Phys 56400 Assignment #3

1. Total radiation length is $20X_0$. For lead, $X_0 = 6.37\ g\cdot cm^{-2}$ and $\rho = 11.35\ g\cdot cm^{-3}$. Hence the total thickness is

$$T = \frac{20X_0}{\rho} = \frac{20\cdot(6.37\ g\cdot cm^{-2})}{11.35 g\cdot cm^{-3}} = 11.22\ cm.$$

For iron, $X_0 = 13.84\ g\cdot cm^{-2}$ and $\rho = 7.874\ g\cdot cm^{-3}$.

$$T = \frac{20X_0}{\rho} = \frac{20\cdot(13.84\ g\cdot cm^{-2})}{7.874\ g\cdot cm^{-3}} = 35.15\ cm.$$

For tungsten, $X_0 = 6.76\ g\cdot cm^{-2}$ and $\rho = 19.3\ g\cdot cm^{-3}$

$$T = \frac{20X_0}{\rho} = \frac{20\cdot(6.76\ g\cdot cm^{-2})}{19.3\ g\cdot cm^{-3}} = 7.01\ cm.$$

2. Total thickness is $16\lambda_I$. For lead, $\lambda_I = 199.6\ g\cdot cm^{-2}$ and $\rho = 11.35\ g\cdot cm^{-3}$. Hence, the total thickness is

$$T = \frac{16\lambda_I}{\rho} = \frac{16\cdot(199.6\ g\cdot cm^{-2})}{11.35\ g\cdot cm^{-3}}$$

$$= 281.4\ cm.$$

For iron, $\lambda_I = 132.1\ g\cdot cm^{-2}$ and $\rho = 7.874\ g\cdot cm^{-3}$ so

$$T = \frac{16\lambda_I}{\rho} = \frac{16\cdot(132.1\ g\cdot cm^{-2})}{7.874\ g\cdot cm^{-3}} = 268.4\ cm.$$

2. (cont.) For brass, which is a mixture of 67% copper and 33% zinc (by mass) the nuclear interaction length is

$$\lambda_I = \left(w_{Cu} \cdot \frac{1}{\lambda_I^{Cu}} + w_{Zn} \cdot \frac{1}{\lambda_I^{Zn}} \right)^{-1}$$

$$= \left(\frac{0.67}{137.3\, g \cdot cm^{-2}} + \frac{0.33}{138.5\, g \cdot cm^{-2}} \right)^{-1}$$

$$= 137.7\, g \cdot cm^{-2}.$$

The density of brass is $8.73\, g \cdot cm^{-3}$ so

$$T = \frac{16 \lambda_I}{\rho} = \frac{16 \cdot (137.7\, g \cdot cm^{-2})}{8.73\, g \cdot cm^{-3}} = 252.4\, cm.$$

3(a) Air has $\left.\frac{dE}{dx}\right|_{min} = 1.815\ \mathrm{MeV \cdot g^{-1} \cdot cm^2}$ and a density of $1.205\ \mathrm{g/\ell} = 1.205 \times 10^{-3}\ \mathrm{g \cdot cm^{-3}}$. Therefore, the rate of energy loss in air is

$$\frac{dE}{dx} = (1.815\ \mathrm{MeV \cdot g^{-1} \cdot cm^2})(1.205 \times 10^{-3}\ \mathrm{g \cdot cm^{-3}})$$
$$= 2.187\ \mathrm{keV \cdot cm^{-1}}$$

The energy lost after travelling 5 km would be

$$\Delta E = \frac{dE}{dx}\Delta x = (2.187\ \mathrm{keV \cdot cm^{-1}})(5\ \mathrm{km})(100{,}000\ \mathrm{cm/km})$$
$$= 1094\ \mathrm{MeV}$$

(b) Water has $\left.\frac{dE}{dx}\right|_{min} = 1.981\ \mathrm{MeV \cdot g^{-1} \cdot cm^2}$ and a density of $1.\ \mathrm{g \cdot cm^{-3}}$. Thus, the rate of energy loss is

$$\frac{dE}{dx} = 1.981\ \mathrm{MeV \cdot cm^{-1}}$$

and the approximate range of a 100 GeV muon would be

$$\Delta x = \frac{\Delta E}{dE/dx} = \frac{100{,}000\ \mathrm{MeV}}{1.981\ \mathrm{MeV \cdot cm^{-1}}} = 50480\ \mathrm{cm}$$
$$= 504.8\ \mathrm{m}.$$

(c) Standard rock has $\left.\frac{dE}{dx}\right|_{min} = 1.688\ \mathrm{MeV \cdot g^{-1} \cdot cm^2}$ and a density of $2.650\ \mathrm{g \cdot cm^{-3}}$. Thus,

$$\frac{dE}{dx} = (1.688\ \mathrm{MeV \cdot g^{-1} \cdot cm^2})(2.650\ \mathrm{g \cdot cm^{-3}}) = 4.47\ \mathrm{MeV \cdot cm^{-1}}$$

and the range would be $\Delta x = \frac{100{,}000\ \mathrm{MeV}}{4.47\ \mathrm{meV \cdot cm^{-1}}} = 223.7\ \mathrm{m}$.

4. The Gluckstern formula is

$$\frac{\sigma_{p_T}}{p_T} = \frac{\sigma_x \cdot p_T}{0.3 \cdot BL^2} \sqrt{\frac{720}{N+4}}$$

In this case, $\sigma_x = 130\,\mu m = 0.013$ cm.

$$L = \frac{1}{2}(D_{out} - D_{in})$$
$$= \frac{1}{2}(370 \text{ cm} - 50 \text{ cm})$$
$$= 160 \text{ cm}.$$

$B = 0.435$ Tesla $= 4.35$ kG
$N = 159$.

For a charged track with $p_T = 50$ GeV,

$$\frac{\sigma_{p_T}}{p_T} = \frac{(0.013 \text{ cm})(50{,}000 \text{ MeV/c})}{(0.3 \text{ MeV/c} \cdot \text{kG}^{-1} \cdot \text{cm}^{-1})(4.35 \text{ kG})(160 \text{ cm})^2} \sqrt{\frac{720}{159+4}}$$

$$= 0.041 \quad \text{or} \quad 4.1\%.$$

(b) If the pressure was reduced to 1 atm, there would be less multiple scattering and hence, σ_x would be reduced. Thus, the momentum resolution would get better.

(c) With less gas (lower pressure) $\frac{dE}{dx}$ would be smaller and $\frac{\sigma_{dE/dx}}{dE/dx}$ would get larger.

In general $\sigma_{dE/dx} \propto \sqrt{dE/dx}$ so one might expect that $\frac{\sigma_{dE/dx}}{dE/dx}$ would be a factor of two worse.