Physics 56400
Introduction to Elementary Particle Physics I

Lecture 10
Fall 2019 Semester
Prof. Matthew Jones
Elementary Particles

• Atomic physics:
  – Proton, neutron, electron, photon

• Nuclear physics:
  – Alpha, beta, gamma rays

• Cosmic rays:
  – Something charged (but what?)

• Relativistic quantum mechanics (Dirac, 1928)
  – Some solutions described electrons (with positive energy)
  – Other solutions described electrons with negative energy
  – Dirac came up with an elegant explanation for to think about this...
The Dirac Sea

- Dirac proposed that all electrons have negative charge.
- Normal electrons have positive energy.
- All negative-energy states are populated and form the Dirac sea.
- The Pauli exclusion principle explains why positive-energy electrons can’t reach a lower energy state — those states are already populated.
- The only way to observe an electron in the sea would be to give it a positive energy.
The Dirac Sea

Lowest positive-energy state corresponds to an electron at rest. It can’t fall into the sea because those states are already filled.
Now we have a positive energy electron, and a hole in the sea. The absence of a negative charge looks like a positive charge. This describes pair production.
Now that there is a hole in the sea, a positive energy electron can fall into it, releasing energy.

This corresponds to electron-positron annihilation.
Positrons

• This explanation was not immediately interpreted as a prediction for a new particle
• However, Carl Anderson observed “positive electrons” in cosmic rays in 1933.
• Nobel prize in 1934.
• Anti-particles are a natural consequence of special relativity and quantum mechanics.
Nuclear Forces

- If all the positive charge of an atom is contained in the tiny nucleus, why doesn’t electrostatic repulsion blow it apart?
- If there is another force that binds the nucleus together, why don’t we observe it in macroscopic experiments?
- Yukawa proposed that it must be a short-range force.
  \[ V(r) = \frac{e^{-r/r_0}}{r} = \frac{e^{-mr/\hbar c}}{r} \]
  - If \( r_0 \sim 1 \text{ fm} \), then \( m \sim 197 \text{ MeV} \)
  - This is consistent with E&M:
    - The photon is massless, making \( V(r) \) observable over macroscopic distances
Searching for Yukawa’s $\pi$-meson

• In 1936, Carl Anderson and Seth Neddermeyer observed a charged particle with intermediate mass in cosmic rays.
• Its mass was consistent with Yukawa’s meson but it did not interact with nuclear material.
• In 1947, the pi meson was observed in cosmic rays
Properties of Elementary Particles

• What distinguishes elementary particles?
  – Mass
  – Charge
  – Spin (intrinsic angular momentum)
  – Lifetime and decays
  – Interactions with other particles
  – Other quantum numbers to be discovered...
# Nucleons

- We already know about protons and neutrons

<table>
<thead>
<tr>
<th></th>
<th>$p$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>938.27 MeV</td>
<td>939.57 MeV</td>
</tr>
<tr>
<td>Charge</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>Spin</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Lifetime</td>
<td>(stable)</td>
<td>882 s</td>
</tr>
<tr>
<td>Decays</td>
<td>—</td>
<td>$n \rightarrow p + e^- + \bar{\nu}$</td>
</tr>
</tbody>
</table>

- When we don’t distinguish between them, we just call them “nucleons”, $N$. 
Pi Mesons

- Pions come in three varieties:

<table>
<thead>
<tr>
<th></th>
<th>$\pi^\pm$</th>
<th>$\pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>139 MeV</td>
<td>135 MeV</td>
</tr>
<tr>
<td>Charge</td>
<td>$\pm 1$</td>
<td>0</td>
</tr>
<tr>
<td>Spin</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lifetime</td>
<td>26 ns</td>
<td>$8.4 \times 10^{-17} , s$</td>
</tr>
<tr>
<td>Decays</td>
<td>$\pi^\pm \rightarrow \mu^\pm \nu$</td>
<td>$\pi^0 \rightarrow \gamma \gamma$</td>
</tr>
</tbody>
</table>

- Produced in nuclear collisions
- Interact strongly with nuclei
Hadrons

• Particles that interact strongly are hadrons.
• There are two types:
  – Baryons (like the proton and neutron)
  – Mesons (like pions)
• Baryon number seems to be a conserved quantity.

\[
\begin{align*}
\text{B} &= +2 \\
\rho + p &\rightarrow p + n + \pi^+ \\
\rho + n &\rightarrow p + n + \pi^0 \\
\text{B} &= +2
\end{align*}
\]
Leptons

- Electrons and muons are somewhat different.

<table>
<thead>
<tr>
<th></th>
<th>$e^\pm$</th>
<th>$\mu^\pm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>0.511 MeV</td>
<td>106 MeV</td>
</tr>
<tr>
<td>Charge</td>
<td>$\pm 1$</td>
<td>$\pm 1$</td>
</tr>
<tr>
<td>Spin</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Lifetime</td>
<td>(stable)</td>
<td>2.2 $\mu$s</td>
</tr>
<tr>
<td>Decays</td>
<td>$-$</td>
<td>$\mu^\pm \rightarrow e^\pm \nu \bar{\nu}$</td>
</tr>
</tbody>
</table>

- Lepton number and flavor seem to be conserved quantities.
- Neither interact strongly with nuclei
- Both are associated with beta decay
Beta Decay

• Nuclear beta decay:
  \[ n \rightarrow p + e^- + \bar{\nu}_e \]
  – The electron anti-neutrino cancels the electron’s lepton-number

• Muon decay:
  \[ \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \]
  – The electron anti-neutrino cancels the electron’s lepton number
  – The muon lepton number is carried by the muon neutrino

• Both are classified as weak decays because the lifetimes are so long
Neutrinos

- Neutrinos do not interact strongly or electromagnetically.
- Their weak interactions are so rare that we almost never observe them directly.
- If nuclear beta decay had a 2-body final state, then the electron would be mono-energetic.
  - Momentum/energy conservation:
    \[ E_e = \frac{m_p^2 + m_e^2 - m_n^2}{2m_p} \]
- In 1930, Pauli postulated the neutrino.
Hadronic Resonances

• Particles that decay strongly have short lifetimes
  – They decay instantaneously

• Strong decays are forbidden when they violate a conservation law
  \[ p \leftrightarrow \pi^+ + \pi^0 \]
  (violates baryon number conservation)

• Elastic scattering cross sections tell us about microscopic structure
  – Hard sphere scattering
  – Coulomb scattering

• Strongly decaying hadrons are observed as resonances in the elastic and inelastic cross sections
Hadronic Resonances

• Elastic pion-proton scattering cross section:

The first peak occurs at $E_{\text{kin}}(\pi) \sim 200$ MeV

$p_p = (m_p, \vec{0})$
$p_\pi = (E_\pi, \vec{p}_\pi)$

$$E_{\text{cm}}^2 = (p_p + p_\pi)^2$$
$$= (m_p + E_\pi)^2 - |\vec{p}_\pi|^2$$
$$= (m_p + E_\pi)^2 - (E_\pi^2 - m_\pi^2)$$
$$= m_p^2 + m_\pi^2 + 2m_p E_\pi$$
$$= m_p^2 + m_\pi^2 + 2m_p(m_\pi + E_{\text{kin}})$$

$E_{\text{cm}} = 1239$ MeV
Hadronic Resonances

- Resonances are often labeled with their mass in MeV.
- Except for charge, their properties are similar.
- In fact, there are four $\Delta(1232)$ resonances.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta^-$</th>
<th>$\Delta^0$</th>
<th>$\Delta^+$</th>
<th>$\Delta^{++}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1232 MeV</td>
<td>1231 MeV</td>
<td>1235 MeV</td>
<td>1231 MeV</td>
</tr>
<tr>
<td>Charge</td>
<td>$-1$</td>
<td>0</td>
<td>+1</td>
<td>+2</td>
</tr>
<tr>
<td>Spin</td>
<td>$3/2$</td>
<td>$3/2$</td>
<td>$3/2$</td>
<td>$3/2$</td>
</tr>
<tr>
<td>Width</td>
<td>117 MeV</td>
<td>117 MeV</td>
<td>117 MeV</td>
<td>117 MeV</td>
</tr>
<tr>
<td>Decays</td>
<td>$n + \pi^-$</td>
<td>$n + \pi^0, p + \pi^+$</td>
<td>$n + \pi^+, p + \pi^0$</td>
<td>$p + \pi^+$</td>
</tr>
</tbody>
</table>

- When we don’t distinguish between them, we just call them $\Delta$... The decays are all just $\Delta \rightarrow N\pi$. 
Isospin

• Electrons have spin $\frac{1}{2}$ but we don’t think of $|e \uparrow\rangle$ and $|e \downarrow\rangle$ as distinctly different particles
  – They are just two states of the same particle
  – They are symmetric unless we put them in a magnetic field

• The properties of the hadron multiplets are almost the same (except for charge):
  – Nucleon doublet
  – Pion triplet
  – Delta quadruplet

• Maybe these are just different states of the same strongly interacting particle

• We can only distinguish between them because of the electromagnetic interaction
Isospin

\[ I = \frac{1}{2} \text{ doublet: } (p_n) = \begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \]

\[ I = 1 \text{ triplet: } \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} = \begin{pmatrix} +1 \\ 0 \\ -1 \end{pmatrix} \]

\[ I = \frac{3}{2} \text{ multiplet: } \begin{pmatrix} \Delta^{++} \\ \Delta^+ \\ \Delta^0 \\ \Delta^- \end{pmatrix} = \begin{pmatrix} +\frac{3}{2} \\ +\frac{1}{2} \\ -\frac{1}{2} \\ -\frac{3}{2} \end{pmatrix} \]
Isospin

- The different charge states have different isospin components along the $I_z$ (or $I_3$) axis.
- This is completely made up and has no geometric meaning (it has nothing to do with the $z$-axis).
- Algebraically, it is the same as angular momentum.
- Fundamentally it is a representation of the group SU(2), which is the same as the group of rotations.
- If the strong interaction conserves isospin, then we can predict branching ratios and relative cross sections.
- You should review Clebsch-Gordon coefficients...
Clebsch-Gordon Coefficients

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND D FUNCTIONS

Note: A square-root sign is to be understood over every coefficient, e.g., for $-8/\sqrt{15}$ read $-\sqrt{8/15}$.

$$Y_{1}^{0} = \sqrt{\frac{3}{4\pi}} \cos \theta$$
$$Y_{1}^{1} = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}$$
$$Y_{2}^{0} = \sqrt{\frac{5}{4\pi}} \cos^2 \theta - \frac{1}{2}$$
$$Y_{2}^{1} = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$$
$$Y_{2}^{2} = \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi}$$

Note: The table format is not clearly visible in the image, but the coefficients are calculated using the formulas above.

Notation:

$$J \quad M \quad M_{1} \quad m_{1} \quad m_{2} \quad \text{Coefficients}$$

The coefficients are calculated using the following formula:

$$d_{m_{0},0} = \sqrt{\frac{4\pi}{2\ell + 1}} Y_{\ell m} e^{-im\phi}$$

and the final expression is:

$$\langle j_{1} j_{2} m_{1} m_{2} | j_{1} j_{2} J M \rangle = (-1)^{J-j_{1}-j_{2}} \langle j_{2} j_{1} m_{2} m_{1} | j_{2} j_{1} J M \rangle$$
Isospin

• Consider $\pi^+ p$ scattering...
• We have to add spin-1 to spin-1/2:

$$|1, +1\rangle |\frac{1}{2}, +\frac{1}{2}\rangle = |\frac{3}{2}, +\frac{3}{2}\rangle$$

• This can proceed only via the $\Delta^{++}$ resonance
• That was easy.
Isospin

- Consider $\pi^- p$ scattering:
- We have to add spin-1 to spin-1/2:

$$ |1, -1\rangle |\frac{1}{2}, +\frac{1}{2}\rangle = \sqrt{\frac{1}{3}} |\frac{3}{2}, -\frac{1}{2}\rangle - \sqrt{\frac{2}{3}} |\frac{1}{2}, -\frac{1}{2}\rangle $$

$$
\begin{array}{c c c c c}
1&1/2 & 3/2 & 3/2 & 1/2 \\
+1&+1/2 & 1 & +1/2 & +1/2 \\
+1&-1/2 & 1/3 & 2/3 & 3/2 \\
0&+1/2 & 2/3 & -1/3 & 1/2 \\
0&-1/2 & 2/3 & 1/3 & 3/2 \\
-1&+1/2 & 1/3 & -2/3 & -3/2 \\
-1&-1/2 & 1 & 1 & 1 \\
\end{array}
$$
Isospin

• Amplitude for observing the $\Delta^{++}$ state:
  \[ \langle \Delta^{++} | \pi^+ p \rangle = 1 \]

• Amplitude for observing the $\Delta^0$ state:
  \[ \langle \Delta^0 | \pi^- p \rangle = \sqrt{1/3} \]

• Cross section $\propto$ probability $\propto |\langle f | i \rangle|^2$

• The $\Delta^0$ cross section in $\pi^- p$ scattering should be $1/3$ the $\Delta^{++}$ cross section in $\pi^+ p$ scattering.

• Remember, the strong interaction doesn’t care. This is all just $\pi N \rightarrow \Delta$ and it conserves isospin.
Isospin

It’s not exact, but what do you expect from a model with almost no content?

And besides, nobody had any better ideas at the time.

Figure 5.35: Total cross section as a function of pion kinetic energy for the scattering of positive and negative pions from protons. (1 mb = 1 millibarn = $10^{-27}$ cm$^2$.)
Weak Decays

• Weak decays do not conserve isospin!
• Examples:

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

\[ I_3 = -\frac{1}{2} \quad I_3 = +\frac{1}{2} \]

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]

\[ I_3 = +1 \quad I_3 = 0 \]

• This is NOT the same as “weak isospin” which we will use to describe the weak interaction.
Other Quantum Numbers

• How do these states change under parity transformations?
  – Even parity: \( \Pi |\psi\rangle = +|\psi\rangle \)
  – Odd parity: \( \Pi |\psi\rangle = -|\psi\rangle \)

• How can we tell?
• The proton is assigned a parity of +1
  – Therefore, the neutron also has a parity of +1
• The deuteron has spin-1 and parity of \((+1)^2\)
• A \( \pi^- \) is captured on deuterium from an S-wave ground state \((L = 0)\) and emits two neutrons
  – Identical fermions must have odd parity and opposite spins
  – Pions have spin-0 so they the deuterons must have \( L = 1 \)
• Parity of initial state: \((+1)^2(\pi)\)
• Parity of final state: \((+1)^2(-1)^L = (-1)\)  \[ \pi = -1 \]

Chinowsky and Steinberger, 1954
Parity

• Parity assignments:
  – $N, \Delta$ have parity +1
  – $\pi$ have parity -1
  – $\gamma$ has parity +1
• Parity is conserved by electromagnetic and strong interactions
• Parity is violated in weak interactions