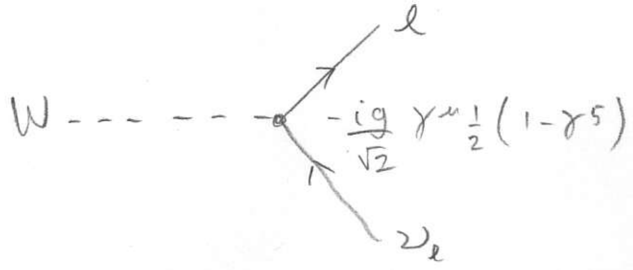
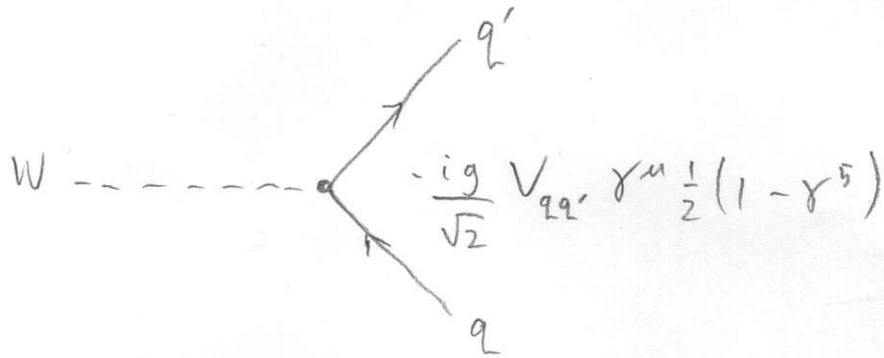
November 22, 2005

Charged current coupling to leptons:



Charged current coupling to quarks:



What are the $V_{qq'}$ parameters?

Quarks found inside hadrons or created by the strong interaction are strong eigenstates.

$$H_{strong}|q\rangle = E|q\rangle$$

The weak interaction couples to weak eigenstates of the quarks. Is there any reason that these should be the same as the strong eigenstates?

No... but they seem to be close.

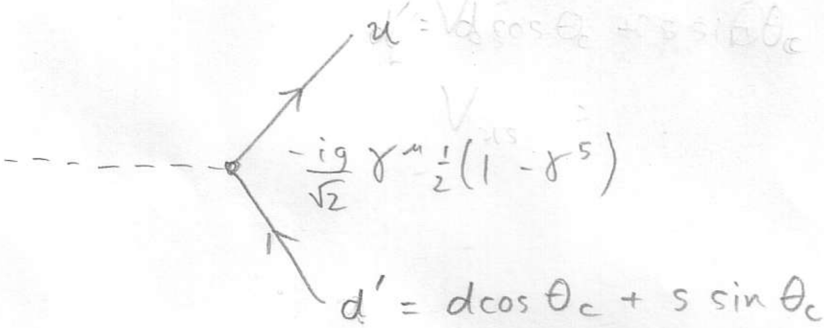
Consider the d and s quarks as strong eigenstates and the d' and s' quarks as weak eigenstates. How are they related?

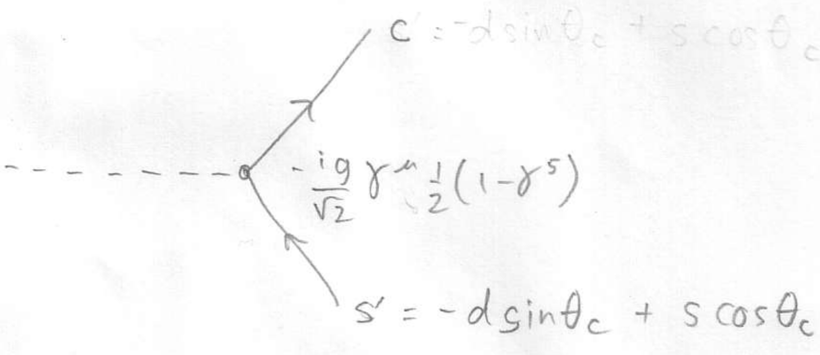
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \mathcal{U} \begin{pmatrix} d \\ s \end{pmatrix}$$

where $\mathcal{U}$ is a 2×2 unitary matrix which we can write

$$\mathcal{U} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

In this case the weak current can be written





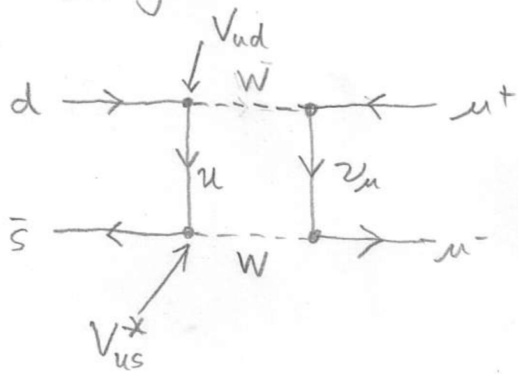
So $V_{ud} = \cos\theta_c \qquad V_{cd} = -\sin\theta_c$

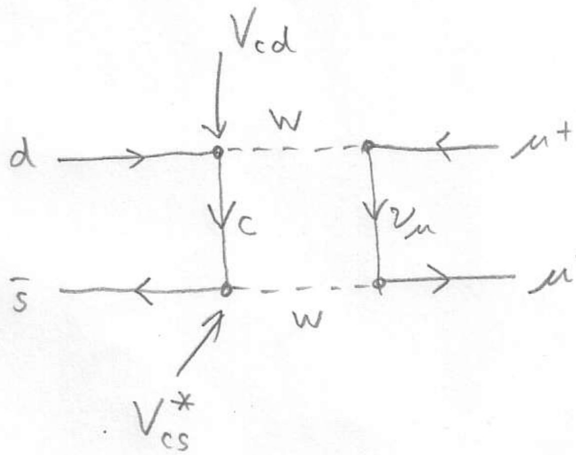
$V_{us} = \sin\theta_c \qquad V_{cs} = \cos\theta_c$.

This scheme has other importent consequences:

If there is a charm quark to complete the $\binom{c}{s}$ doublet, then Flavor Changing Neutral currents are cancelled.

Eg: $\underset{d\bar{s}}{K^{\circ}} \rightarrow \mu^{+}\mu^{-}$ might proceed via the diagrams:





$\propto V_{ud}V_{us}^{*} = \cos\theta_c \sin\theta_c$ $\qquad \propto V_{cd}V_{cs}^{*} = -\cos\theta_c \sin\theta_c$

If there were no charm quark, the second diagram would not be present and would not cancel the first diagram.

In fact, $Br(K^{\circ}_{L} \rightarrow \mu^{+}\mu^{-}) = 7.27 \times 10^{-9}$ only can take place because $m_c >> m_u$.

What are the couplings for 3 quarks Families?

Now we need

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad \text{where } U^{+}U = I.$$

weak eigenstates

strong eigenstates

U is called the Cabibbo-Kobayashi-Maskawa matrix, denoted V_{CKM}.

A 3×3 unitary matrix can be expressed in terms of 3 real parameters and one complex phase in various ways. The "standard representation" uses angles θ_{12}, θ_{23}, θ_{13} and a phase δ_{13}:

$$V_{CKM} = \begin{pmatrix} \cos\theta_{12}\cos\theta_{13} & \sin\theta_{12}\cos\theta_{13} & \sin\theta_{13} e^{-i\delta_{13}} \\ \vdots & \vdots & \sin\theta_{23}\cos\theta_{13} \\ \vdots & \vdots & \cos\theta_{23}\cos\theta_{13} \end{pmatrix}$$

Measured values of the magnitudes are

$$V_{CKM} = \begin{pmatrix} \sim .97 & \sim .22 & \sim .004 \\ \sim .22 & \sim .97 & \sim .04 \\ \sim .01 & \sim .04 & \sim 1 \end{pmatrix}$$

So U is "almost" diagonal $\Rightarrow$ strong and weak eigenstates are "almost" the same for some reason.

An easier form to work with is the Wolfenstein parameterization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

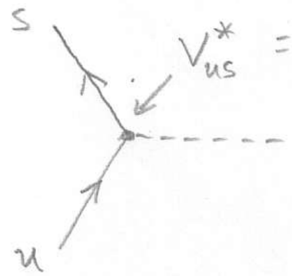
where $\lambda = \sin\theta_c \simeq 0.22$

$A \simeq 0.83$

$\left.\begin{array}{l} \rho \sim .2 \\ \eta \sim .3 \end{array}\right\}$ still some uncertainty.

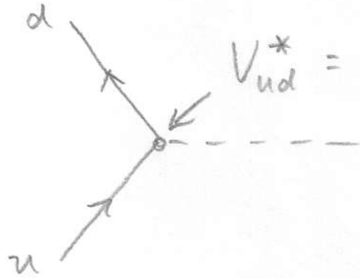
Examples:

$K^+ \to \mu^+ \nu_\mu$



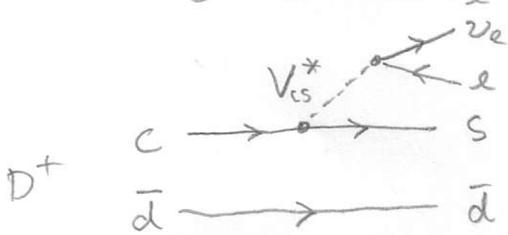
$V_{us}^* = \sin\theta_c$

$\pi^+ \to \mu^+ \nu_\mu$

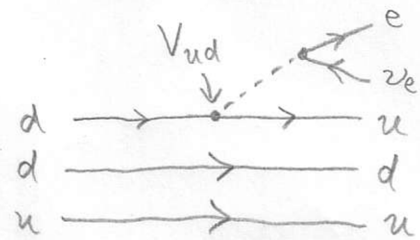


$V_{ud}^* = \cos\theta_c$

$D^+ \to K^0 \ell^+ \nu_e$



$n \to p e^- \bar{\nu}_e$



When to use $V_{qq'}$ or $V_{qq'}^*$?

Incomming u-type, outgoing d-type lines : $V_{qq'}^*$

Incomming d-type, outgoing u-type lines : $V_{qq'}$

Incomming $\bar{u}$-type, outgoind $\bar{d}$ type lines : $V_{qq'}$

Incomming $\bar{d}$-type, outgoing $\bar{u}$-type lines : $V_{qq'}^*$

$B^0 \to K^- \pi$ b

Flavor changing neutral currents can proceed via higher order diagrams called penguin diagrams:

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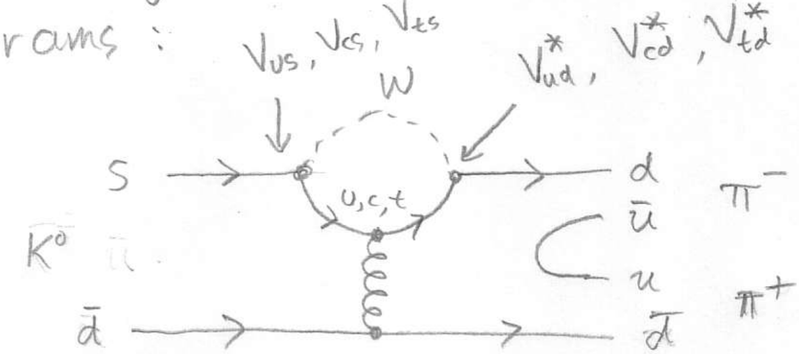
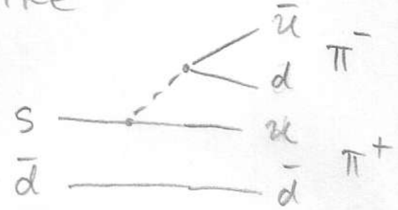
Eg,



In many cases there are other competing processes like



In some cases the transition is a pure penguin process: