Physics 564 - Pauli and Dirac Matrices

The Pauli matrices are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (1)$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (2)$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (3)$$

They satisfy the following identity:

$$\sigma_i \sigma_j = i\epsilon_{ijk}\sigma_k + \delta_{ij} \quad (4)$$

where

$$\epsilon_{ijk} = \begin{cases} 1 & \text{for even permutations of } (ijk) \\ -1 & \text{for odd permutations of } (ijk) \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

It follows that

$$(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = (\sigma_i a_i)(\sigma_j b_j) \quad (6)$$

$$= i a_i b_j \epsilon_{ijk}\sigma_k + a_i b_j \delta_{ij} \quad (7)$$

$$= i(\vec{a} \times \vec{b}) \cdot \vec{\sigma} + \vec{a} \cdot \vec{b} \quad (8)$$

$$= i(\vec{\sigma} \times \vec{a}) \cdot \vec{b} + \vec{a} \cdot \vec{b} \quad (9)$$

We will use the following representation for the Dirac matrices:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (10)$$

$$\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}. \quad (11)$$

They satisfy

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu \quad (12)$$

$$= 2g^{\mu\nu} \quad (13)$$

$$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0. \quad (14)$$

The tensor $\sigma^{\mu\nu}$ is defined:

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] = \frac{i}{2}\left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu\right) \quad (15)$$

The matrix γ^5 is defined

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{16}$$

and satisfies the identities

$$\left\{\gamma^\mu, \gamma^5\right\} = \gamma^\mu\gamma^5 + \gamma^5\gamma^\mu = 0 \tag{17}$$

$$\gamma^{5\dagger} = \gamma^5 \tag{18}$$

$$(\gamma^5)^2 = 1 \tag{19}$$