Physics 565 - Spring 2011, Assignment #2, Due Feburary 4$^{\text{th}}$

1. Consider the Lagrangian density

$$\mathcal{L} \;=\; (\partial^\mu \phi^*)(\partial_\mu \phi) - m^2 \phi^* \phi$$

describing a system of two fields, $\phi(x)$ and $\phi^*(x)$.
(a) Find the expression for the canonical energy momentum tensor corresponding to this Lagrangian.
(b) If the fields are represented in terms of creation and annihilation operators as follows,

$$\phi(x) \;=\; \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left(\alpha(k) e^{-ik\cdot x} + \beta^\dagger(k) e^{ik\cdot x} \right)$$
$$\phi^*(x) \;=\; \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left(\beta(k) e^{-ik\cdot x} + \alpha^\dagger(k) e^{ik\cdot x} \right)$$

show that the i'th component of the momentum operator, corresponding to the conserved "charge" of the spatial parts of the energy momentum tensor, can be written:

$$P^i \;=\; \int d^3x T^{0i} = \int \frac{d^3k}{(2\pi)^3} \frac{k^i}{2\omega_k} \left(N_\alpha(k) + N_\beta(k) \right)$$

where $N_\alpha(k) = \alpha^\dagger(k)\alpha(k)$ and $N_\beta(k) = \beta^\dagger(k)\beta(k)$ are the number operators corresponding to particles of type α and β, respectively.
2. Express the operator,

$$Q = \int d^3x J^0(x),$$

corresponding to the time component of the conserved current,

$$J^\mu \;=\; (\partial^\mu \phi^*)\phi - \phi^*(\partial^\mu \phi)$$

in terms of the number operators $N_\alpha(k)$ and $N_\beta(k)$ and interpret the meaning of the resulting expression.