Physics 56400
Introduction to Elementary Particle Physics I

Now in PowerPoint!

Lecture 24
Fall 2018 Semester
Prof. Matthew Jones
Neutrino Physics

- Early evidence for neutrinos:
  - Continuous spectrum of beta energies
  - Angular momentum conservation arguments
  - Proposed by Pauli in 1930

\[ ^{210}\text{Bi} \rightarrow ^{210}\text{Po} + \beta^- + ??? \]
Pauli’s Neutrino Hypothesis

Dear Radioactive Ladies and Gentlemen,

... The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton mass. - The continuous beta spectrum would then make sense with the assumption that in beta decay, in addition to the electron, a neutron is emitted such that the sum of the energies of neutron and electron is constant.

I admit that my remedy may seem almost improbable because one probably would have seen those neutrons, if they exist, for a long time.

Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball on the night from December 6 to 7.

... Signed W. Pauli
The shape of β energy spectrum near the endpoint is sensitive to the neutrino mass.

\[
m_\nu < 1 \text{ keV}
\]
Detecting Anti-Neutrinos

Fermi theory: \( n \rightarrow p + \beta^- + \bar{\nu} \)

Inverse reaction: \( \bar{\nu} + p \rightarrow n + \beta^+ \)

1952 – Reines & Cowan propose to detect the \( \beta^+ \) annihilation in liquid scintillator

Only sensitive to cross sections \( \sigma \sim 10^{-40} \text{ cm}^2 \)

Expected cross section is \( 10^{-44} \text{ cm}^2 \)

Sensitivity dominated by background count rate.

The same sensitivity would be achieved at a reactor.
Detecting Anti-Neutrinos

Delayed coincidence – neutron capture on cadmium...

\[
\bar{\nu} + p \rightarrow n + e^+ \quad e^+e^- \rightarrow \gamma\gamma \quad (1.05 \text{ MeV})
\]

\[
n + ^{108}\text{Cd} \rightarrow ^{109}\text{Cd}^* \rightarrow ^{109}\text{Cd} + \gamma \quad (9 \text{ MeV})
\]

Detect neutron capture within 5 µs of \(e^+e^-\) annihilation.

\(\text{CdCl}_2\) dissolved in water
Detecting Neutrinos

• How do we know that $\nu$ and $\bar{\nu}$ are different?

• Pontecorvo’s method:

$$\nu + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar}$$

  – Natural abundance of $^{37}\text{Cl}$ is 24%
  – Tetrachloroethylene ($C_2\text{Cl}_4$) is non-flammable, inexpensive
  – $^{37}\text{Ar}$ is unstable with a half-life of 35 days
  – Production and decay rates in equilibrium
  – Small numbers of $^{37}\text{Ar}$ atoms can be extracted from a large volume of $C_2\text{Cl}_4$.

• Described in a lecture to physics students at McGill and written up in NRC Report P.D.-205

  – Immediately classified by the US Atomic Energy Commission
Detecting Neutrinos

Ray Davis, 1955

\[ \nu + ^{37}Cl \rightarrow e^- + ^{37}Ar \]
\[ \bar{\nu} + p \rightarrow n + e^+ \]

Unlike photons and $\pi^0$ mesons, which are their own anti-particles, neutrinos and anti-neutrinos are different!

Argon Extraction

- Flush with helium
- Freeze onto charcoal trap cooled with LN$_2$
- Count $^{37}Ar$ with G-M tube
- Demonstrated greater than 90% extraction efficiency

55 gallons of CCl$_4$

A second system consisted of 1000 gallons of CCl$_4$

Attempt to Detect the Antineutrinos from a Nuclear Reactor by the Cl$^{37}(\nu,e^-)A^{37}$ Reaction*

RAYMOND DAVIS, JR.
Department of Chemistry, Brookhaven National Laboratory, Upton, Long Island, New York
(Received September 21, 1954)
1959 – Bruno Pontecorvo considered the production of neutrinos at accelerators:

\[
\begin{align*}
\pi^+ & \rightarrow \mu^+ \nu_\mu \\
\nu_\mu + n & \rightarrow p + \mu^- \\
\pi^- & \rightarrow \mu^- \bar{\nu}_\mu \\
\bar{\nu}_\mu + p & \rightarrow n + \mu^+
\end{align*}
\]
Elastic Neutrino Scattering

- Sufficient to use the Fermi 4-point interaction
- First, consider elastic scattering $\nu_e + e^- \rightarrow \nu_e + e^-$

\[
\begin{align*}
J^\mu &= \bar{u}_2 \frac{1}{2} \gamma^\mu (1 - \gamma^5) u_1 \\
J_\mu &= \bar{u}_4 \frac{1}{2} \gamma_\mu (1 - \gamma^5) u_2
\end{align*}
\]

\[
|\tilde{\mathcal{M}}|^2 = 64G_F^2 (p_1 \cdot p_2) (p_3 \cdot p_4) = 16G_F^2 s^2
\]

\[
s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \approx 2p_1 \cdot p_2 \approx 2p_3 \cdot p_4
\]
Elastic Neutrino Scattering

- Next, consider elastic $\bar{\nu}_e$ scattering $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$

$$|\vec{M}|^2 = 64G_F^2(p_4 \cdot p_2)(p_3 \cdot p_1)$$

$$= 16G_F^2 t^2$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$

$$\approx -2p_1 \cdot p_3 \approx -2p_2 \cdot p_4$$

This is the same diagram except with $1 \leftrightarrow 4$
Elastic Neutrino Scattering

- In both cases we can consider the cross section in the center-of-mass frame:

\[ p_1 = \left( \frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2} \right) \]

\[ p_3 = \left( \frac{\sqrt{s}}{2}, \frac{\sqrt{s}}{2} \sin \theta, 0, \frac{\sqrt{s}}{2} \cos \theta \right) \]

\[ t = -\frac{1}{2} s(1 - \cos \theta) \]

- Incident flux: \( F = 2s \)

- Phase space: \( dQ = \frac{1}{32\pi^2} d\Omega \)

\[ d\sigma = \frac{|\overline{M}|^2}{F} dQ \]
Elastic Neutrino Scattering

• Differential scattering cross section:

\[
d\sigma = \frac{4G_F^2 s}{64\pi^2} (1 - \cos \theta)^2 d\Omega
\]

\[
G_F^2 S \frac{(1 - y)^2 dy}{8\pi}
\]

\( (y = \cos \theta) \)

• Total cross section:

\[
\sigma = \frac{G^2 s}{3\pi}
\]
Elastic Neutrino Scattering

- Normally this experiment would be carried out in the lab frame with the electron initially at rest.
  \[ p_{\nu} = (E_{\nu}, \hat{p}_{\nu}), \ |\hat{p}_{\nu}| = E_{\nu} \]
  \[ p_{e} = (m_{e}, \vec{0}) \]
  \[ s = (p_{\nu} + p_{e})^2 \approx 2E_{\nu}m_{e} \]

- Total cross section:
  \[ \sigma = \frac{2G_{F}^{2}m_{e}E_{\nu}}{3\pi} \]
  - If \( E_{\nu} \sim 5 \text{ MeV} \) then \( \sigma = 2.9 \times 10^{-44} \text{ cm}^{2} \)
  - This small cross section is typical of neutrino interactions.
Muon Neutrino Beams

- A wide-band neutrino beam is produced by a high energy proton beam on a target
  - Lots of pions and kaons are produced
  - These decay to muons and muon neutrinos
  - Muons decay to electrons and neutrinos
- A narrow-band beam is produced from decays of pions and kaons that are selected with a specific range of momenta.
- Muon neutrino beams can interact with thick nuclear targets
  \[ \nu_\mu + N \rightarrow \mu^- + X \]
  \[ \bar{\nu}_\mu + N \rightarrow \mu^+ + X \]
- The muons will ionize the instrumented parts of the target and are easily detected.
Probing Nuclear Structure

• The muon neutrino will either interact with a $d$-quark or a $\bar{u}$-quark.

• These diagrams are the same as the elastic scattering diagrams except with the momenta re-labeled.
Probing Nuclear Structure

\[
\frac{d\sigma}{d\Omega} (\nu_\mu d \rightarrow \mu^- u) = \frac{G_F^2 S}{4\pi^2}
\]

\[
\frac{d\sigma}{d\Omega} (\bar{\nu}_\mu u \rightarrow \mu^+ d) = \frac{G_F^2 S}{16\pi^2} (1 + \cos \theta^*)^2
\]

\[
\frac{d\sigma}{d\Omega} (\nu_\mu \bar{u} \rightarrow \mu^- \bar{d}) = \frac{G_F^2 S}{16\pi^2} (1 + \cos \theta^*)^2
\]

\[
\frac{d\sigma}{d\Omega} (\bar{\nu}_\mu \bar{d} \rightarrow \mu^+ \bar{u}) = \frac{G_F^2 S}{4\pi^2}
\]

- To calculate cross sections in the lab frame, express these in terms of \( y = (E_\mu - E_\nu)/E_\nu \)

\[
1 - y = \frac{1}{2} (1 + \cos \theta^*)
\]

- But what is \( s \) when we are colliding with quarks inside a nucleus?
Parton Density Functions

- Suppose that a quark carries a fraction, $x$, of the total momentum of the nucleus

$$\begin{align*}
  p_v &= \left( \frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2} \right) \\
  p_q &= \left( \frac{x\sqrt{s}}{2}, 0, 0, -\frac{x\sqrt{s}}{2} \right) \\
  \hat{s} &= (p_v + p_q)^2 = xs
\end{align*}$$

- The differential cross sections can be expressed

$$\frac{d\sigma}{dxdy}(\nu_\mu N \to \mu X) = \sum_i f_i(x) \left( \frac{d\hat{\sigma}_i}{dy} \right)$$

- $f_i(x)dx$ is the probability for finding a quark of type $i$ with momentum fraction between $x$ and $x + dx$. 
Probing Nuclear Structure

• We can’t *a priori* calculate these probability densities

• Isospin symmetry:

\[
d_n(x) = u_p(x) \\
\bar{u}_n(x) = \bar{d}_p(x)
\]

• For an isoscalar target like liquid deuterium, we will average over protons and neutrons:

\[
\frac{d\sigma}{dxdy}(\nu N) = \frac{G_F^2 xS}{2\pi} \left[ u(x) + d(x) + (\bar{u}(x) + \bar{d}(x)) (1 - y)^2 \right]
\]

\[
\frac{d\sigma}{dxdy}(\bar{\nu} N) = \frac{G_F^2 xS}{2\pi} \left[ \bar{u}(x) + \bar{d}(x) + (u(x) + d(x))(1 - y)^2 \right]
\]
Probing Nuclear Structure

• In the lab frame,
  \[ s = (p_\nu + p_N)^2 = 2M_N E_\nu \]
• These cross sections can be made relatively large using very high-energy neutrino beams.
• Integrate over the parton densities to define:
  \[ Q \equiv \int_0^1 x (u(x) + d(x)) dx \]
  \[ \bar{Q} = \int_0^1 x (\bar{u}(x) + \bar{d}(x)) dx \]
• And recall that
  \[ \int_0^1 (1 - y)^2 dy = \frac{1}{3} \]
Probing Nuclear Structure

\[
\sigma(\nu N) = \frac{G_F^2 M_N E_\nu}{\pi} \left( Q + \frac{1}{3} \bar{Q} \right)
\]

\[
\sigma(\bar{\nu} N) = \frac{G_F^2 M_N E_\nu}{\pi} \left( \bar{Q} + \frac{1}{3} Q \right)
\]

• If we were to measure $\sigma/E_\nu$ as a function of $E_\nu$, we would expect it to remain constant.

• Furthermore, we expect that

\[
R = \frac{\sigma(\bar{\nu} N)}{\sigma(\nu N)} = \frac{1 + 3 \bar{Q}/Q}{3 + \bar{Q}/Q}
\]
Probing Nuclear Structure

• If there were no anti-quarks in the nucleon, then we would expect that $R = 1/3$.
• The cross sections are indeed found to be directly proportional to the beam energy but the observed ratio is
  \[ R \sim 0.45 \Rightarrow \frac{\bar{Q}}{Q} \sim 0.14 \]
• This provides direct evidence that neutrinos can interact with the anti-quarks from the sea of virtual $\bar{q}q$ pairs within the nucleon.
Probing Nuclear Structure

Fig. 5.13. Neutrino and antineutrino cross-sections on nucleons. The ratio $\sigma/E_\nu$ is plotted as a function of energy and is indeed a constant, as predicted in (5.45) and (5.46).