PEAK VALUES OF CONDUCTIVITY IN INTEGER AND FRACTIONAL QUANTUM HALL EFFECT

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The diagonal conductivity \( \sigma_{xx} \) was measured in the Corbino geometry in both integer and fractional quantum Hall effect (QHE). We find that peak values of \( \sigma_{xx} \) are approximately equal for transitions in a wide range of integer filling factors \( 3 < \nu < 16 \), as expected in scaling theories of QHE. This fact allows us to compare peak values in the integer and fractional regimes within the framework of the law of corresponding states.

Keywords: D. quantum Hall effect, D. fractional quantum Hall effect

Since the discovery of the quantum Hall effect (QHE) considerable attention has been devoted to the transition region between quantum Hall plateaus. Experiments on low mobility samples showed that peak values of the diagonal conductivity are Landau-level dependent (\( \sigma_{xx}^{\text{peak}} \propto \nu \)), as predicted by Ando and Uemura. Similar dependence was observed in high mobility samples for transitions at high filling factors. However, the scaling theory of QHE predicts that peak values of \( \sigma_{xx} \) in the integer QHE (IQHE) should be universal, independent of the filling factor. A crossover from the Ando to the scaling regime should occur at magnetic fields such that the magnetic length is comparable to the range of the disorder potential. In the scaling regime the extended states exist only near a single energy in each Landau level; a phase transition occurs when the chemical potential crosses from the localized states of i-th Landau level (\( \nu = i \) QHE plateau) through the extended states (peak of \( \sigma_{xx} \) at \( \nu \approx i + 1/2 \)) to the localized states of the (i + 1)-st Landau level (\( \nu = i + 1 \) QHE plateau).

The scaling regime was extensively studied theoretically, however universality of peak values of \( \sigma_{xx} \) has not been observed experimentally. Recently, several authors calculated peak values for both IQHE and fractional QHE (FQHE) regimes. In this paper we present an experimental study of the peak values of \( \sigma_{xx} \). We have observed that the peak values of \( \sigma_{xx} \) are nearly equal for a wide range of transitions between IQHE states. This observation allows us to compare peak values of \( \sigma_{xx} \) in the integer and fractional QHE regimes for the same sample. We show that transitions between FQHE states can be successfully mapped onto transitions between IQHE states, according to the law of corresponding states.

We chose a Corbino geometry (see inset in Fig. 1) in order to avoid edge channels connecting sample contacts. In samples with Hall-bar geometry the longitudinal resistance \( R_L \) is not proportional to the local resistivity \( \rho_{xx} \) because of non-local transport. This effect makes measurements of \( R_L \) geometry and sample dependent.
In the Corbino geometry the measured two-terminal resistance $R_{2T}$ is inversely proportional to the local conductivity: $R_{2T} = \Box/\sigma_{xx}$, where $\Box = 1/2\pi \ln(r_2/r_1)$ is a geometrical factor (“number of squares”). We present all our data in terms of $\sigma_{xx}$, calculated according to this formula from measured $R_{2T}$. Samples were fabricated from high mobility ($1.5 < \mu < 2.0 \times 10^{6} \text{ cm}^2/\text{V s}$) GaAs/AlGaAs heterojunction wafers. Corbino geometry was defined by circular Ohmic contacts with the inner radius $0.2 \leq r_1 \leq 0.6 \text{ mm}$ and the outer radius $r_2 = 1.5 \text{ mm}$. Two-dimensional electron systems (2DES) were prepared by illuminating samples with a red LED. Temperature was measured with a calibrated Ruthenium Oxide chip resistor and the absolute values are believed to be accurate to 5%. Measurements were done using standard lock-in technique at 2.5 Hz with an applied current 50 pA rms; no heating effects were observed at this current.

Representative magnetoconductivity data are shown in Figs. 1 and 2. In IQHE (Fig. 1) minima in $\sigma_{xx}$ are well developed up to filling factor $\nu = 40$. At a temperature $T < 100 \text{ mK}$ we observe a remarkable result: peaks of $\sigma_{xx}$ form an approximately flat region between $\nu = 3$ and 16; the values of $\sigma_{xx}^{\text{peak}}$ fall within the dashed lines in Fig. 1, which are displaced by $\pm 7\%$ from the average value of $0.22 \, e^2/h$. The average peak value varies in different samples between 0.20 and 0.35 $e^2/h$ at the lowest $T = 13 \text{ mK}$. The range of $\nu$ where $\sigma_{xx}^{\text{peak}}$ is $\nu$ independent also changes a little from sample to sample. The flat region is well defined up to $T \sim 40-100 \text{ mK}$ and disappears at higher temperatures (compare data for 20 mK and 1.8 K in Fig. 1). At low fields ($\nu > 20$), values of $\sigma_{xx}^{\text{peak}}$ follow the Ando dependence $\sigma_{xx}^{\text{peak}} \approx \beta T^2 \nu$ in the full experimental temperature range ($13 \text{ mK} \leq T \leq 1.6 \text{ K}$) with the coefficient $\beta \approx 0.1$. The same value of $\beta$ is obtained from the amplitude of Shubnikov - de Haas oscillations, following Coleridge et al.\(^3\).

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Fig. 1. Magnetoconductivity in the IQHE regime at 20 mK (solid line) and 1.8 K (dotted line) for Sample A ($n = 1.05 \times 10^{11} \text{ cm}^{-2}$). Horizontal solid line gives the average peak conductivity $\sigma_{xx}^{\text{peak}} = 0.22 \, e^2/h$ (for $\nu$ between 3 and 16) and the dashed lines are offset $\pm 7\%$ from the average value. Sample layout in the Corbino geometry is shown in the inset.

Fig. 2. Magnetoconductivity in the FQHE regime for Sample B ($n = 1.17 \times 10^{11} \text{ cm}^{-2}$) at 14 mK (solid line). Also shown are peak values of conductivity $\sigma_{xx}$ calculated from $\sigma_{xx}^{\text{peak}}$ for composite fermions using Eq. 1 with (i) theoretically expected value of 0.5 $e^2/h$ ($\star$) and (ii) the experimentally obtained value $\sigma_{xx}^{\text{peak}} = 0.35 \, e^2/h$ ($\circ$).
The relation between the peak values of conductivity in IQHE and FQHE is understood naturally within the framework of the "law of corresponding states". This law predicts equivalence between two QHE systems with filling factors \( \nu^* \) and \( \nu \) related through:

\[
i) \text{Landau level addition transformation } \nu^* \leftrightarrow \nu + 1;
\]
\[
ii) \text{particle-hole conjugation } \nu^* \leftrightarrow 1 - \nu;
\]
\[
iii) \text{flux attachment transformation } (\nu^{-1})^\ast \leftrightarrow \nu^{-1} + 2m,
\]

where \( m \) is an integer. Universality of \( \sigma_{\text{ex}}^{\text{peak}} \) in IQHE is a manifestation of the first transformation and was discussed above, while \( iii) \) relates the system of composite fermions (CF) in FQHE and electrons in IQHE. The peak conductivity for CF, \( \sigma_{\text{cf}}^{\text{peak}} \), can be related to the experimentally measurable value of \( \sigma_{\text{ex}}^{\text{peak}} \) through equations derived in Ref. 11.

\[
\rho_{xx} = \frac{h}{e^2} \frac{\sigma_{\text{cf}}^{\text{peak}}}{(\sigma_{\text{cf}}^{\text{peak}})^2 + (\nu_{\text{cf}})^2},
\]
\[
\rho_{xy} = -\frac{h}{e^2} \frac{\nu_{\text{cf}}}{(\sigma_{\text{cf}}^{\text{peak}})^2 + (\nu_{\text{cf}})^2 + 2}.
\]

Here \( \nu_{\text{cf}} \) is the filling factor of CF at which the transition occurs, \( \sigma_{\text{cf}}^{\text{peak}} \) is in units of \( e^2/h \) and the conductivity tensor can be obtained by matrix inversion from the resistivity tensor. Note that for the \( m = 1 \) main FQHE sequence, \( \nu_{\text{cf}} = \nu_{\text{cf}}/(2\nu_{\text{cf}} + 1) \); for example, the transition between \( \nu_{\text{cf}} = 1 \) and \( 2 \) for CF (at \( \nu_{\text{cf}} \approx 3/2 \)) corresponds to the transition between \( \nu = 1/3 \) and \( 2/5 \) for electrons (at \( \nu \approx 3/8 \)).

Peak values for CF conductivity are expected to be the same as for electrons in IQHE. Thus, Eq. 1 relates \( \sigma_{\text{ex}}^{\text{peak}} \) in IQHE and FQHE regimes. Such comparison is meaningful only because the peak \( \sigma_{\text{ex}} \) values are nearly the same in IQHE in our experiments. We introduce \( \sigma_{\text{IQHE}}^{\text{peak}} = < \sigma_{\text{ex}}^{\text{peak}} > \), where averaging is done over IQHE transitions for which \( \sigma_{\text{ex}}^{\text{peak}} \) differs less than 15% from the mean value. The correspondence between IQHE and FQHE is demonstrated in Fig. 2: values for open and solid circles are calculated from Eq. 1 using experimental \( \sigma_{\text{IQHE}}^{\text{peak}} = 0.34 e^2/h \) and theoretical values, respectively. There is no adjustable parameters in this calculation; open circles fit the data better, as could be expected.

Peak conductivity for CF \( \sigma_{\text{cf}}^{\text{peak}} \) can be obtained directly from the experimental FQHE conductivity data via Eq. 1, without considering a priori any particular relation between \( \sigma_{\text{cf}}^{\text{peak}} \) and \( \sigma_{\text{IQHE}}^{\text{peak}} \). In Fig. 3 thus determined \( \sigma_{\text{cf}}^{\text{peak}} \) is plotted for several transitions between FQHE states together with \( \sigma_{\text{IQHE}}^{\text{peak}} \) for IQHE regime in a wide temperature range. Again, this comparison involves no adjustable parameters; we interpret the close agreement of the absolute values of \( \sigma_{\text{cf}}^{\text{peak}} \) and \( \sigma_{\text{IQHE}}^{\text{peak}} \) as well as their very similar temperature dependence as an experimental confirmation of the law of corresponding states.

Although the peak values of \( \sigma_{\text{ex}} \) for electrons in IQHE and CF in FQHE are nearly the same, they are not saturating to \( 0.5 e^2/h \) at \( T \to 0 \) (Fig. 3), as expected for non-interacting particles. In contrast, the peak conduc-

![Fig. 3. The temperature dependence of the peak values \( \sigma_{\text{ex}}^{\text{peak}} \) for IQHE transitions (solid lines) and \( \sigma_{\text{cf}}^{\text{peak}} \) for composite fermions for transitions between FQHE states 1/3 \( \leftrightarrow \) 2/5 (○) and 2/5 \( \leftrightarrow \) 3/7 (*). \( \sigma_{\text{cf}}^{\text{peak}} \) is obtained from the experimental data using Eq. 1.](image-url)
tivity decreases at low temperatures. The similarity in the $T$-dependence of peak conductivities $\sigma_{ij}^{\text{peak}}$ and $\sigma_{\text{IQHE}}^{\text{peak}}$ in the full experimental range of temperatures suggests that the underlying physics is essentially the same in both regimes. In low mobility samples a similar reduction of conductivity was attributed to interaction effects. It should be noted that in QHE regime weak localization is not expected to alter $\sigma_{\text{peak}}$ because high $B$ breaks time reversal symmetry.

To summarize our results, we observed, for the first time, that the peak values of $\sigma_{xx}$ are nearly equal for a wide range of transitions between IQHE states as was predicted in the scaling theories of QHE. This observation allows us to compare peak values of $\sigma_{xx}$ in the integer and fractional QHE regimes for the same sample. We show that transitions between FQHE states can be successfully mapped onto transitions between IQHE states, according to the law of corresponding states. In other words, peak values of diagonal conductivity for electrons in IQHE are the same as for composite fermions in FQHE.

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REFERENCES

6. The reason for experimental difficulties is that either high disorder in low mobility samples or FQHE in high mobility samples limit the range of filling factors with scaling behavior in the IQHE regime; this range may be too narrow to be observed.