Physics 536 - Assignment #4

1. Digital data is frequently transmitted on cables that consist of pairs of conductors twisted together. If each conductor carries equal and opposite currents, then their radiated electromagnetic fields cancel. However, if there is an imbalance in the current, energy can be radiated and cause interference in other circuits.

(a) Consider the following model for a twisted pair, driven by two voltage sources, \( v_1(t) = V_1 e^{i\omega t} \) and \( v_2(t) = V_2 e^{i\omega t} \) as shown:

![Twisted Pair Diagram]

in which \( R_S \) and \( R'_S \) represent the impedances of the driving voltage sources and \( R_L \) and \( R'_L \) represent resistive loads at the receiving end of the cable. Show that, in general, equal and opposite currents will flow in the two conductors only when \( V_1 = -V_2 \) and \( R_S + R_L = R'_S + R'_L \).

Considering that manufacturing tolerances on components are typically only 1-5%, argue that it would be possible, if not likely, that an imbalanced current would flow and become a source of electromagnetic interference.
(b) A similar circuit uses a transformer to terminate the twisted pair in the following configuration:

Assuming \( L_1 = L_2 \) and perfect coupling between the transformer windings, and that \( \omega \gg R/L \), show that the currents that flow in the two conductors are now equal and opposite, regardless of whether \( V_1 = -V_2 \) or whether \( R_S + R_L = R'_S + R'_L \).

In this configuration, the transformer is sometimes called a “balun”, and it forces equal and opposite currents to flow in the two conductors, minimizing radiated electromagnetic energy. These magnetic components are sometimes integrated into the RJ45 connectors used for Ethernet.
2. This problem investigates the parameters that affect the physical size of transformers. Consider the transformer circuit:

\[ V_{out} = \frac{i\omega MV_{in}R_L}{R_SR_L + i\omega(L_1R_L + L_2R_S)} \]  

which applies in the case of perfect coupling, \( M = \sqrt{L_1L_2} \).

(a) Calculate the frequency, \( \omega_0 \), at which the magnitude of the ratio \( V_{out}/V_{in} \) is \( 1/\sqrt{2} \) its value when \( \omega \to \infty \).

(b) Suppose a transformer with a turns ratio of 10:1 was used to convert 110 V AC to 11 V AC. If the voltage source had an impedance \( R_S = 0.5 \, \Omega \) and the load had a resistance of 1 \, \Omega, what primary inductance, \( L_1 \), would be required such that \( \omega_0 < 190 \, s^{-1} \), which is about half the 60 Hz line frequency?

(c) By arguing that the primary winding can be thought of as an ideal solenoid, show that the physical volume of a transformer is directly proportional to its inductance and therefore inversely proportional to the frequency, \( \omega_0 \).

(d) Explain why it might be desirable to use an AC voltage source with a frequency of 400 Hz instead of 60 Hz, in such an application. In fact, it is not uncommon to use a motor-generator to produce 400 Hz AC power for industrial applications.
3. Consider the following circuit:

\[ v_{in}(t) = \begin{cases} 
0 \text{ V} & \text{for } t < 0 \\
1 \text{ V} & \text{for } t > 0 
\end{cases} \] (2)

(a) Sketch, as accurately as possible, the voltage observed at \( v_{out}(t) \). This configuration is sometimes used to produce short pulses without the use of fast timing circuits.

(b) At some point the voltage source \( v_{in}(t) \) will return to zero volts. Sketch the voltage observed at \( v_{out}(t) \) when this happens.