Supplementary Materials

Formation of helical domain walls in the fractional quantum Hall regime, a step toward realization of high order non-Abelions

Taihong Wu, Zhong Wan, Aleksandr Kazakov, Ying Wang, George Simion, Jingcheng Liang, Kenneth W. West, Kirk Baldwin, Loren N. Pfeiffer, Yali Lyanda-Geller, and Leonid P. Rokhinson

Comparison of spin transition in different wafers

Parameters of four different wafers used in this study are summarized in Table S1 and gate control of ferromagnetic spin transition in these wafers is demonstrated in Fig. S1.

<table>
<thead>
<tr>
<th>wafer</th>
<th>$d_1$ [nm]</th>
<th>$d_2$ [nm]</th>
<th>$n \times 10^{11}$ [cm$^{-2}$]</th>
<th>$\mu \times 10^6$ [cm$^2$/Vs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>135</td>
<td>110</td>
<td>1.3</td>
<td>3.8</td>
</tr>
<tr>
<td>B</td>
<td>135</td>
<td>70</td>
<td>1.6</td>
<td>4.1</td>
</tr>
<tr>
<td>C</td>
<td>185</td>
<td>270</td>
<td>0.8</td>
<td>3.7</td>
</tr>
<tr>
<td>D</td>
<td>135</td>
<td>160</td>
<td>0.8</td>
<td>5.0</td>
</tr>
</tbody>
</table>

TABLE S1. Parameters of several wafers used in the study. Distance $d_1$ is the thickness of GaAs layer, and $d_2$ is the $\delta$-doping setback below the heterointerface.

FIG. S1. Filling factor $\nu = 2/3$ spin transition measured in single gate Hall bar samples in different wafers. Dashed magenta line marks the spin transition.
Quantum Hall ferromagnets exhibit a phase transition between spin-polarized and spin-unpolarized states [1]. It was shown in [2, 3] that using electrostatic gates that make exchange interactions in a system position-dependent, it is possible to create both polarized and unpolarized state in a single sample with domain wall separating regions with different spin polarization. In Ref [3], an analytic theory of domain walls for integer quantum hall effect ferromagnets was presented. It was further demonstrated that these domain walls lead to topological superconductivity when proximity-coupled to an s-type superconductor, with Majorana fermions forming at the boundary between trivial and topological superconducting phases.

In investigation of fractional quantum Hall effect, edge states at the boundaries of the quantum Hall system and edge modes flowing around antidots and constrictions has been intensively studied over the years [47 –7]. In particular, at a fractional filling factor $\nu = 2/3$, edge modes have been studied in both polarized and various kinds of unpolarized phases [8–11]. However, microscopic consideration of the boundary between polarized and unpolarized phases presents a challenge. Here we demonstrate emergence of a domain wall between polarized and unpolarized states in fractional quantum Hall ferromagnets by using a method of exact diagonalization in a system with small number of particles.

In order to understand the range of parameters we first obtain the quantum phase transition between spin-polarized and spin-unpolarized states using exact diagonalization on the Haldane sphere with a monopole charge $2Q$ producing a magnetic field[12]. The Haldane pseudopotentials, characterizing the interactions on the sphere, are defined by electron wavefunctions in the triangular quantum well and depend on the electric field. For $N$ particles on the sphere at a filling factor $\nu = 2/3$, we obtain that the polarized state emerges at $2Q = 3/2N$ and unpolarized state arises at $2Q = 3/2N - 1$. As these monopoles are different, we introduce a regularization procedure yielding the difference between energies of polarized and unpolarized states $\Delta = (E_p - E_u)_{3/2N}$. Where $E_p$ and $E_u$ are ground state energies for $2Q = 3/2N$ and $2Q = 3/2N - 1$, correspondingly. At $\Delta > 0$ the unpolarized state is the ground state. We find that an electric field of $4.8 \times 10^4$ V/cm induces the spin polarization transition.

For creating reconfigurable network of helical domain walls in fractional quantum Hall ferromagnets, it is crucial to understand physics of edge states generated by using electrostatic gates or in the presence of varying g-factor. In previous works edge states have been studied numerically only in ideal fractional quantum Hall systems near sample boundaries [8], but not at the boundary between polarized and unpolarized phases. The Haldane sphere cannot be used for simulation of such a boundary because spin-polarized and spin unpolarized states appear at different monopole strengths. Investigation of edge states and their control using gates or inhomogeneous g-factors can be numerically studied using disk geometry [13, 14].

We simulate a system of electrons in a magnetic field in a quantum Hall ferromagnet on the disk. Long range Coulomb interactions among electrons are introduced using Haldane pseudopotentials. A neutralizing background and a confinement potential are used to hold the electrons together. We use a parabolic confinement $U(r) = Cr^2$, where $C = 0.036e^2/\epsilon l_m^3$. A realistic confinement involves placing a neutralizing background close to the 2D electron gas. However, Laughlin states are observed only if the distance between the electron gas and the background is smaller than one magnetic length, which is much smaller than the characteristic experimental separation of the doping layer from the 2D gas. Small separation of background charge from the disk is a limitation of the disk model.

In our modeling we use spatially-dependent Zeeman effect to control the spin polarization of the 2DEG. As is evident from experiments, spin transition can be controlled by modulation of either Coulomb or Zeeman energies interchangeably, see e.g. Fig. 77. Introducing spatially-dependent Zeeman splitting has computational advantages. In our model, the central region of the disk of radius $R_1 = 2.9l_m$ is characterized by a large Zeeman term $E_Z^{max}$, while the outer region with the outer diameter $R_2 = 4.8l_m$ is set to $E_Z^{min} = 0$. The Zeeman term is varied smoothly within the region $R_1 < r < R_1 + \Delta R$, where $\Delta R = 0.4l_m$, resulting in a smooth variation of wavefunctions across the disk and avoiding spurious effects originating from abrupt changes. Note that due to a strong penetration of electron wavefunctions from the $R_1 < r < R_2$ region into the $r < R_1$ region the difference of the average spin splitting $\int \psi(r)^+ E_Z(r) \psi(r) d^2r$ for the modes on the two sides of the domain wall is $< 6\%$, similar to the experimental conditions. Therefore, our model reflects soft edge characterizing the experiment.

The total electron Hamiltonian is given by

$$\mathcal{H} = \frac{1}{2m} \sum_i (\mathbf{p} + eA_i)^2 + \frac{\hbar^2}{2m} \frac{\nabla^2}{L_y} + U_i$$

(S1)

The Hamiltonian is diagonalized using a configuration interaction method. The states are classified according to their (conserved) projection of total angular momentum on $z$-axis, $L_z$, and total spin of electrons. The main challenges are the consideration of spin unpolarized states as their Hilbert space is significantly increased by the presence of both spin...
orientations of electrons, and the consideration of a system that includes both spin-polarized and spin-unpolarized regions.

We place $N$ electrons on a disk, allowing only single particle states with $0 \leq m \leq 3N/2 - 1$, resulting in $R_2 = \sqrt{3N - 2l_m}$. Only states from the lowest Landau level $n = 0$ with spin up and spin down polarizations are included. The region of polarized states is characterized by a radius $R_1 = l_m\sqrt{3N/2 - 1}$. The typical density distribution has two contributions: from the spin polarized region in the interior of the disk and from the spin-unpolarized region in the exterior of the disk. The electron density for $r > R_1$ is composed of the tails of the wavefunctions and is a manifestation of soft confinement of charge carriers that we choose. Spin polarization varies from 1 inside the disk to zero closer to $R_2$.

We have included up to 12 electrons in the exact diagonalization calculation for the polarized state at a filling factor $2/3$, and 8 electrons for unpolarized phases and modeling of polarized and unpolarized phases in a disk with spatially dependent Zeeman term. We identified the ground state, which is spin-polarized in the center and unpolarized in the outer region of the disk, as well as the edge states flowing close to the boundary between spin-polarized and spin-unpolarized regions. Energies and wavefunctions for edge states, their density and spin density distributions on the disk have been calculated. For 8 particles, the ground state corresponds to the total angular momentum projection $L_z = 46$ and total spin of 8 particles $S_z = 2$. That is, ground state of the disk with inhomogeneous Zeeman splitting is given by $3N/4$ electrons in the spin-up states and $N/4$ electrons in the spin-down states. In the composite fermion transformation, two vortices are attached to each of them and they completely fill the lowest composite fermion $\Lambda$-level with spin up ($0 \leq m \leq N/2 - 1$) and partially fill two composite fermion $\Lambda$-levels with spin up ($-1 \leq m \leq N/2 - 2$) and spin down ($N/2 \leq m \leq N - 1$). The composite fermion transformation defines the angular momentum of the ground state:

$$L_z = L_z^{CF} + pN(N-1) = -\frac{N(N-3)}{4} + N(N-1) = \frac{N(3N-1)}{4}.$$  \hspace{1cm} (S2)

For $N = 8$, $L_z = 46$ indeed, coinciding with the result of our numerical simulation. The ground state is separated by the gap from the rest of the spectra, Fig. S3, and does not carry current. Ground state is spin polarized in the interior part of the disk and spin-unpolarized in the exterior area, as expected Fig. S4.

The lowest energy excitations which have spin polarization of the ground state and correspond to a single flux addition or subtraction from the ground state have $L_z = 45$ and $L_z = 47$, see Fig. S3. These are the modes that carry electrical current. In a disk geometry these two modes have different angular velocities. When mapped onto a plane, these states will have different linear velocities, i.e. have counter-propagating components. In Fig. S5, we show the results for the difference of spin densities of the two modes near the domain wall between polarized and unpolarized region. Despite finite size effects in a small system, the exact diagonalization allows clearly identify that the two edge states in the domain wall region have components of spin density with opposite orientation. Thus, having opposite components of velocity and spin, these states can potentially be coupled to an s-type superconductor, a pre-requisite for generating topological superconductivity. In integer quantum Hall ferromagnets [3] proximity superconducting coupling has resulted in topological superconductivity in the domain wall region and Majorana zero modes at the boundaries between superconducting regions with non-trivial topological and trivial s-type order. In the fractional quantum Hall ferromagnet, because of the difference in the degeneracy of composite fermion $\Lambda$-levels, the parafermion modes are expected to arise at the boundary of topological and non-topological superconductor[15].
FIG. S2. Model for simulation of edge states near electrostatically induced boundary; (a) disk with three regions; (b) profile of Zeeman interactions.

FIG. S3. Spectra of 8 electrons on the disk with profile of Zeeman interactions shown in Fig.S2. The spectra are characterized by total angular momentum $L_z$ and total spin of particles $S_z$. Ground state at $L_z = 46$ and $S_z = 2$ is circled red. Edge excitations with the same $S_z = 2$ as in the ground state and with $L_z = 45, 47$, which correspond to the addition or subtraction of a single flux, are circled black.
FIG. S4. The ground state electron density (red) and spin density (blue) for 8 electrons on a disk containing the domain wall between polarized and unpolarized states at a filling factor 2/3 in a magnetic field.

FIG. S5. Difference of spin densities of the two edge states with $L_z = 45$ and $L_z = 47$ in the region of the disk containing the domain wall between polarized and unpolarized states at a filling factor 2/3 in a magnetic field.


