Assignment # 1 Solutions.

1. π^- beam has momentum of 500 GeV/c.
Target nucleon is at rest. Suppose it is a proton.

$p_\pi = (E_\pi, \vec{p}_\pi)$; $E_\pi = \sqrt{|p_\pi|^2 + m_\pi^2} = \sqrt{(500\ \text{GeV})^2 + (.1396\ \text{GeV})^2}$
$= 500$ GeV

$p_p = (M_p, \vec{0})$

$$s = (p_\pi + p_p)^2 = p_\pi^2 + p_p^2 + 2p_\pi \cdot p_p$$
$$= m_\pi^2 + m_p^2 + 2E_\pi M_p$$

$$\sqrt{s} = (m_\pi^2 + m_p^2 + 2E_\pi m_p)^{1/2}$$
$$= ((.1396\ \text{GeV})^2 + (.9383\ \text{GeV})^2 + 2(500\ \text{GeV})(.9383\ \text{GeV}))^{1/2}$$
$$= 30.65\ \text{GeV}$$

2. $$E_{D^0} = \frac{s + m_{D^0}^2 - m_X^2}{2\sqrt{s}}$$

$$E_{D^0}^{max} = \frac{s + m_{D^0}^2}{2\sqrt{s}} = \frac{(30.65\ \text{GeV})^2 + (1.864\ \text{GeV})^2}{2(30.65\ \text{GeV})}$$
$$= 15.38\ \text{GeV}$$

$$p_{D^0}^{max} = \sqrt{(E_{D^0}^{max})^2 - m_{D^0}^2}$$
$$= \sqrt{(15.38\ \text{GeV})^2 - (1.864\ \text{GeV})^2}$$
$$= 15.27\ \text{GeV}$$

3. See figures and example program.

4. $p_z' = \gamma p_z^{cm} + \gamma\beta E^{cm}$

$$\gamma = \frac{E_{lab}}{\sqrt{s}} = \frac{(E_\pi + m_p)}{\sqrt{s}}$$

$$= \frac{(500 \text{ GeV} + .9383 \text{ GeV})}{30.65 \text{ GeV}}$$

$$= 16.34$$

$$\gamma\beta = \sqrt{\gamma^2 - 1}$$

$$= \sqrt{(16.34)^2 - 1}$$

$$= 16.31$$

$$p_z^{lab} = (16.34)\, p_{D^0}^{cm} + (16.31)\, E_{D^0}^{cm}$$

See figure for distribution of p_z^{lab} when $p_z^{cm} = x_F\, p_{D^0}^{max}$ and x_F is distributed like $6x(1-x)$.

5. When the D^0 has momentum p_z^{lab} in the lab frame, the probability density for the decay time, t, in the lab is

$$\frac{dN}{dt} = \frac{1}{\gamma\tau} e^{-t/\gamma\tau} = \frac{c}{\gamma c\tau} e^{-ct/\gamma c\tau}$$

and since the decay length in the lab is $y = \beta c t$, $\frac{dN}{dy} = \frac{dN}{dt}\frac{dt}{dy} = \left(\frac{c}{\gamma c\tau}\right)\left(\frac{1}{\beta c}\right) e^{-y/\gamma\beta c\tau}$

$$= \left(\frac{1}{\gamma\beta c\tau}\right) e^{-y/\gamma\beta c\tau}$$

where $\gamma\beta = \frac{p_{lab}}{m_{D^0}}$.

Fraction with $y < 1\text{ cm}$ is 0.497.

6. $$E_K^* = \frac{m_{D^0}^2 + m_K^2 - m_\pi^2}{2\,m_{D^0}}$$

$$= \frac{(1.864\text{ GeV})^2 + (.4937\text{ GeV})^2 - (.1396\text{ GeV})^2}{2(1.864\text{ GeV})}$$

$$=$$

$$E_\pi^* = \frac{m_{D^0}^2 + m_\pi^2 - m_K^2}{2\,m_{D^0}}$$

$$= \frac{(1.864\text{ GeV})^2 + (.1396\text{ GeV})^2 - (.4937\text{ GeV})^2}{2(1.864\text{ GeV})}$$

$$=$$

7. $\cos\theta^*$ is uniformly distributed between -1 and $+1$. So, if x is uniformly distributed between 0 and 1 then $\cos\theta^* = 2x - 1$ will have the appropriate probability distribution. If $y = \cos\theta^* = 2x - 1$ then $\theta^* = \cos^{-1}(y)$.

See figures for distributions of $\cos\theta^*$ and θ^*.

8. In the D^0 rest frame,

$$p_{zK}^* = p_K^* \cos\theta^*$$
$$p_{TK}^* = p_K^* \sin\theta^* = p_K^* \sqrt{1-\cos^2\theta^*}$$
$$E_K^* = \sqrt{(p_K^*)^2 + m_K^2}$$
$$p_{z\pi}^* = -p_\pi^* \cos\theta^*$$
$$p_{T\pi}^* = -p_\pi^* \sin\theta^* = -p_\pi^* \sqrt{1-\cos^2\theta^*}$$
$$E_\pi^* = \sqrt{(p_\pi^*)^2 + m_\pi^2}$$

where $p_K^* = \sqrt{(E_K^*)^2 - m_K^2} = p_\pi^* = \sqrt{(p_\pi^*)^2 - m_\pi^2}$

8. (cont.)

If $\gamma = \dfrac{E_{D^0}^{lab}}{m_{D^0}}$ and $\gamma\beta = \dfrac{P_{D^0}^{lab}}{m_{D^0}}$

then
$$p_{z_K}^{lab} = \gamma\, p_{z_K}^{*} + \gamma\beta E_K^{*}$$
$$p_{T_K}^{lab} = p_{T_K}^{*}$$
$$E_K^{lab} = \gamma E_K^{*} + \gamma\beta\, p_{z_K}^{*}$$
$$\theta_{lab}(K^-) = \tan^{-1}\left(\frac{p_{T_K}^{lab}}{p_{z_K}^{lab}}\right)$$

Similarly,
$$p_{z_\pi}^{lab} = \gamma\, p_{z_\pi}^{*} + \gamma\beta E_\pi^{*}$$
$$p_{T_\pi}^{lab} = p_{T_\pi}^{*}$$
$$E_\pi^{lab} = \gamma E_\pi^{*} + \gamma\beta\, p_{z_\pi}^{*}$$
$$\theta_{lab}(\pi^+) = \tan^{-1}\left(\frac{p_{T_\pi}^{lab}}{p_{z_\pi}^{lab}}\right)$$

See figures for $p_{z_{K^-}}^{lab}$, $p_{z_\pi}^{lab}$, $\theta_{lab}(K)$, $\theta_{lab}(\pi)$.

Calculate $p_K = \sqrt{(p_{z_K}^{lab})^2 + (p_{T_K}^{lab})^2}$

then $\sigma_{p_K} = 0.0002(p_K^{lab})^2$.

Likewise, $p_\pi^{lab} = \sqrt{(p_{z_\pi}^{lab})^2 + (p_{T_\pi}^{lab})^2}$

$$\sigma_{p_\pi} = 0.0002\,(p_\pi^{lab})^2$$

If we assume the angles are measured accurately then the measured momenta will be $p_K^{measured} = p_K^{lab} + \sigma_{p_K} X$

where X has a Gaussian distribution with mean 0 and width 1.

Similarly $p_\pi^{measured} = p_\pi^{lab} + \sigma_{p_\pi} X$

Then

$$p_{z_K}^{meas.} = p_K^{meas} \cos\theta_K$$
$$p_{T_K}^{meas} = p_K^{meas} \sin\theta_K$$

$$p_{z_\pi}^{meas} = p_\pi^{meas} \cos\theta_\pi$$
$$p_{T_\pi}^{meas} = -p_\pi^{meas} \sin\theta_\pi \qquad (\text{when } \theta_\pi \text{ is } > 0)$$

The energies are calculated using an assumed mass:

$$E_K^{meas} = \sqrt{(p_K^{meas})^2 + m_K^2}$$
$$E_\pi^{meas} = \sqrt{(p_\pi^{meas})^2 + m_\pi^2}$$

The measured invariant mass is

$$(m_{K\pi}^{meas})^2 = (E_K^{meas} + E_\pi^{meas})^2 - (p_{z_K}^{meas} + p_{z_\pi}^{meas})^2 - (p_{T_K}^{meas} + p_{T_\pi}^{meas})^2$$

The distribution of $m_{K\pi}^{meas}$ is shown in the figures.

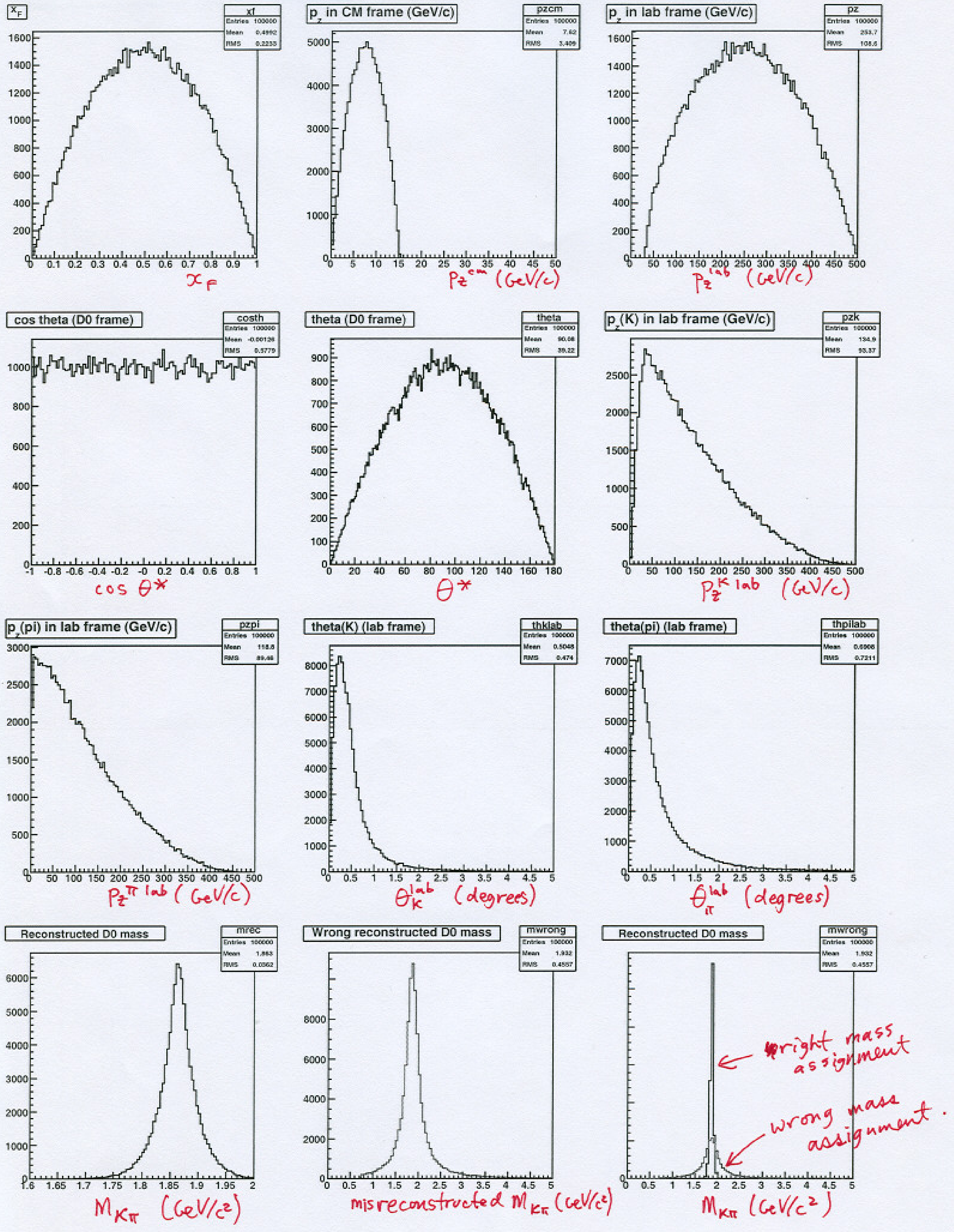
10. With the masses of K and π misassigned we would have

$$E_K'^{meas} = \sqrt{(p_\pi^{meas})^2 + m_K^2}$$

$$E_\pi'^{meas} = \sqrt{(p_K^{meas})^2 + m_\pi^2}$$

Again, $(m_{K\pi}'^{meas})^2 = (E_K'^{meas} + E_\pi'^{meas})^2 - (p_{zK}^{meas} + p_{z\pi}^{meas})^2 - (p_{TK}^{meas} + p_{T\pi}^{meas})^2$

and $m_{K\pi}'^{meas}$ is plotted in the figures.



x_F
xf
p_z in CM frame (GeV/c)
pzcm
p_z in lab frame (GeV/c)
pz
cos theta (D0 frame)
costh
theta (D0 frame)
theta
p_z(K) in lab frame (GeV/c)
pzk
p_z(pi) in lab frame (GeV/c)
pzpi
theta(K) (lab frame)
thklab
theta(pi) (lab frame)
thpilab
Reconstructed D0 mass
mrec
Wrong reconstructed D0 mass
mwrong
Reconstructed D0 mass
mwrong
x_F
p_z^{cm} (GeV/c)
p_z^{lab} (GeV/c)
$\cos\theta^*$
θ^*
$p_z^{K\,lab}$ (GeV/c)
$p_z^{\pi\,lab}$ (GeV/c)
θ_K^{lab} (degrees)
θ_π^{lab} (degrees)
$M_{K\pi}$ (GeV/c²)
misreconstructed $M_{K\pi}$ (GeV/c²)
$M_{K\pi}$ (GeV/c²)
right mass assignment
wrong mass assignment.