1. The differential partial decay width for a 2-body decay can be expressed

\[ d\Gamma = \frac{1}{32\pi^2} |\vec{M}|^2 \frac{|\vec{k}|}{M^2} d\Omega \]

where \( M \) is the mass of the decaying particle and the two final state particles which have momentum \( \pm \vec{k} \).

(a) Ignoring the masses of the final state particles, show that the partial width for \( W^+ \to \mu^+ \nu_\mu \) can be expressed

\[ \Gamma = \frac{G_F M_W^3}{6\sqrt{2}\pi} \]

(b) Ignoring the masses of the final state particles, calculate the partial widths for \( Z^0 \to \mu^+ \mu^- \) and \( Z^0 \to \nu_\mu \bar{\nu}_\mu \).

2. (a) Show that the \( Z^0 f \bar{f} \) vertex factor

\[ \frac{-ig}{\cos \theta_W} \gamma^\mu \frac{1}{2} (c^f_V - c^f_A \gamma^5) \]

can also be written

\[ \frac{-ig}{\cos \theta_W} \gamma^\mu \left( c^f_L \frac{1}{2} (1 - \gamma^5) + c^f_R \frac{1}{2} (1 + \gamma^5) \right) \]

where the left- and right-handed couplings, \( c^f_L \) and \( c^f_R \), are expressed in terms of the vector and axial vector couplings, \( c^f_V \) and \( c^f_A \).

(b) The general expressions for \( c^f_V \) and \( c^f_A \) are

\[ c^f_V = T^3_f - 2 \sin^2 \theta_W Q_f \]
\[ c^f_A = T^3_f \]

where \( T^3_f \) is the third component of the weak isospin and \( Q_f \) is the charge of the fermion, in units where \( Q_e = -1 \). Complete the entries in the following table of couplings for quarks and leptons in the Standard Model:

<table>
<thead>
<tr>
<th>( f )</th>
<th>( Q_f )</th>
<th>( T^3_f )</th>
<th>( c^f_A )</th>
<th>( c^f_V )</th>
<th>( c^f_L )</th>
<th>( c^f_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu_e, \nu_\mu, \nu_\tau )</td>
<td>0</td>
<td>1/2</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>( e^-, \mu^-, \tau^- )</td>
<td>-1</td>
<td>-1/2</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>( u, c, t )</td>
<td>2/3</td>
<td>1/2</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>( d, s, b )</td>
<td>-1/3</td>
<td>-1/2</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
</tbody>
</table>

Express the couplings both symbolically, and numerically, using \( \sin^2 \theta_W = 0.231 \).