Hadron Spectroscopy

The J/ψ and its excited states were identified as bound states of $c\bar{c}$ quarks.
In 1977, 78 a similar set of resonances was discovered in proton fixed target experiments at Fermilab (400 GeV protons on a Cu / Pb target).
The Υ resonances were associated with $b\bar{b}$ production.

$c\bar{c}$	$^{2S+1}L_J$	mass	$b\bar{b}$		
η_c	$^1S_0$	2980 MeV	η_b	$^1S_0$	?
J/ψ	$^3S_1$	3097	$\Upsilon(1S)$	$^3S_1$	9460 MeV
$\chi_{c_0}(1P)$	$^3P_0$	3415	$\chi_{b_0}(1P)$	$^3P_0$	9860 MeV
$\chi_{c_1}(1P)$	$^3P_1$	3511	$\chi_{b_1}(1P)$	$^3P_1$	9893 MeV
$\chi_{c_2}(1P)$	$^3P_2$	3556	$\chi_{b_2}(1P)$	$^3P_2$	9913
$\psi(2S)$	$^3S_1$	3686	$\Upsilon(2S)$	$^3S_1$	10.023 GeV
			$\chi_{b_0}(2P)$	$^3P_0$	10.232
			$\chi_{b_1}(2P)$	$^3P_1$	
			$\chi_{b_2}(2P)$	$^3P_2$	
			$\Upsilon(3S)$		
			$\Upsilon(4S)$		10.580 GeV
			$\Upsilon(5S)$		
			⋮		

The rich structure in the mass spectrum is due to spin-spin and spin-orbit interactions.

Hyperfine splitting

The spin-spin splitting is of the form

$$\Delta H_{Hyp} = \frac{\alpha}{m_1 m_2}\left[\delta^3(\vec{r}) \cdot \frac{8\pi}{3} \vec{s}_1 \cdot \vec{s}_2 + \frac{1}{r^3} S_{12} \right]$$

The $\vec{s}_1 \cdot \vec{s}_2$ term only acts on the S-wave states since these are the only ones for which the radial wavefunctions are finite at $r = 0$. It has expectation values:

$$\langle {}^1S_0 | \vec{s}_1 \cdot \vec{s}_2 | {}^1S_0 \rangle = -3/4$$

$$\langle {}^3S_1 | \vec{s}_1 \cdot \vec{s}_2 | {}^3S_1 \rangle = +1/4$$

(Recall $\vec{s}_1 \cdot \vec{s}_2 = \frac{1}{2}\left[S(S+1) - s_1(s_1+1) - s_2(s_2+1) \right]$)

The S_{12} is called the "Tensor Operator" and it has expectation values

$$\langle {}^3P_2 | S_{12} | {}^3P_2 \rangle = -2/5$$

$$\langle {}^3P_1 | S_{12} | {}^3P_1 \rangle = 2$$

$$\langle {}^3P_0 | S_{12} | {}^3P_0 \rangle = -4$$

$$\langle {}^1S_0 | S_{12} | {}^1S_0 \rangle = 0$$

$$\langle {}^3S_0 | S_{12} | {}^3S_0 \rangle = 0$$

The spin-orbit interaction has the form:

$$\Delta H_{S.O.} = \frac{-1}{2r}\frac{\partial V}{\partial r}\left[\frac{\vec{L}\cdot\vec{S}_1}{m_1^2} + \frac{\vec{L}\cdot\vec{S}_2}{m_2^2}\right]$$

$$+ \frac{\alpha}{r^3}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\left(\frac{\vec{L}\cdot\vec{S}_1}{m_1^2} + \frac{\vec{L}\cdot\vec{S}_2}{m_2^2}\right)$$

Recall $\vec{L}\cdot\vec{S} = \frac{1}{2}\left[J(J+1) - L(L+1) - S(S+1)\right]$

for quarks with equal mass,

$$\frac{\vec{L}\cdot\vec{S}_1}{m_1^2} + \frac{\vec{L}\cdot\vec{S}_2}{m_2^2} = \frac{1}{m_q^2}\vec{L}\cdot\vec{S}$$

where $\vec{S} = \vec{S}_1 + \vec{S}_2$.

$$\langle {}^3P_2 | \vec{L}\cdot\vec{S} | {}^3P_2 \rangle = +1$$

$$\langle {}^3P_1 | \vec{L}\cdot\vec{S} | {}^3P_1 \rangle = -1$$

$$\langle {}^3P_0 | \vec{L}\cdot\vec{S} | {}^3P_0 \rangle = -2$$

H_{SO} also depends on the shape of the potential that binds the quarks together.

If heavy vector mesons ($c\bar{c}$ or $b\bar{b}$) can really be thought of as nonrelativistic systems like the Hydrogen atom, then what is the form of the potential that binds the quarks together?

For Hydrogen it is $V(r) \sim \frac{-1}{r}$.

Is it the same for $Q\bar{Q}$ states?

It was an emperical observation that the partial widths of the $^3S_1$ states decaying to e^+e^- were the same for all vector mesons, after adjusting for the quark charges:

$$\frac{\Gamma(^3S_1 \to e^+e^-)}{Q_q^2} \approx 10 \text{ keV}$$

for the ρ, ω, ϕ, J/ψ and Υ states.

This process can be calculated:

$$\Gamma(^3S_1 \to e^+e^-) = 16\pi\alpha^2 Q_q^2 \frac{|\psi(0)|^2}{m_V^2}$$

where m_V is the mass of the vector meson. This is called the Van Royen Weisskopff formula.

The fact that $\frac{\Gamma}{Q_a^2} \approx 10 \text{ keV}$

tells us that $\frac{|\psi(0)|^2}{m_V^2}$ must be

the same for all vector mesons.

So $|\psi(0)|^2 \propto m_V^2$.

Recall that for a Hydrogen atom

$$\psi(r) = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0} \right)^{3/2} e^{-r/a_0}$$

where $a_0 = \frac{1}{\alpha m_e}$ or $\frac{1}{\alpha \underbrace{m_a/2}_{\text{reduced mass}}}$.

So for the Hydrogen atom $|\psi(0)|^2 \propto m^3$.

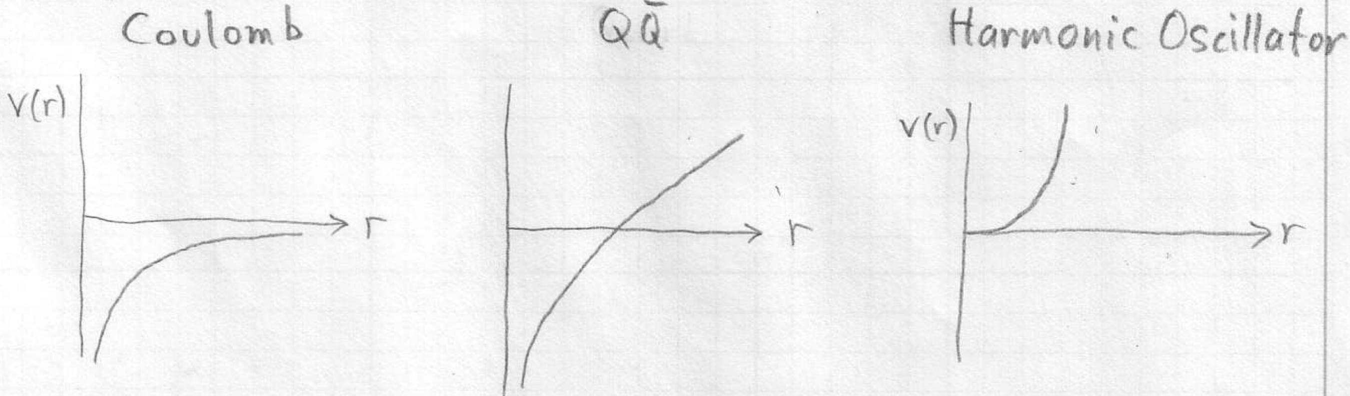
For the harmonic oscillator problem the spatial representation of the ground state wavefunction was

$$\psi(x) = \left(\frac{\beta^2}{\pi} \right)^{1/4} e^{-\beta^2 x^2/2}$$

where $\beta = \sqrt{\kappa m}$.

So in this case $|\psi(0)|^2 \propto m^{1/2}$

The fact that $|\psi(0)|^2 \propto m^2$ for vector mesons suggests that the form of the potential is somewhere in between the coulomb potential and the harmonic oscillator potential.



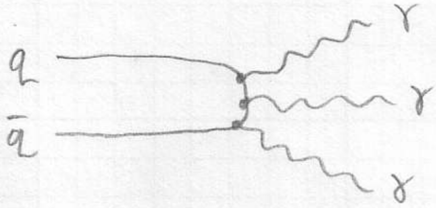
Phenomenologists usually parameterize the potential as being linear + coulomb:

$$V(r) \sim \frac{-4/3\,\alpha_s}{r} + c + br$$

where $4/3\,\alpha_s$ replaces the electromagnetic coupling constant α in the coulomb potential.

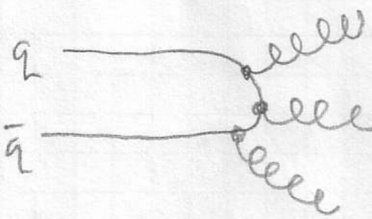
We know that $\alpha = \frac{1}{137}$ for low energy EM processes. What is α_s?

Some one calculated the partial width for the process $V \to \gamma\gamma\gamma$:



$$\Gamma({}^3S_1 \to \gamma\gamma\gamma) = \frac{2(\pi^2-9)}{9\pi} \alpha^6 m$$

The equivalent of the photon for strong interactions is the gluon :



$$\Gamma({}^3S_1 \to \text{hadrons}) = \frac{2(\pi^2-9)}{9\pi} \left(\frac{4}{3}\alpha_s\right)^6 m$$

Hence,
$$\alpha_s^6 = \frac{\Gamma({}^3S_1 \to \text{hadrons})}{m_c} \cdot \frac{9\pi}{2(\pi^2-9)} \left(\frac{3}{4}\right)^6$$

The total width of the J/ψ is 87 keV and the hadronic branching fraction is 88% . This gives

$$\alpha_s^6 = \frac{0.222}{m_c}$$

If $m_c \approx \frac{1}{2} m_{J/\psi}$ this gives

$$\alpha_s = \left(\frac{.087 \text{ MeV} \times 0.88}{1500 \text{ MeV}} \times \frac{9\pi}{2(\pi^2-9)} \right)^{1/6} \times \frac{3}{4}$$

$$= 0.23$$

So $\alpha_s \gg \alpha$. That's why strong interactions are so strong.