Physics 565 - Spring 2011, Assignment #6, Due March 23rd

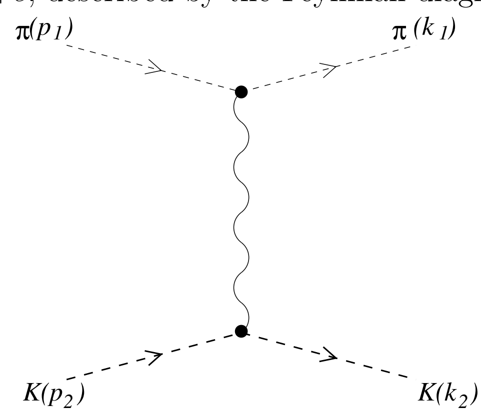
1. Consider a $2 \to 2$ scattering process where initial state particles with mass m and M have 4-momenta p_1 and p_2, respectively and final state 4-momenta k_1 and k_2. Show that the products of 4-momenta can be written in terms of the Mandlestam variables as follows:

$$\begin{aligned} k_1 \cdot k_2 = p_1 \cdot p_2 &= \frac{1}{2}(s - m^2 - M^2) \\ p_1 \cdot k_2 = p_2 \cdot k_1 &= \frac{1}{2}(m^2 + m^2 - u) \\ p_1 \cdot k_1 &= m^2 - \frac{1}{2}t \\ p_2 \cdot k_2 &= M^2 - \frac{1}{2}t \end{aligned}$$

and that

$$s + t + u = 2m^2 + 2M^2.$$

2. Consider the elastic scattering of distinct (ie, not identical) spin-0 particles (hypotheticalally pions and kaons) with charge $+e$, described by the Feynman diagram:



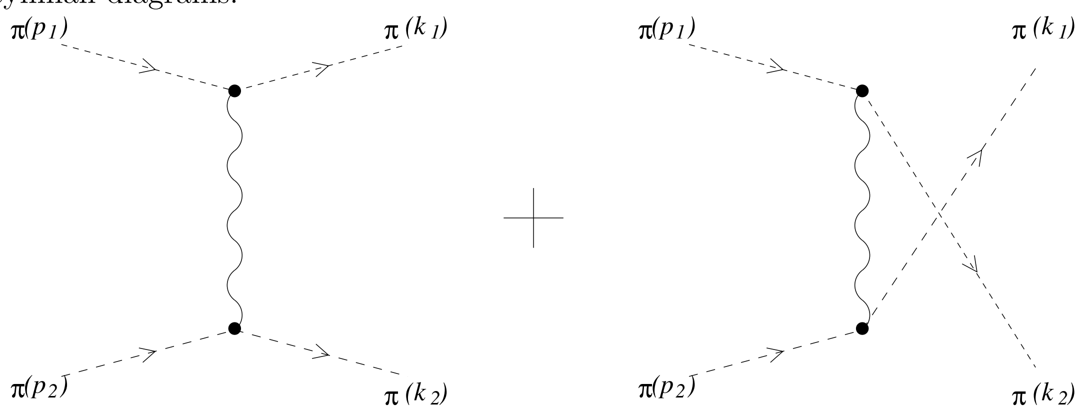
(a) Show that the reduced matrix element for this process can be written

$$-i\mathcal{M} = -ie^2 \frac{(p_1 + k_1) \cdot (p_2 + k_2)}{(p_1 - k_1)^2} \tag{1}$$

(b) Show that this can be expressed in terms of the Mandlestam variables as follows:

$$-i\mathcal{M} = -ie^2 \frac{s - u}{t} \tag{2}$$

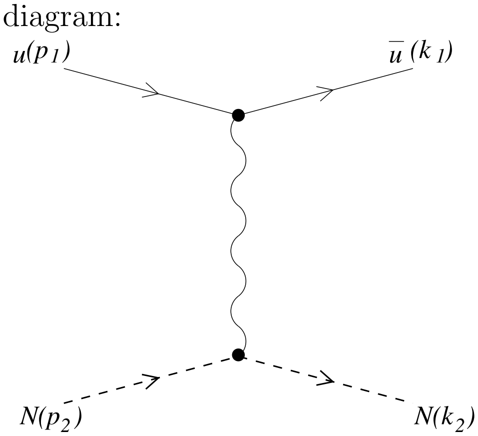
3. Consider the elastic scattering of indistinguishable spin-0 particles with charge $+e$, described by the Feynman diagrams:



Show that the reduced matrix element can be written

$$-i\mathcal{M} = -ie^2\left(\frac{s-u}{t} + \frac{s-t}{u}\right) \quad (3)$$

4. Consider the elastic scattering of an electron from a point-like particle with spin 0 and charge Ze, described by the Feynman diagram:



Show that the spin-averaged reduced matrix element squared can be expressed

$$|\overline{\mathcal{M}}| = 8Z^2e^4\frac{(s(s+t)+m^2(M^2+t)-m^4}{t^2} \quad (4)$$

where m is the electron mass and M is the mass of the spin-0 particle.