Formation of helical domain walls in the fractional quantum Hall regime as a step toward realization of high-order non-Abelian excitations

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We propose an experimentally feasible platform to realize parafermions (high-order non-Abelian excitations) based on spin transitions in the fractional quantum Hall effect regime. As a proof of concept we demonstrate a local control of the spin transition at a filling factor 2/3 and formation of a conducting fractional helical domain wall (fhDW) along a gate boundary. Coupled to an s-wave superconductor these fhDWs are expected to support parafermionic excitations. We present exact diagonalization numerical studies of fhDWs and show that they indeed possess electronic and magnetic structures needed for the formation of parafermions. A reconfigurable network of fhDWs will allow manipulation and braiding of parafermionic excitations in multigate devices.

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Topological quantum computation can be performed with Majorana fermions (MF) [1], but MF-based qubits are not computationally universal [2]. Parafermions (PFs), higher-order non-Abelian excitations, are predicted to have denser rotation group and their braiding enables two-qubit entangling gates [3,4]. A two-dimensional array of parafermions can serve as a building block for a system which supports Fibonacci anyons with universal braiding statistics [5], a holy grail of topological quantum computing. In an important conceptual paper Clark et al. proposed that PF excitations can emerge in the fractional quantum Hall effect (FQHE) regime if two counterpropagating fractional chiral edge states with opposite polarization are brought into close proximity in the presence of superconducting coupling [6]. Here we propose that domain walls formed at spin phase transitions in the FQHE regime have the prerequisite helical structure to support PF excitations when coupled to an s-wave superconductor. We demonstrate experimentally that in a triangular quantum well a 2D system can be tuned across a spin transition at a filling factor \( \nu = 2/3 \) using electrostatic gating. We also demonstrate formation of conducting channels at boundaries between incompressible polarized and unpolarized \( \nu = 2/3 \) states. These channels are formed from two counterpropagating \( \nu = 1/3 \) states with opposite spin orientation; we will refer to them below as fractional helical domain walls (fhDWs) in analogy to helical channels formed along the edges in the quantum spin Hall effect. Local control of polarization allows formation of a reconfigurable network of fhDWs with fractionalized charge excitations and, potentially, parafermion manipulation and braiding. We present exact diagonalization numerical studies of fhDWs.

Helical channels are commonly associated with the quantum spin Hall effect [7], topological insulators [8], or nanowires with spin-orbit interactions [9,10], where Coulomb interactions are not strong enough to fractionalize charges. A natural system to look for PFs is a 2D electron gas (2DEG) in the FQHE regime, where edge states support fractionally charged excitations. In the conventional QHE setting, though, edge modes are chiral. Helical channels can potentially emerge as domain walls during a quantum Hall ferromagnetic transition. It has been predicted that domain walls formed in the integer QHE regime at a filling factor \( \nu = 1 \) have helical magnetic order [11]. Experimentally, local electrostatic control of domain walls in the integer QHE regime at \( \nu = 2 \) was recently demonstrated in magnetic semiconductors [12], and their electronic and magnetic structure has been calculated [13].

In the FQHE regime spin transitions have been observed at a filling factor \( \nu = 2/3 \) and other fractions [14,15]. At the transition, the 2DEG spontaneously phase separates into regions of different spin polarizations, and conducting domain walls are formed along the domain boundaries [16,17]. An experimental challenge is to devise a system where spin transitions in the FQHE regime can be controlled locally, allowing formation and manipulation of DWs. Theoretically, neither magnetic nor electronic structure of these domain walls is known.

It has been realized that hard \( \nu = 2/3 \) edge states at the 2D gas boundaries are complex objects with chiral downstream propagation of charge excitations and neutral energy-carrying modes propagating both up- and downstream [18–20]. Recent experiments show that two copropagating \( \nu = 1/3 \) charge modes at the edge of a sample are weakly interacting and can be spatially separated [21]. Thus, a domain wall formed between two spin polarized domains at \( \nu = 2/3 \) ferromagnetic transition can be formally constructed from two counterpropagating
Calculated for two different values of the wave-function extent at a GaAs/AlGaAs heterojunction interface. Solid and dotted lines correspond to the two different gate voltages and show the change of \( z_0 \). Note the break in the horizontal axis.

\[
v = 1/3 \text{ chiral charge modes with opposite spin polarization, similar to the domain walls formation in the integer quantum Hall ferromagnetic transition [13].}
\]

Spin transitions in the FQHE regime can be readily understood within the framework of the theory of composite fermions (CFs) [22], where FQHE states at filling factors \( v = v_f/(2v_a - 1) \) for \( 1/2 < v < 1 \) are mapped onto integer QHE states with a filling factor \( v \) for CFs. The energy spectrum of CF \( \Lambda \) levels with an index \( p = 1, 2, 3 \ldots \) can be written as

\[
E_p^{\Lambda} = \hbar \omega_c \left( \frac{p - 1}{2} \right) \pm g \mu_B B. \tag{1}
\]

The CF cyclotron energy \( \hbar \omega_c \) is proportional to the charging energy \( E_c = e^2/\sqrt{r^2 + \frac{e}{g}} \), where \( r_m \propto \sqrt{B} \) is the magnetic length, \( B \perp z \) is the out-of-plane component of the magnetic field, and \( z_0 \) is the extent of the wave function in the out-of-plane direction. The second term is the Zeeman energy. Due to the difference in \( B \) dependencies of the two terms, levels \( \Lambda_{p,1} \) and \( \Lambda_{p+1,1} \) cross at some \( B^* > 0 \); see Fig. 1. Thus, for \( v > 1 \) (two \( \Lambda \) levels are filled) the top level energies undergo a spin transition at \( B^* \). The \( v = 2/3 \) state is unpolarized for \( B < B^* \) and fully polarized for \( B > B^* \).

Conventionally, FQHE spin transitions are studied in tilted magnetic fields, where global control of the field angle \( \theta \) changes the ratio of Zeeman and cyclotron energies. For a triangular confinement, though, \( z_0 \) is gate dependent, \( z_0 = z_0(V_g) \) (see Fig. 1(b)), and local control of \( E_c \) and \( B^* \) at a fixed \( B \) becomes possible. Within the Fang-Howard approximation of the wave function in a triangular well, \( z_0 = 3/b \), where \( b \propto n^{1/3} \) is a function of electron density. For GaAs parameters and \( B^* \approx B_{\nu=2/3} \approx 4–6 \text{T} \), the field \( B^* \) becomes density and gate dependent: \( \delta B^*/B^* \approx 0.38 n/\nu \), \( n/\nu = \delta V_x/V_x \). The field position of the \( v = 2/3 \) state is also density and gate dependent, \( \delta B_{\nu=2/3}/B_{\nu=2/3} \approx 0.38 n/\nu \). Thus, for a well-developed \( v = 2/3 \) state and a sharp spin transition there should be a range of magnetic fields where spin polarization of the top level can be tuned locally by electrostatic gating.

In order to demonstrate electrostatic control of polarization we have grown a number of wafers where high mobility 2D electron gas is confined at a single GaAs/AlGaAs interface.

Inverted GaAs/AlGaAs heterojunctions are grown by molecular beam epitaxy, the top layer is 130–230 nm thick GaAs, Si \( \delta \)-doping placed 70–300 nm beneath the heterojunction interface. The top 25 nm of GaAs are lightly doped to reduce the surface pinning potential. In the main text we present data on devices fabricated from wafers A (Fig. 2) and D (Fig. 3), wafers parameters can be found in [23]. Inverted heterostructures allow electrostatic gating of a shallow 2D gas with no hysteresis, also in similar wafers proximity-induced superconductivity has been reported [24]. Ohmic contacts are formed by annealing Ni/AuGe/Ni/Au 6 nm/120 nm/20 nm/20 nm in a H\(_2\)/N\(_2\) atmosphere. 10-mm-thick Ti gates are separated from GaAs and from each other by 50 nm Al\(_2\)O\(_3\) grown by an atomic layer deposition (ALD). The gates are semitransparent and a 2D electron gas is created by shining red LED at \( \approx 4 \text{ K} \). Measurements were performed in a dilution refrigerator with the base temperature \( T \approx 30 \text{ mK} \) using a standard lock-in technique with excitation current \( I_{ac} \) = 0.1–10 nA.

Magnetoresistance in the vicinity of \( v = 2/3 \) plateau is shown in Fig. 2(a). At the base temperature a 0.35-T-wide incompressible state is interrupted by a small peak at \( B^* = 4.94 \text{ T} \). This peak has all the characteristics of a spin phase transition studied in the past, including strong current dependence and hysteresis with respect to the field sweep direction, which appears at high bias currents [15]. We identify this peak with the spin transition; unpolarized \((u)\) and polarized \((p)\) \( v = 2/3 \) states are highlighted with yellow and cyan on the plot. The most important data is shown in Fig. 2(b), where resistance is plotted as a function of a gate voltage \( V_g \) measured at a fixed magnetic field. Similarly to the field scan, an 18-mV-wide \( v = 2/3 \) plateau is interrupted by a small peak at \( V_g^* = -151 \text{ mV} \), which we identify with the spin transition. The transition peak is narrow, \( \approx 6 \text{ mV} \) with well-defined \( v = 2/3 \) states with different polarization on the two sides of the peak. Thus, it is possible to control spin polarization locally by electrostatic gating.

Formation of a conducting domain wall at a boundary between unpolarized and polarized \( v = 2/3 \) regions is shown in Fig. 3. A 2D gas is separated into two regions where density is independently controlled by gates \( G_1 \) and \( G_2 \). The length of the gate boundary is \( 7 \mu \text{m} \) for the shown device. Resistances \( R_1 \) and \( R_2 \) for the 2D gases under the two gates are combined into a single plot in (c) in order to visualize
regions in the \((V_{g1}-V_{g2})\) plane where FQHE states on both sites of the boundary overlap (a small coupling between the gates results in slightly nonorthogonal evolution of the features). A small bump in the middle of the \(\nu = 2/3\) state is the spin transition (the data is taken with high \(I_{ac} = 1.3\) nA in order for the transition to be visible above the noise level), and separates the 2/3 region into four quadrants with different polarizations across the gate boundary. In (d) resistance measured across the gate boundary is plotted as a function of both gate voltages. In the region outlined red incompressible 3/5 states are formed on both sites of the gate boundary and \(R = 0\). A chiral channel is formed between 2/3 and 3/5 states (two regions outlined black). In this case resistance is gradient and field direction dependent: \(R = 0\) or \(R = 1/6R_q\), where \(R_q = h/e^2\). Within the 2/3 state \(R = 0\) in the \((pp)\) and \((uu)\) quadrants, indicating formation of a well defined incompressible state under the gate boundary. When polarization of the 2/3 state changes across the gate boundary \(R\) becomes nonzero indicating formation of a conducting channel. Resistance \(R \sim 3–5\) kΩ does not depend significantly on the direction of the density and polarization gradient \((up\ or\ pu)\) nor on the magnetic field direction, consistent with the formation of a helical domain wall. In the current geometry resistance of the fhDW is not measured directly, within Landauer-Büttiker formalism we extract \((10–20)R_q\) channel resistance for 2–7-μm-long fhDWs with no clear scaling with the length. The lack of scaling may indicate that scattering predominantly occurs in hot spots formed at trijunctions where fhDWs merge with edge states.

In order to investigate the structure of domain walls formed between spin-unpolarized and spin-polarized regions we performed exact diagonalization studies of a system with small number of particles. To simulate edge states we use the disk geometry [25,26] shown schematically in Fig. 4. Long-range Coulomb interactions between electrons are introduced using Haldane pseudopotentials. A neutralizing background and a confinement potential are used to hold electrons inside the disk. As is evident from experiments, the spin transition can be controlled by modulation of either Coulomb or Zeeman energies interchangeably; see, e.g., Fig. 2. In our modeling we use spatially dependent Zeeman effect to control spin polarization of the 2DEG. The central region of the disk of radius \(R_1 = 2.9l_m\) is characterized by a high Zeeman energy term \(E_{Z}^{\text{max}}\), while the outer region with the outer diameter \(R_2 = 4.8l_m\) is set to \(E_{Z}^{\text{max}} = 0\). The Zeeman term varies smoothly within \(R_1 < r < R_1 + \Delta R\), where \(\Delta R = 0.4l_m\), resulting in a smooth variation of wave functions across the disk and avoiding spurious effects originating from abrupt changes. Note that due to a strong penetration of electron wave functions...
of the spatially dependent Zeeman interaction used to form the domain wall. The ground state for eight particles has the total angular momentum projection \( L_z \) for unpolarized states or coexisting polarized and unpolarized states will emerge at the boundary of topological and trivial \( s \)-wave superconductors, as predicted in Ref. [6].

In summary, we propose that domain walls formed during ferromagnetic spin transitions in the fractional quantum Hall effect regime can be used as building blocks to form topological superconductors that support parafermion excitations. Exact diagonalization study of spin transitions in a disk geometry confirms that domain walls, formed between spin-up and spin-down domains at \( v = 2/3 \), indeed possess electronic (two counterpropagating modes) and magnetic (opposite spin orientation for the two modes) structure needed to couple to an \( s \)-wave superconductor. We demonstrate that in triangular quantum wells spin transitions can be controlled locally by electrostatic gating and conducting helical domain walls can be formed in multigate devices. Such local control allows formation of reconfigurable networks of domain walls. In the presence of proximity-induced superconducting coupling the system becomes a reconfigurable network of one-dimensional topological superconductors with parafermion excitations.


T.W., Z.W., and A.K. contributed equally to the experimental part of the work.


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