the thickness of the shear layer $\Delta = 2a$ as follows; at $y = 0$ in the bottom left panel of Figure 4, the original shear layer position is shifted as a whole in the $x$ direction by $\xi \approx 2a$. Therefore, if we define this as the amplitude of the boundary oscillation, we obtain for this case the ratio of the wavelength to the amplitude of the oscillation $\lambda/\xi \approx 13$, which is consistent with observational results of the boundary oscillation [Lepping and Burlaga, 1979]. Also, by dividing $\xi \approx 2a$ by $\Delta \approx 40 \times 2a/V_0$, the time it took to shift the boundary layer by $\xi$, we obtain the speed of the boundary oscillation $\nu_{osc} \approx V_0/40$ at $y = 0$. For a typical velocity jump across the magnetospheric boundary, $V_0 \approx 500$ km/s, we obtain $\nu_{osc} \approx 12$ km/s. This speed is within the range of the observed speed of the boundary oscillation or motion of the magnetospheric boundary [e.g., Aubry et al., 1971; Russell and Elphic, 1979]. These reasonable agreements between the simulated nonlinear K-H mode and the observed characteristics of boundary oscillations seems to suggest that the Kelvin-Helmholtz instability indeed occurs at the magnetospheric boundary, and it provides us with one explanation of the highly dynamic motion of the magnetospheric boundary. Without knowing the Alfvén mach number defined by the magnetic field parallel to the sheared flow, it is difficult to determine which of Figures 4–6, varying from a slightly bent magnetic field line to a tangled magnetic field line, is more applicable to the magnetospheric boundary. However, we should point out that the magnetic field line bending seen in the bottom right panel of Figure 4 resembles the observed magnetic field variations along the magnetopause [Southwood, 1979], which show clear finite amplitude sinusoidal variations of the magnetic field coherent within the distance of two ISEE satellites. The observed large amplitude nature of the wave $B_N \sim B_0 \geq 10\%$ (for the background magnetic field of $\sim 20\%$) is also consistent with simulation results in Figure 3, where $B_{max}/B_0 < 0.7$, although there has been no confirmation yet that the observed wave is a slow magnetosonic mode. In this respect, it is interesting to note that Lepping et al. [1981] have found a slow mode wave at Saturn’s dayside magnetopause with localized enhancement of the magnetic field [Smith et al., 1980], which seems to be consistent with a strongly localized enhancement of the magnetic field shown in Figure 8.

At the solar wind-magnetosphere interface, where the velocity changes from $V_o$ to zero, the excited wave has a real frequency $\omega_r = k V_o/2$. For the fastest growing mode, this becomes $\omega_r \sim V_o/(2\Delta)$ [Ong and Roderick, 1972; Miura and Prit-
chet, 1982]. Therefore for a reasonable thickness of the boundary layer, \( A \), geomagnetic pulsations with \( Pc \) 5 frequency range inside the magnetosphere, having an azimuthal mode number of the order of 10, and a phase velocity of a few 100 km/s (a fraction of the solar wind speed), may be understood as a consequence of the field line resonance tapped by the K-H instability at the magnetospheric boundary [Southwood, 1974; Chen and Hasegawa, 1974]. The present self-consistent nonlinear treatment of the instability has demonstrated (Figure 17) that at the expense of the flow kinetic energy in the magnetosphere, the nonlinear K-H modes yield substantial Poynting fluxes directed into the magnetosphere, which reach 1.4% (parallel configuration) and 0.4% (transverse configuration) of the flow kinetic energy \( \frac{1}{2} \rho V_a^2 \) of the magnetosphere. Notice that these energy fluxes associated with the evanescent eigenmode are different from a linear energy flux associated with the propagating mode, which exists only when the compressibility is present [Fu and Kivelson, 1983].

From Figure 2 we see that the fastest growing K-H mode has a growth rate typically given by \( \gamma \sim 0.1V_a/2a \). Hence

\[
\gamma/\omega_a = 0.2/(2k_a) = 0.2 - 0.4
\]

for \( 0.5 < 2k_a < 1.0 \). The e-folding distance \( l_e \), defined as the distance the K-H mode propagates during one e-folding time, can be calculated as

\[
l_e = (1/\gamma)(\psi/\omega_a) \sim (5/2)n \sim \lambda
\]

If we use the wavelength typical of the K-H mode observed at the magnetospheric boundary, \( \lambda \sim 2 \times 10^7 \) km [e.g., Lepping and Burlaga, 1979], the exponentialiation distance \( l_e \sim \lambda \) is much shorter than the dimension typical of the magnetosphere, \( \sim 60R_E \). Therefore, if we reasonably assume that the Kelvin-Helmholtz instability starts from near the subsolar point of the magnetopause, the K-H mode has enough travel time to develop fully into the nonlinear stage before reaching dawn and dusk meridians. Such an expectation is consistent with observations of Williams [1980], which suggest that the occurrence of a large amplitude K-H mode was observed period 0840–1040 LT of the magnetospheric boundary. Observation of large plasma vortices in the near-earth tail of the magnetosphere [Hones et al., 1981] may also further evidence that the K-H instability has experienced a large exponential growth before reaching the near-earth tail of the magnetosphere.

Finally, we should point out the relevance of the Kelvin-Helmholtz instability to reconnection on the magnetopause boundary. Our model has neglected the presence of a current layer at the magnetospheric boundary and thus the evolution of the current layer by the K-H instability is not clear from the present simulation. Nevertheless, the magnetic field structure seen in Figures 4–6 may be of interest in that the K-H driven vortices amplify the magnetic field. Such a dynamo action due to a change of the flow configuration from the initial parallel shear flow to the vortex flow yielding converging flow pattern pressing up the magnetic field lines might set a flow configuration favorable for the magnetic field reconnection process. The formation of the slow rarefaction layer might also be important in the reconnection process by increasing the Alfvén velocity and thus possibly the merging rate [Vasyliunas, 1975; Zwan and Wolf, 1976].

### 6.3. Anomalous Transport

We have seen in Table 2 that the momentum flux transferred by the instability into the magnetosphere reaches \( \leq 0.02 \rho RV_a^2 \) in the parallel configuration and \( \leq 0.01 \rho RV_a^2 \) in the transverse configuration. Notice that in contrast to the anomalous viscosity discussed below, these anomalous tangential stresses obtained for both configurations are independent of the thickness of the velocity shear layer and may well give the most reliable measures of the strength of the solar wind–magnetosphere interaction. These values of anomalous momentum fluxes (tangential stress) reaching 1~2% of the incident solar wind momentum flux seem to be just the right magnitude to drive the magnetospheric convection [Hill, 1979].

Let us now estimate the contribution of the K-H instability at the flank sides of the boundary to the magnetospheric convection by using the convection potential drop over the polar cap, which is a useful measure of the viscous interaction driving the magnetospheric convection; if we assume that the flank side of the low-latitude magnetospheric boundary (where the antisunward plasma flow velocity changes continuously from the magnetosheath speed to zero) is on the closed field line, the total potential drop across the boundary layer should be mapped onto the polar cap edge and would give a contribution to the antisunward plasma convection over the polar cap. We give a detailed analysis to derive a convection formula in another paper (A. Miura, unpublished manuscript, 1983) and here present only the essence of the analysis: if we assume that the momentum flux transferred into the magnetosphere in the transverse configuration (modeling the low-latitude boundary) is equal to the increase of the antisunward \( E \times B \) plasma momentum inside the low-latitude boundary layer, by simply integrating the electric field across the boundary layer we obtain

\[
\phi_{KH} = K(\mu_0 \rho V_a)^{1/2}(\rhoV_a^2/\mu_0)^{1/4}V_0^2 \sin 2\psi
\]

where \( \phi_{KH} \) is the integral across the boundary layer of the electric field component normal to the boundary layer, \( K \) is the value of the tangential stress divided by \( \rhoV_a^2 \), \( \lambda \) is the wavelength of the K-H mode, \( v_e \) is the magnetosheath speed, \( \psi \) is the angle between the solar wind flow and the boundary normal vector, and \( \rho \) and \( \mu \) are mass densities in the magnetosheath and in the boundary layer, respectively. For reasonable parameters set at the magnetopause (see A. Miura, unpublished manuscript, 1983) for details), we obtain from this \( \phi_{KH} \sim 10 \text{ kV} \). Assuming equal dawn and dusk contributions and taking into account the variability of the parameters used, we multiply this by 2 and obtain 10~30 kV as a rough bound of the contribution of the K-H instability to the convection potential drop over the polar cap. Such a convection potential drop independent of the interplanetary magnetic field (IMF) and hence independent of the dayside reconnection might cause the observed residual convection potential drop over the polar cap, which is observed to be \( \sim 35 \text{ kV} \) [Reiff et al., 1981]. As we discussed previously, such a viscous interaction would be set up near dawn and dusk meridians, where the K-H instability has developed fully into a nonlinear stage to exert a large viscous tangential stress. It is important to emphasize here that the above potential drop of 10~30 kV was obtained by using the anomalous momentum flux of the transverse configuration assuming that the low-latitude boundary is well represented by the transverse configuration. But if the boundary involves the magnetic field parallel to the flow, the anomalous tangential stress \( \sigma \) is doubled, and the above potential drop may also be doubled.

We have seen in section 5 that the anomalous viscosity \( \nu_{sao} \) of the instability at \( x \sim 0 \) becomes \( \sim 2\times10^{-2} aV_0 \) in the
parallel configuration and $10^{-2} a V_0$ in the transverse configuration. If we substitute these anomalous viscosities into the effective Reynolds number $Re = a V_0 / \nu_{\text{me}}$, we obtain $Re \sim 40$ for the parallel configuration and $Re \sim 100$ for the transverse configuration at the magnetospheric boundary. Hence, it is expected that the global magnetospheric convection induced inside the magnetosphere is close to what one would expect from the viscous interaction at the magnetospheric boundary with $Re \sim 40-100$. In the magnetopause boundary layer, the velocity shear scale length $\Delta = 2a$ is usually larger than the scale of the magnetic field change [Eastman and Hones, 1979], which is typically $\gtrsim 5 \rho_{Li}$ [e.g., Berchem and Russell, 1982], where $\rho_{Li}$ is the typical ion Larmor radius. Therefore, in order to obtain the lower bound for the anomalous viscosity, we use $a > \rho_{Li}$ and $V_0 \sim v_t$ (typical ion thermal speed) to obtain $\nu_{\text{me}} \gtrsim 0.01 \sim 0.03 \rho_{Li}$ [Eastman and Hones, 1980]. Under conditions typical in the magnetopause boundary layer, $T_i \sim 10 T_e$ [e.g., Paschmann et al., 1978], this value of the anomalous viscosity becomes as large as or even larger than the Bohm diffusion $D_B = T_i / (16 e B_0)$, which is usually regarded as the upper bound of anomalous particle diffusion in the low $\beta$ plasma. The ratio of the anomalous viscosity by the K-H instability to the classical ion- ion viscosity across the magnetic field [Braginskii, 1966] is given by $\nu_{\text{me}} / \nu_{\text{i-i}} = 10^{-2} (\alpha_i - \tau_i)^3$, where $\alpha_i$ is the ion gyrofrequency and $\tau_i$ is the ion-ion collision time. At the magnetospheric boundary, where $\alpha_i \tau_i \sim 10^{14}$, the classical ion-ion viscosity is, of course, negligible compared with the anomalous viscosity by the Kelvin-Helmholtz instability. For parameters typical at the boundary, $a > \rho_{Li} \sim 250$ km and $V_0 \sim v_t \sim 400$ km/s, we obtain $\nu_{\text{me}} \gtrsim 10^{13}$ erg cm$^{-2}$/s for both configurations. This anomalous viscosity is comparable to the kinematic viscosity required for the viscous-like interaction hypothesis [Axford and Hines, 1961]. Therefore, from this estimate of the viscosity, we again obtain the important indication that the Kelvin-Helmholtz instability contributes importantly to the magnetospheric convection.

Here, we would point out the following regarding the interpretation of the anomalous transport in the solar wind-magnetosphere interaction. We have assumed in the initial setting of the simulation that the initial equilibrium involves a finite thickness of the velocity shear layer (2a). Actually, the lower limit of the thickness of the velocity shear layer near the nose of the magnetopause may be set by the finite ion Larmor radius of typical ions [e.g., Parker, 1967; Lee and Kan, 1979]. Therefore our model should take the original shear layer thickness to correspond to a thickness of $\gtrsim \rho_{Li}$. Consequently, we have implicitly assumed an existence of an initial small momentum transport corresponding to the potential drop of the order of the typical ion thermal energy divided by the electronic charge (which is typically $\sim 1$ keV), which cannot be self-consistently taken into account in the present MHD simulation. Accordingly we do not address the formation of the boundary layer of thickness $\sim \rho_{Li}$ in this paper, but we have addressed the widening of and the transport through the initial thickness of the velocity shear (boundary) layer by the K-H instabilities to form a boundary layer of a thickness much larger than $\sim \rho_{Li}$. Since the observational results show that the boundary layer is much wider than the ion Larmor radius [Eastman and Hones, 1979], it seems quite likely that the observed boundary (velocity shear) layer has experienced a widening of the velocity shear layer by the nonlinear saturation of the K-H instabilities as we have seen in Figure 16.

Albeit our simulation model is rather simple, the present results illuminate the nature of the anomalous transport and show that the MHD Kelvin-Helmholtz instabilities are quite different for the two distinct configurations of flow and magnetic field that we have considered. In the transverse configuration the viscous interaction is due to the hydrodynamic Reynolds stress. If it is applied to the low-latitude boundary, it could account for a convection potential drop over the polar cap of $\sim 10^{-3}$ to $3 \times 10^{-4}$, which is approximately $10^{14}$ erg cm$^{-2}$/s to satisfy the requirements for the viscous-like interaction hypothesis by Axford and Hines [1961]. But where the configuration involves the magnetic field, the shear flow, the viscous interaction is essentially due to the hydromagnetic Maxwell (magnetic) stress, which gives rise to transport 2–3 times larger than that in the transverse configuration. Since the actual magnetospheric boundary appears to be constantly in motion and highly dynamic or turbulent in both flow and magnetic field components, the important role of the magnetic stress in the transport process should particularly be emphasized. The extension of the present simulation model to a more realistic configuration, including current layer, at the magnetospheric boundary is straightforward and will be discussed elsewhere.

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