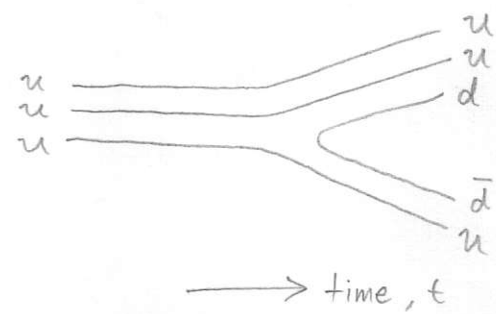
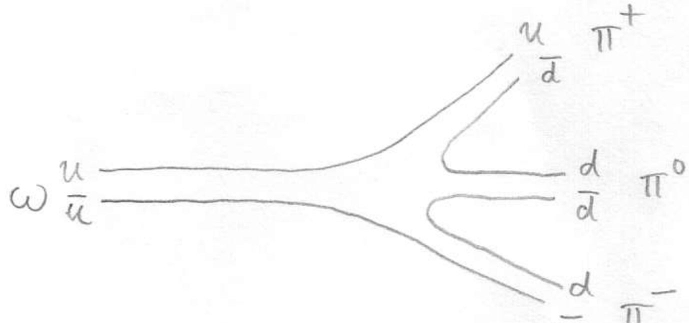
"Quark current diagrams" can be used to describe the qualitative features of strong transitions.

eg, The decay $\Delta^{++} \to p\pi^+$



Quark flavor quantum numbers are conserved at any time, t. Creation of new quarks can only occur in $q\bar{q}$ pairs.

Other examples:



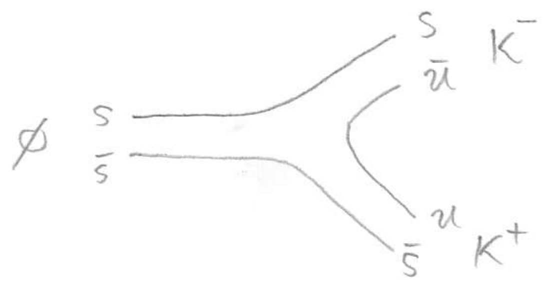
In general, you pay a penalty for each $q\bar{q}$ creation or annihilation. (ie, one power of α_s, the strong coupling constant.)

But you also have to keep track of phase space and angular momentum conservation.

For this reason,

$\omega \to \pi^+ \pi^- \pi^0$ is favored over $\omega \to \pi^+ \pi^-$, the $\pi^+ \pi^-$ must be in an orbital P-wave state with $l = 1$ and is suppressed by powers of q^{2l+1}.

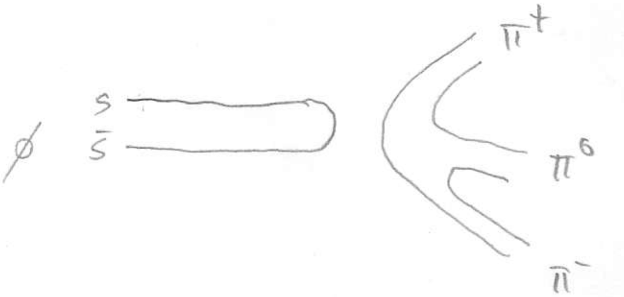
However, these arguments do explain why $\phi \to K^+ K^-$ or $K^0 \overline{K^0}$ but not $\pi^+ \pi^- \pi^0$:



$\Delta m = m_\phi - 2 m_K$

≈ 20 MeV

not much phase space.



suppressed by 3 powers of α_s compared with $\phi \to K\bar{K}$.

$\Delta m = m_\phi - 3 m_\pi$

≈ 600 MeV

lots of phase space

This is called the OZI rule.

"disconnected quark current diagrams are suppressed."

Because the $\phi \to 3\pi$ channel is suppressed, the width of the ϕ resonance is small.

$$\Gamma(\phi) \approx 4.3 \text{ MeV}$$

compared with $\Gamma(\omega) = 8.5$ MeV

or $\Gamma(\rho) = 150$ MeV.

In 1974 a narrow resonance was observed in e^+e^- collisions and in p Be fixed target experiment.

$$e^+e^- \to \psi \to e^+e^-$$

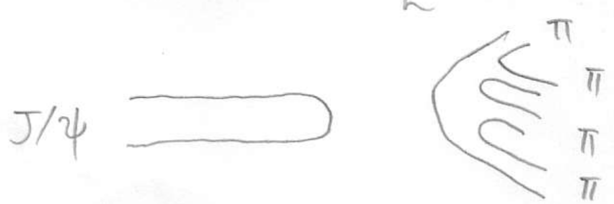
$$p + Be \to J\, X$$
$$\hookrightarrow e^+e^-$$

$$M(J/\psi) \sim 3097 \text{ MeV}$$

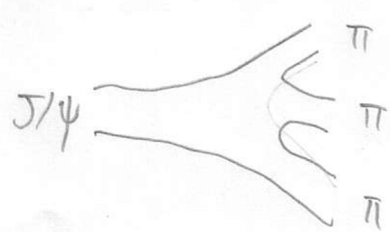
Lots of phase space available but the width was extremely narrow:

$$\Gamma(J/\psi) \sim 91 \text{ keV}.$$

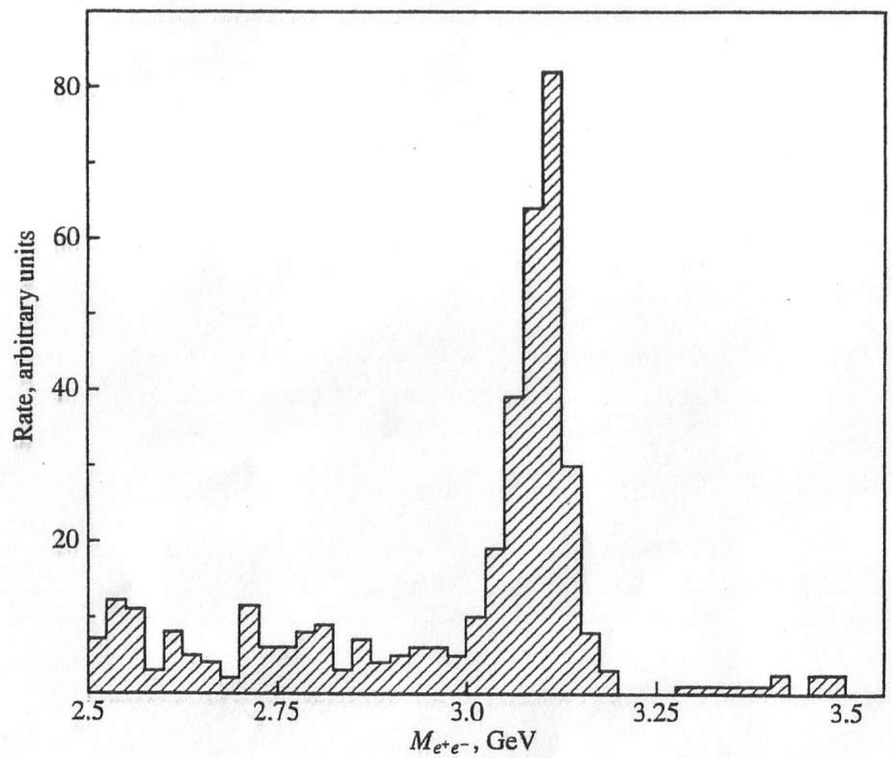
OZI rule suggests that there are no connected quark current diagrams:

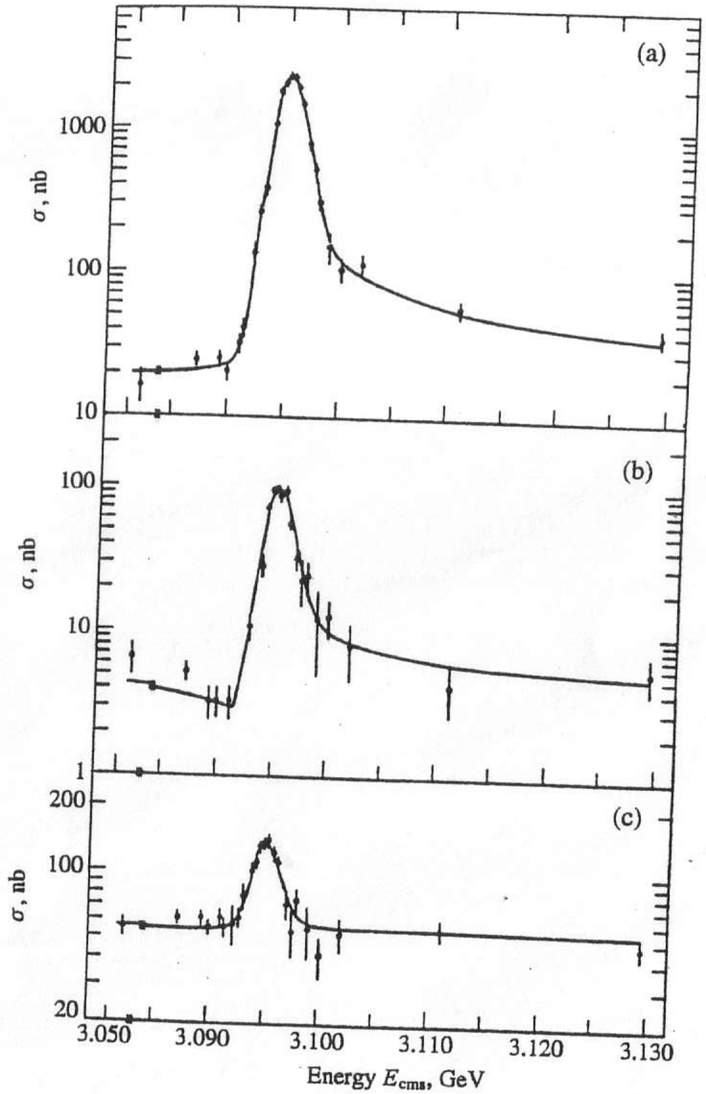


ok



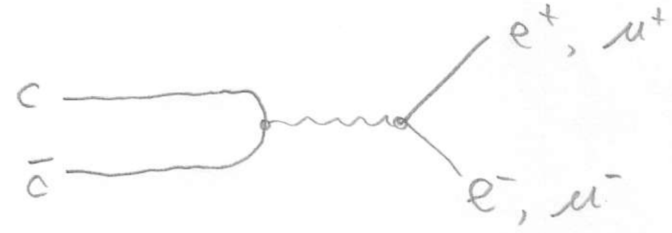
Does not occur.


Rate, arbitrary units
80
60
40
20
2.5
2.75
3.0
3.25
3.5
$M_{e^+e^-}$, GeV

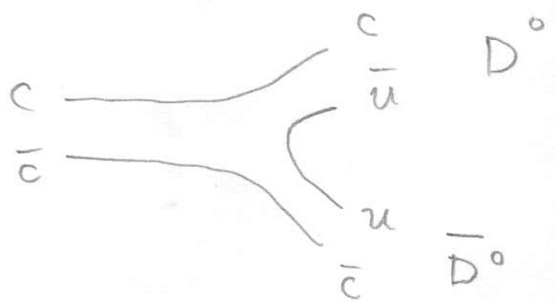

(a)
(b)
(c)
σ, nb
1000
100
10
100
10
1
200
100
20
3.050
3.090
3.100
3.110
3.120
3.130
Energy E_{cms}, GeV

(4)

This suggests that there is a new flavor of quark that is not present in the final state particles. "Hidden Charm".



Why not this diagram?



Subsequently, "open charm" was descovered.

$m(D^0) = 1865 \text{ MeV}$

$$2 m_{D^0} > m_{J/\psi}$$
$$(3730 \text{ MeV}) > (3097 \text{ MeV})$$

So the narrowness of the J/ψ resonance suggested that a new type of $q\bar{q}$ pair was being produced.

But other resonances were also discovered:

J/ψ mass 3097 MeV

$\chi_{c_0} \to J/\psi\ \gamma$ mass 3415 MeV

$\chi_{c_1} \to J/\psi\ \gamma$ mass 3511 MeV

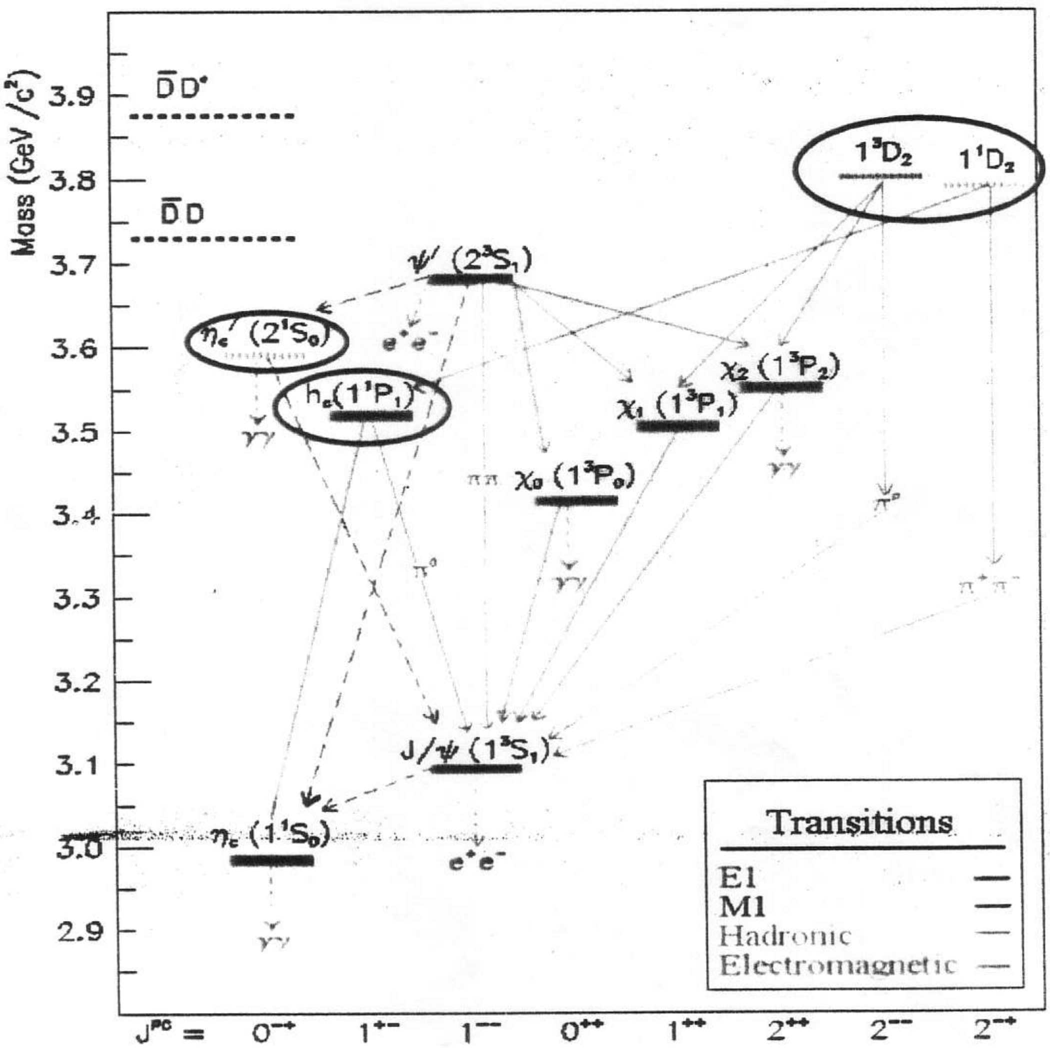
$\chi_{c_2} \to J/\psi\ \gamma$ mass 3556 MeV

$\psi(2s) \to J/\psi\ \pi^+\pi^-$ mass 3689 MeV

Finally, $\psi(3770) \to D\bar{D}$ mass 3770 MeV $> 2m_D$.

These masses and transitions resemble those of the Hydrogen atom or positronium.

The charmonium spectrum



Reminder of Hydrogen atom wave functions:

Two particles, the proton and an electron.

Schrödenger's equation:

$$H|\psi_n\rangle = E_n|\psi_n\rangle$$

where $H = \frac{p_p^2}{2m} + \frac{p_e^2}{2m} + V(\vec{r}) \quad (\vec{p} \to -i\nabla)$

Transform to coordinates with respect to center of mass, the two particles decouple:

$$(H_0 + V)|\psi_n\rangle = E_n|\psi_n\rangle$$

where $H_0 = \frac{p^2}{2\mu}$, $V(r) = -\frac{e^2}{r}$

$$\mu = \frac{m_p m_e}{m_p + m_e} = \frac{m_e}{1 + m_e/m_p} \approx m_e$$

In the position representation the wavefunction decouples into a radial and an orbital part:

$$\langle \vec{r}|\psi_{nlm}\rangle = R_{nl}(r)\, Y_l^m(\theta,\phi)$$

where n, l, m are radial and orbital quantum numbers.

The S-wave states have $l = 0$
P-wave states have $l = 1,\ m = 0, \pm 1$
D-wave states have $l = 2$, etc.

When spin-orbit and spin-spin coupling is ignored, all the l, m states are degenerate. These can be treated as perturbations:

$$H_{Hyp} = \frac{\alpha}{m_1 m_2} \left[\delta^3(\vec{r}) \cdot \frac{8\pi}{3} \vec{S}_1 \cdot \vec{S}_2 + \frac{1}{r^3} S_{12} \right]$$

Recall that $\vec{S}_1 \cdot \vec{S}_2 = \begin{cases} +1/4 & \text{when } S = 1 \\ -3/4 & \text{when } S = 0 \end{cases}$

S_{12} is called the "Tensor Operator" and has the expectation values:

$$\langle {}^3P_2 | S_{12} | {}^3P_2 \rangle = -2/5$$
$$\langle {}^3P_1 | S_{12} | {}^3P_1 \rangle = 2$$
$$\langle {}^3P_0 | S_{12} | {}^3P_0 \rangle = -4$$
$$\langle {}^1S_0 | S_{12} | {}^1S_0 \rangle = 0$$
$$\langle {}^3S_0 | S_{12} | {}^3S_0 \rangle = 0$$

where the commonly used notation is ${}^{2S+1}L_J$

Spin-Orbit coupling :

The circulating electron current creates a magnetic field that interacts with the spin of the proton (and vice versa).

$$H_{so} = -\frac{1}{2r}\frac{\partial V}{\partial r}\left[\frac{\vec{L}\cdot\vec{S}_1}{m_1^2} + \frac{\vec{L}\cdot\vec{S}_2}{m_2^2}\right]$$

$$+ \frac{\alpha}{r^3}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\left(\frac{\vec{L}\cdot\vec{S}_1}{m_1^2} + \frac{\vec{L}\cdot\vec{S}_2}{m_2^2}\right)$$

$$\vec{L}\cdot\vec{S} = \frac{1}{2}\left[J(J+1) - L(L+1) - S(S+1)\right]$$

$$= \begin{cases} 1 & \text{for } {}^3P_2 \\ -1 & \text{for } {}^3P_1 \\ -2 & \text{for } {}^3P_0 \end{cases}$$

H_{so} depends on the potential that binds the particles together.

Emperical observation:

$$\frac{\Gamma(^3S_1 \to e^+e^-)}{Q_q^2} \simeq 10 \text{ keV}$$

for ρ, ω, ϕ and J/ψ states where Q_q is the charge of the quark.

This can be calculated using the Hydrogen atom analogy:

$$\Gamma(^3S_1 \to e^+e^-) = Q_q^2 \cdot 16\pi\alpha^2 \frac{|\psi(0)|^2}{m_v^2}$$

This tells us that $\frac{|\psi(0)|^2}{m_v^2}$ must be a constant.

Recall that for the Hydrogen atom

$$\psi(r) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$$

where $a_0 = \frac{1}{\alpha m_e}$ or $\frac{1}{\alpha \underbrace{m_q/2}_{\text{reduced mass}}}$.

So this would mean that $|\psi(0)| \propto m^3$.

For a harmonic oscillator,

$$\psi(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4} e^{-\beta^2 x^2/2}$$

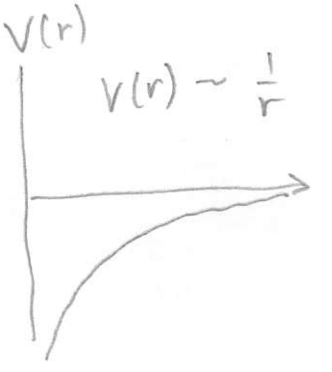
where $\beta^2 = \sqrt{\kappa m}$

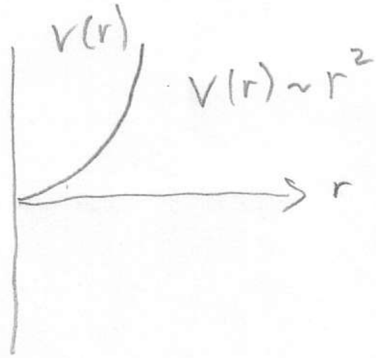
So $|\psi(0)|^2 \propto m^{1/4}$.

The fact that for the vector mesons,

$$|\psi(0)|^2 \propto m_v^2$$

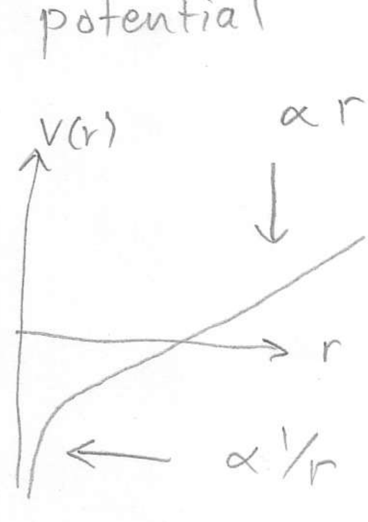
suggests that $V(r)$ is in between the $1/r$ and the r^2 dependence.





A commonly used emperical potential for quarks is

$$V(r) \sim \frac{-\frac{4}{3}\alpha_s}{r} + C + br$$



The comparison of the charmonium and positronium spectra suggests the following qualitative properties of quarks and the potential :

- massive quarks can be treated nonrelativistically
- spin-spin and spin-orbit splittings are important and can be used to assign quantum numbers to the states
- the potential is intermediate between $1/r$ and r^2.
  $\Rightarrow$ Confining at large r.
  free at small r.
- quarks are just as real as electrons and positrons.