1. Consider the following transformer circuit:

(a) Show that the exact expression for the voltage gain is given by

\[ A = \frac{|V_{\text{out}}|}{|V_{\text{in}}|} = \frac{n}{\sqrt{\left(1 + n^2 \frac{R_1}{R_2}\right)^2 + \left(\frac{R_1}{\omega L_1}\right)^2}} \] (1)

where \( n = n_2/n_1 = \sqrt{L_2/L_1} \) and where we assume perfect coupling between the two coils in the transformer, \( \text{i.e. } M = \sqrt{L_1L_2} \).

(b) Describe the low-frequency and high-frequency behavior of this circuit and calculate the frequency, \( \omega_{-3 \text{ db}} \), at which \( A(\omega_{-3 \text{ db}}) = A_{\text{max}}/\sqrt{2} \).

(c) Suppose that \( R_1 = 50 \, \Omega \), \( R_2 = 100 \, \Omega \). What value of \( n \) will maximize the power dissipated in \( R_2 \) at high frequencies?

(d) In this case, what value of \( L_1 \) would be required to give \( f_{-3 \text{ db}} = 1 \, \text{MHz} \) where \( \omega = 2\pi f \)? What value of \( L_2 \) would then be required to give the desired value of \( n \)?

(e) Use SPICE to perform an AC sweep analysis of the circuit using the component values determined in part (d). In SPICE, the transformer in this circuit is described using coupled inductors as follows:

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L1  2  0  value
L2  3  0  value
K12 L1 L2  k
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in which \( k \) is the constant in the expression \( M = k\sqrt{L_1L_2} \). For ideal coupling, as is the case in this problem, use \( k = 1 \).

Hand in the graph of the magnitude of the voltage across \( R_2 \) as a function of frequency, between 1 Hz and 10 MHz. Verify that \( f_{-3 \text{ db}} \) occurs at the frequency calculated in part (d).