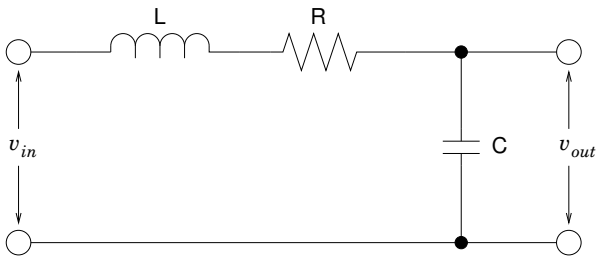
Physics 536 - Assignment #3

1. A first-order low-pass RC filter has a frequency response that falls off with a slope of 20 dB per decade or 6 dB per octave. Show that the second-order RLC filter shown below has a frequency response that falls off with a slope of 40 dB per decade or 12 dB per octave.

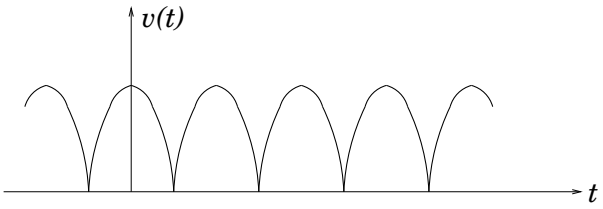


2. What inductance would be needed to make a second-order low-pass RLC filter with a cut-off frequency of $f = 2\pi\omega_0 = 1$ MHz if a 10 nF capacitor was to be used? In principle, this filter could act as an RLC resonant circuit near this frequency. What value of R would ensure that the Q-factor for this oscillator is equal to 1?

3. Consider a periodic voltage source of the form

$$v(t) = V_0|cos(\omega t)|$$

which looks something like this:

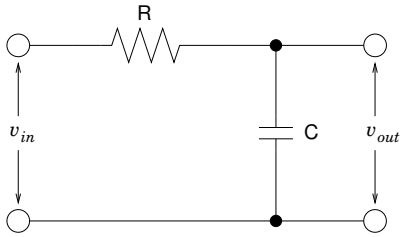


This waveform can be represented by the Fourier series:

$$v(t) = \frac{4V_0}{\pi}\left(\frac{1}{2} + \frac{1}{3}\cos 2\omega t - \frac{1}{15}\cos 4\omega t + \frac{1}{35}\cos 6\omega t - \frac{1}{63}\cos 8\omega t + \cdots\right)$$

In general, the coefficient in front of a term of the form $\cos 2k\omega t$ will be $\frac{(-1)^{k+1}}{(2k-1)(2k+1)}$ where $k = 1, 2, \ldots$.

Suppose a voltage source of this form, with $V_0 = 10$ V and a frequency of $f = \omega/2\pi = 120$ Hz was applied to the low-pass RC circuit shown below in which $R = 10\ \Omega$ and $C = 2000\ \mu$F.



(a) Write down an analytic expression for the magnitude and phase shift of an output waveform when the input waveform is of the form $v_{in}(t) = V_0 \cos(\omega t)$.

(b) Calculate the numerical values of the magnitude and phase shift of the output voltage for each of the discrete frequency components in the Fourier representation of the input voltage, up to $n = 4$.

(c) By whatever method you can come up with, produce a plot of the input and output waveforms calculated by adding up the response of circuit to each of the frequency components found in (b). Be sure to show the waveform over at least three cycles.