PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 35 – Interference

Spring 2016 Semester

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Interference

- Electric field:

$$\vec{E}(\vec{x},t) = \vec{E}_0 \cos(\vec{k}\cdot\vec{x} - \omega t)$$

- Light intensity:

$$I = c\epsilon \left\langle \left|\vec{E}\right|^2 \right\rangle_T$$

- Two electric fields:

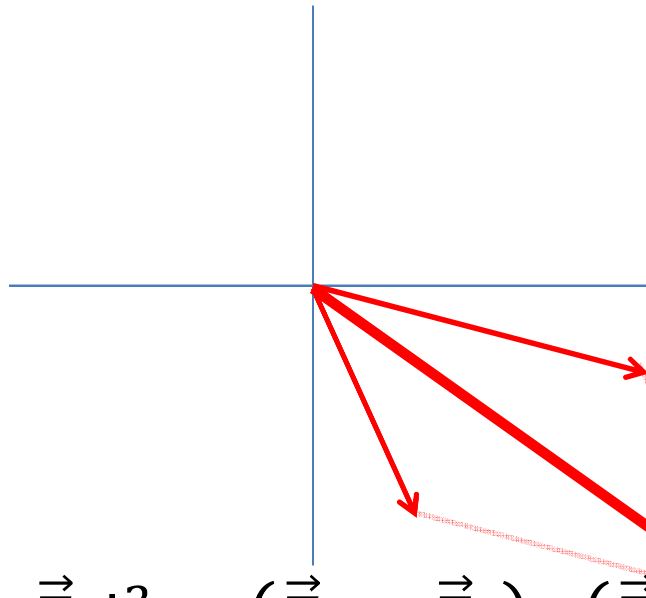
$$\vec{E}_1(\vec{x},t) = \vec{E}_{10} \cos(\vec{k}\cdot\vec{x} - \omega t + \xi_1)$$
$$\vec{E}_2(\vec{x},t) = \vec{E}_{20} \cos(\vec{k}\cdot\vec{x} - \omega t + \xi_2)$$

- Light intensity:

$$I = v\epsilon \left\langle |\vec{E}_1 + \vec{E}_2|^2 \right\rangle_T$$

Interference

$$I = v\epsilon\left\langle |\vec{E}_1 + \vec{E}_2|^2 \right\rangle_T$$



$$|\vec{E}_1 + \vec{E}_2|^2 = \left(\vec{E}_1 + \vec{E}_2\right)\cdot\left(\vec{E}_1 + \vec{E}_2\right)$$

$$= \left|\vec{E}_1\right|^2 + \left|\vec{E}_2\right|^2 + 2\vec{E}_1\cdot\vec{E}_2$$

$$I = v\epsilon\left\langle\left|\vec{E}_1\right|^2\right\rangle_T + v\epsilon\left\langle\left|\vec{E}_2\right|^2\right\rangle_T + 2v\epsilon\left\langle\vec{E}_1\cdot\vec{E}_2\right\rangle_T$$

Interference

$$I = v\epsilon \left\langle \left|\vec{E}_1\right|^2 \right\rangle_T + v\epsilon \left\langle \left|\vec{E}_2\right|^2 \right\rangle_T + 2v\epsilon \langle \vec{E}_1 \cdot \vec{E}_2 \rangle_T$$

$$= I_1 + I_2 + I_{12}$$

$$I_{12} = v\epsilon \vec{E}_{01} \cdot \vec{E}_{02} \cos\delta$$

Phase difference: $\delta = \vec{k}_1 \cdot \vec{x} - \vec{k}_2 \cdot \vec{x} + \xi_1 - \xi_2$

- Why didn't we care about I_{12} when discussing geometric optics?
  - Incoherent light: $\langle I_{12} \rangle = 0$
  - Random polarizations
  - Path lengths long compared with λ: $\langle I_{12} \rangle = 0$
  - Many possible paths for light to propagate along

Interference

- Another way to have $I_{12} = 0$ is when the electric fields are orthogonal:

$$I = I_1 + I_2 + I_{12}$$

$$I_{12} = v\epsilon \vec{E}_{01} \cdot \vec{E}_{02} \cos\delta$$

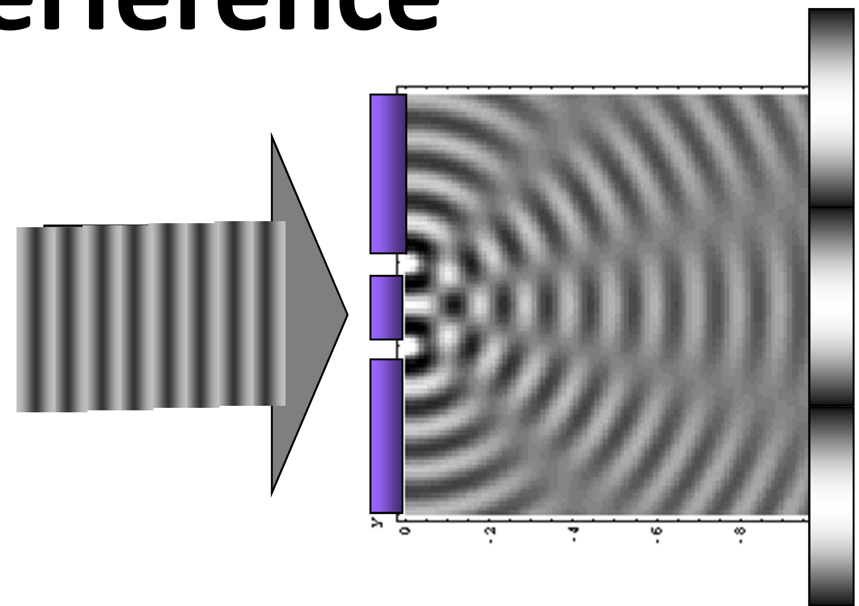
- If the two electric fields correspond to opposite polarization, then there is no interference
- If the two electric fields are parallel (same polarization), then

$$I_{12} = 2\sqrt{I_1 I_2} \cos\delta$$

- Interference depends on the phase difference

Interference

- Two point sources:



$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

- Constructive interference: $\cos\delta > 0$
- Total constructive interference: $\cos\delta = 0, \pm 2\pi, \ldots$
- Destructive interference: $\cos\delta < 0$
- Total destructive interference: $\cos\delta = \pm\pi, \pm 3\pi, \ldots$
- Special case when $\vec{E}_{01} = \vec{E}_{02}$:

$$I = 2I_0(1 + \cos\delta) = 4I_0\cos^2\frac{\delta}{2}$$

Conservation of Energy

- Energy should be conserved…
- The intensity is greater than the incoherent sum in some places, but less than the incoherent sum in other places:

$$I = I_1 + I_2 + I_{12}$$
$$I_{12} = v\epsilon \vec{E}_{01} \cdot \vec{E}_{02} \cos\delta$$

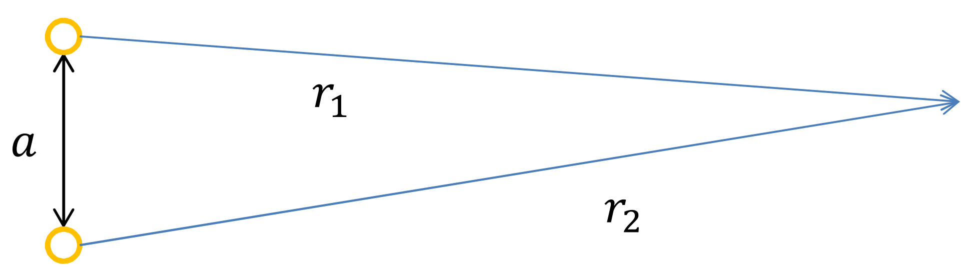
- Positive definite: I_1 and I_2
- Positive and negative: I_{12}
- Spatial average of I_{12} is zero

Interference Maxima and Minima

$$I = 2I_0(1 + \cos\delta) = 4I_0\cos^2\frac{\delta}{2}$$

$$(\text{when } \vec{E}_{01} = \vec{E}_{02})$$

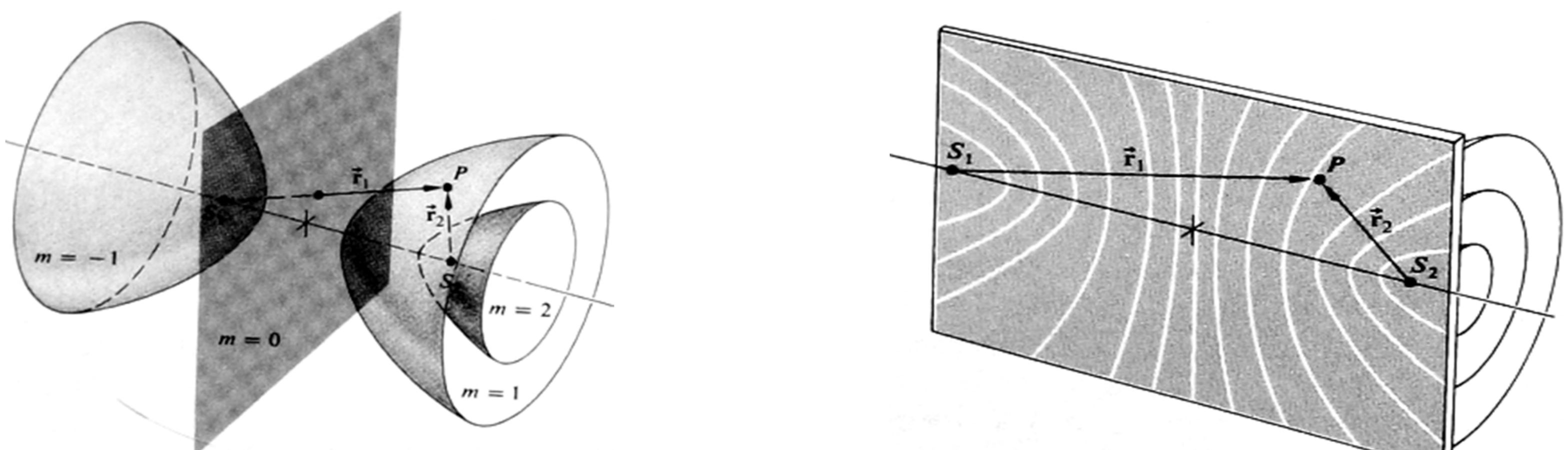
- Recall that $\delta = \vec{k}_1 \cdot \vec{x} - \vec{k}_2 \cdot \vec{x} + \xi_1 - \xi_2$
- Consider the following case:
  - the sources are at different positions
  - $|\vec{k}_1| = |\vec{k}_2| = k$
  - the sources are in phase, $\xi_1 - \xi_2 = 0$



Interference Maxima and Minima

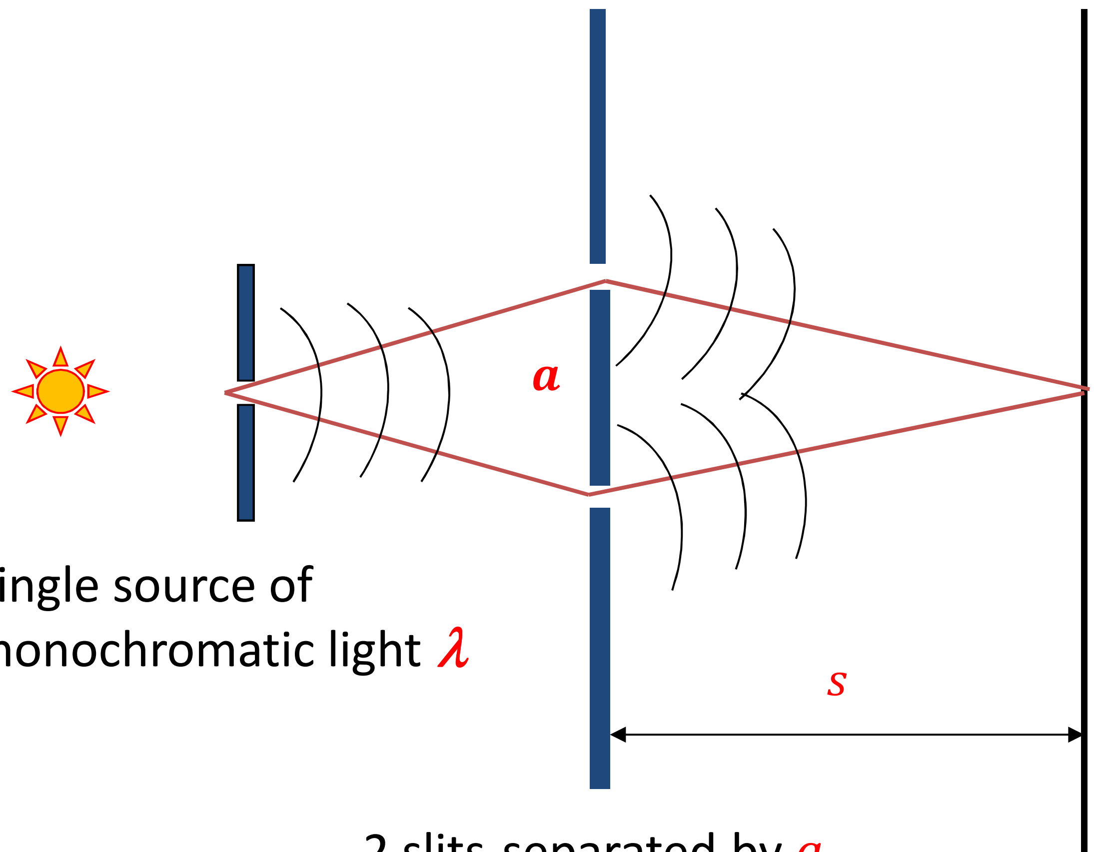
$$\delta = \vec{k}_1 \cdot \vec{x} - \vec{k}_2 \cdot \vec{x} + \xi_1 - \xi_2$$
$$= k(r_1 - r_2)$$
$$I = 4I_0 \cos^2 \frac{\delta}{2} = 4I_0 \cos^2 \frac{1}{2} k(r_1 - r_2)$$

- Maximum when $(r_1 - r_2) = \frac{2\pi m}{k} = m\lambda,\ m = 0, \pm 1, \pm 2, \ldots$
- Minimum when $(r_1 - r_2) = \frac{\pi m'}{k} = \frac{m'}{2}\lambda,\ m' = \pm 1, \pm 3, \ldots$



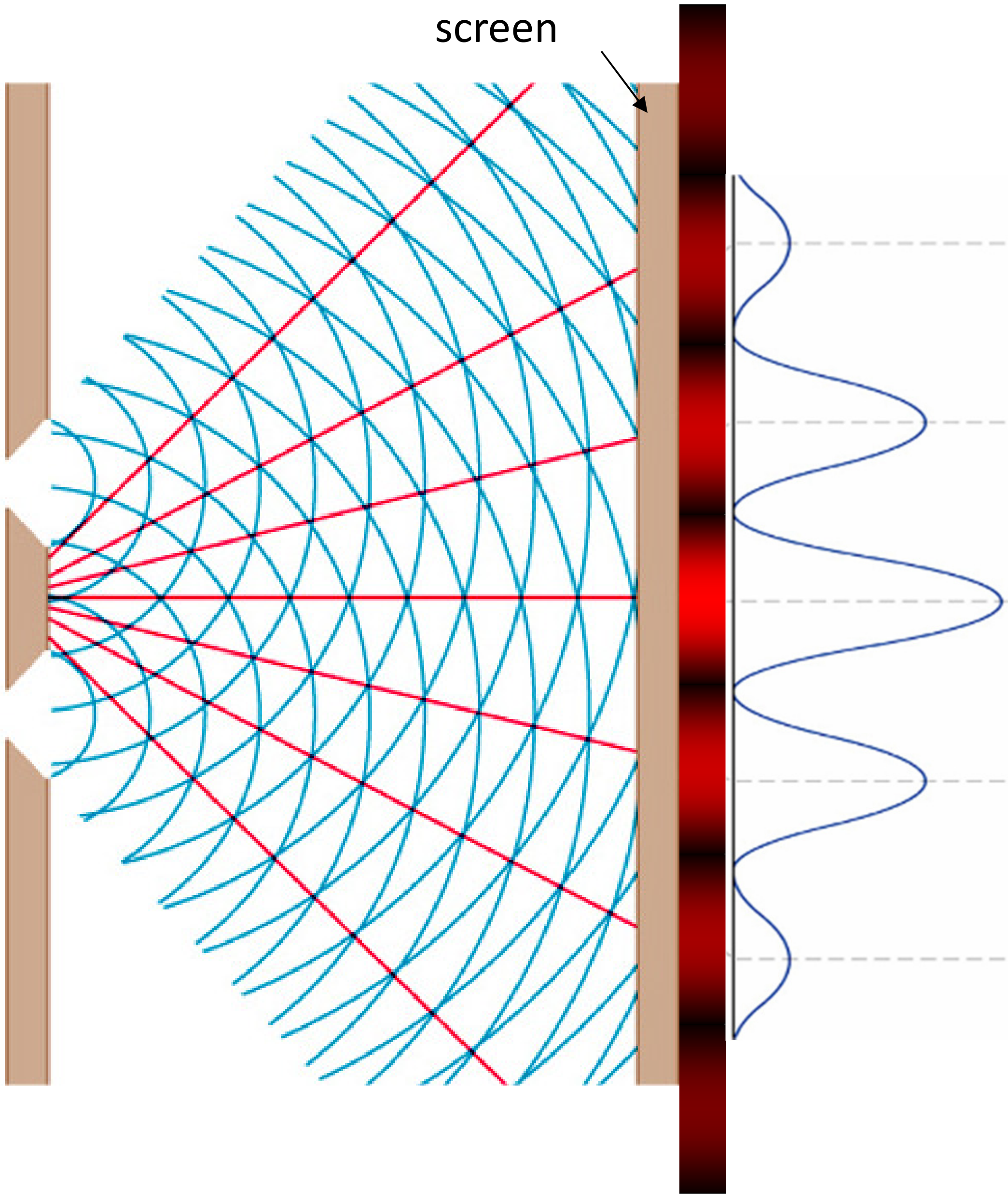
hyperboloid of revolution

Young's Double-Slit Experiment



The rays start in phase, and travel the same distance, so they will arrive in phase.

Young's Double-Slit Experiment: Screen



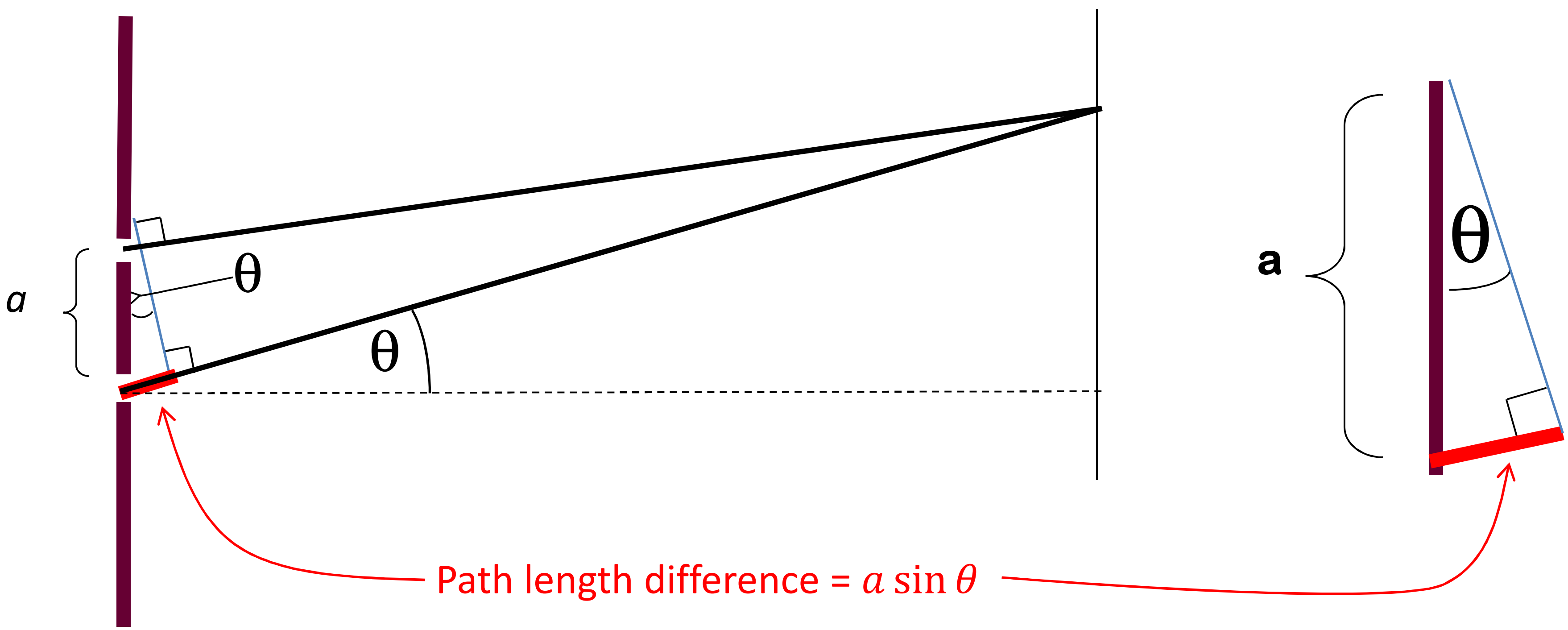
Path difference changes across the screen:

Sequence of minima and maxima

At points where the difference in path length is $0, \pm\lambda, \pm 2\lambda, \ldots,$ the screen is bright, (constructive).

At points where the difference in path length is $\pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \ldots$, the screen is dark, (destructive).

Young's Double-Slit Experiment



Constructive interference $a\sin\theta = m\lambda$

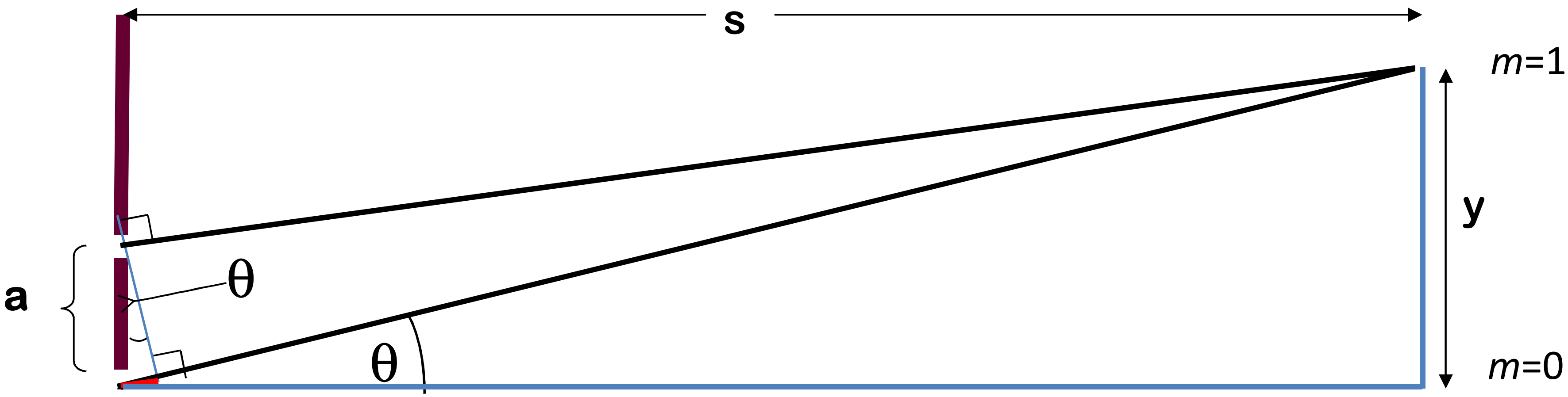
Destructive interference $a\sin\theta = \left(m + \frac{1}{2}\right)\lambda$

where m = 0, ±1, ± 2, …

Need $\lambda < a$ for distinct maxima

Example

Two slits 0.46 mm apart are 500 cm away from the screen. What would be the distance between the zero'th and first maximum for light with λ=532 nm?



$$a \sin\theta = m\lambda$$

Geometry: $\sin\theta \approx \tan\theta = y/s$

(for small θ)

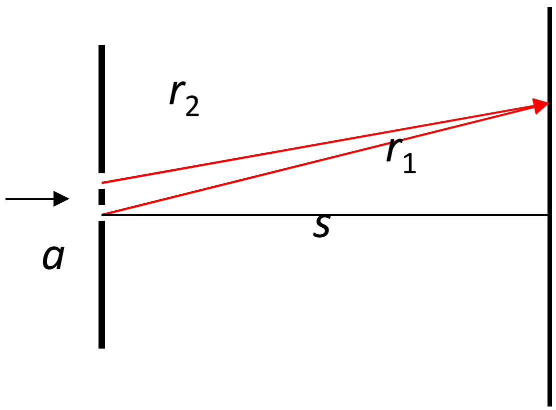
$$\downarrow$$

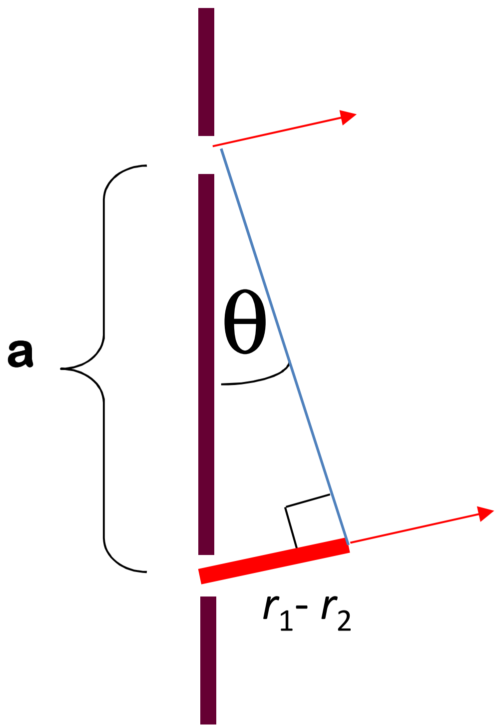
$$ay/s = m\lambda$$

$$\downarrow$$

$$y = \frac{m\lambda s}{a} \longrightarrow y = \frac{(1)(532 \times 10^{-7}\ cm)(500\ cm)}{(0.046\ cm)} = 0.58 \text{ cm}$$

Young's Double Slit Experiment





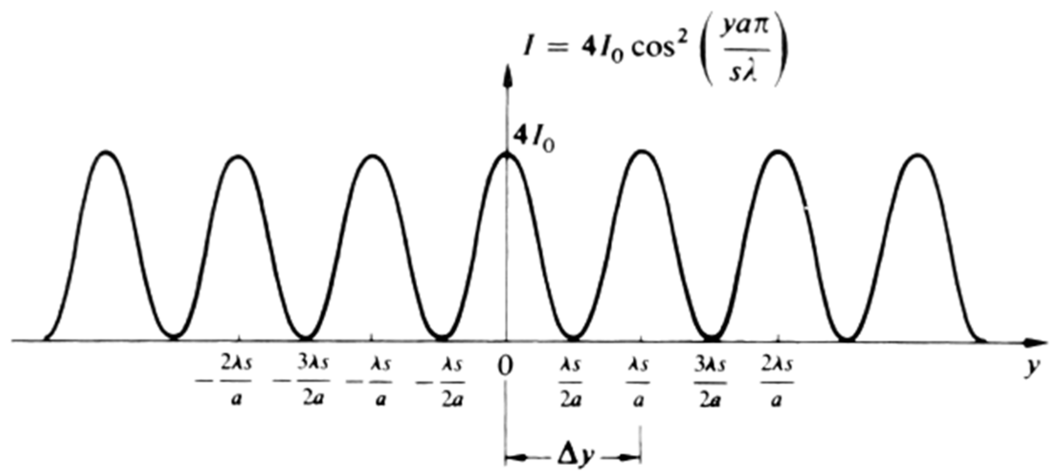
Far from the source, $s \gg a$,

$$I = 4I_0 \cos^2 \frac{\delta}{2}$$

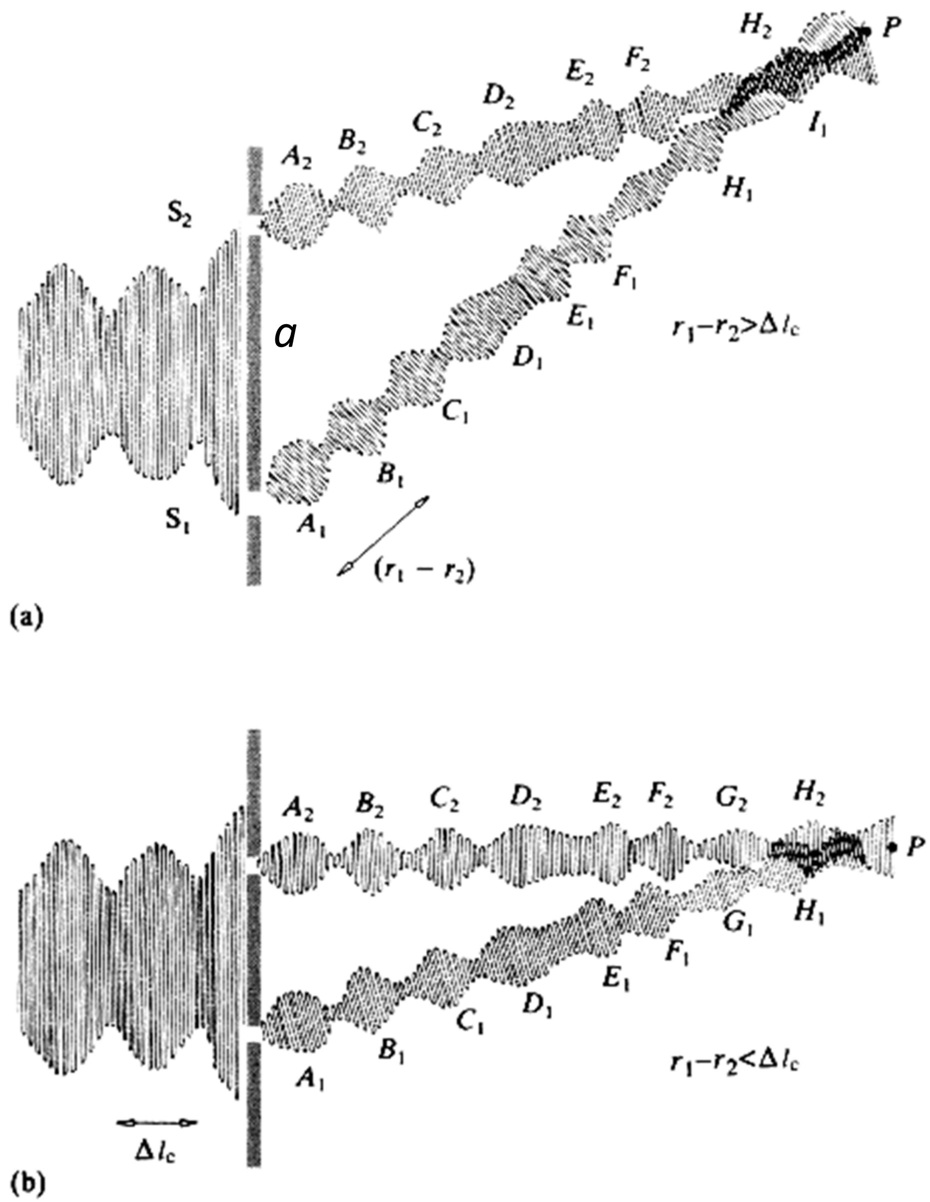
$$= 4I_0 \cos^2 \left(\frac{k(r_1 - r_2)}{2}\right)$$

$$r_1 - r_2 = a \sin\theta \approx a \tan\theta \approx \frac{ay}{s}$$

$$I \approx 4I_0 \cos^2 \frac{kay}{2s} = 4I_0 \cos^2 \frac{\pi a y}{s\lambda}$$



Coherence Length



1. Spatial coherence: wave front should be coherent over distance *a*
2. Spatial coherence:

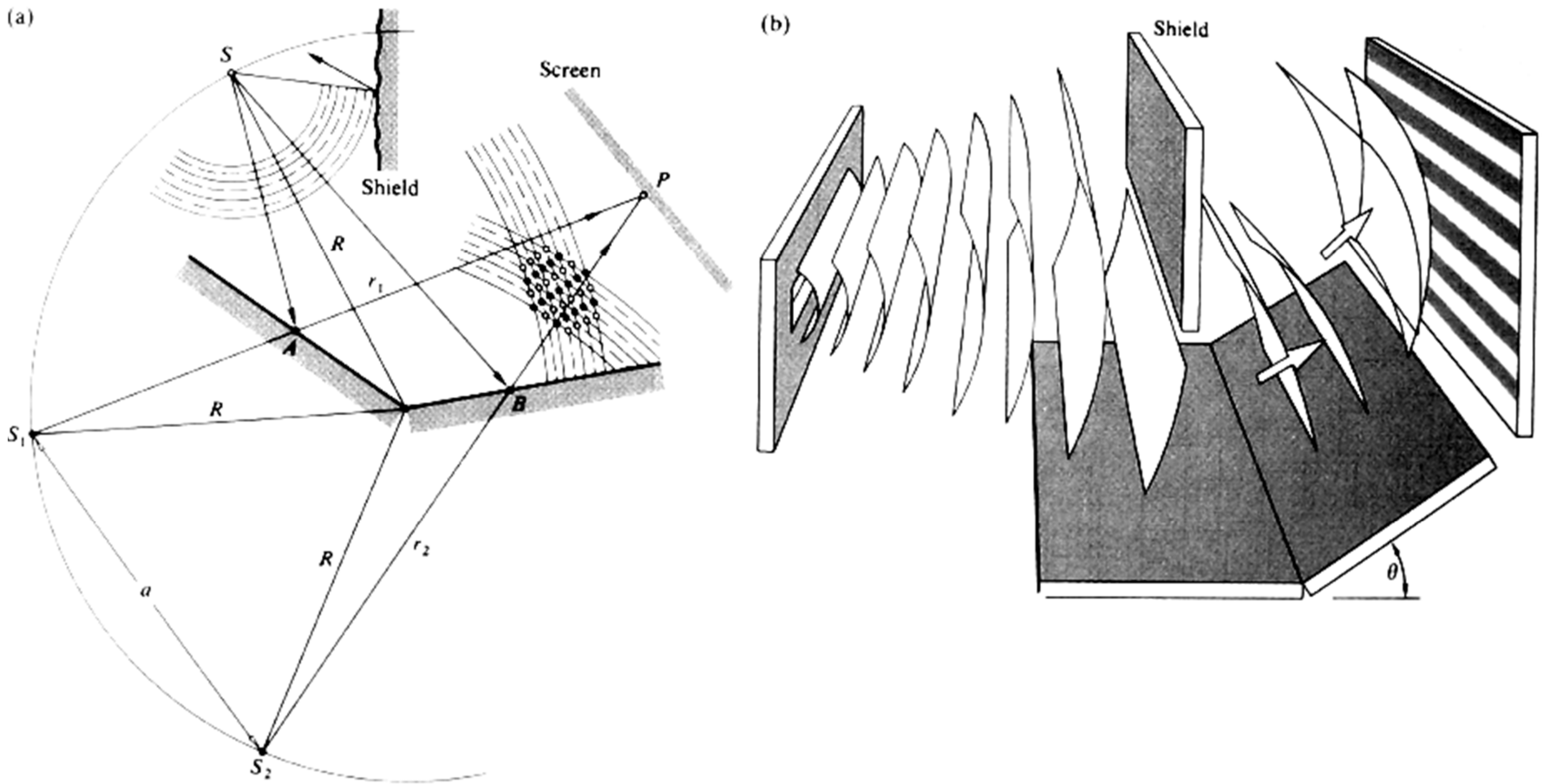
$$r_1 - r_2 < l_c$$

3. Waves should not be orthogonally polarized

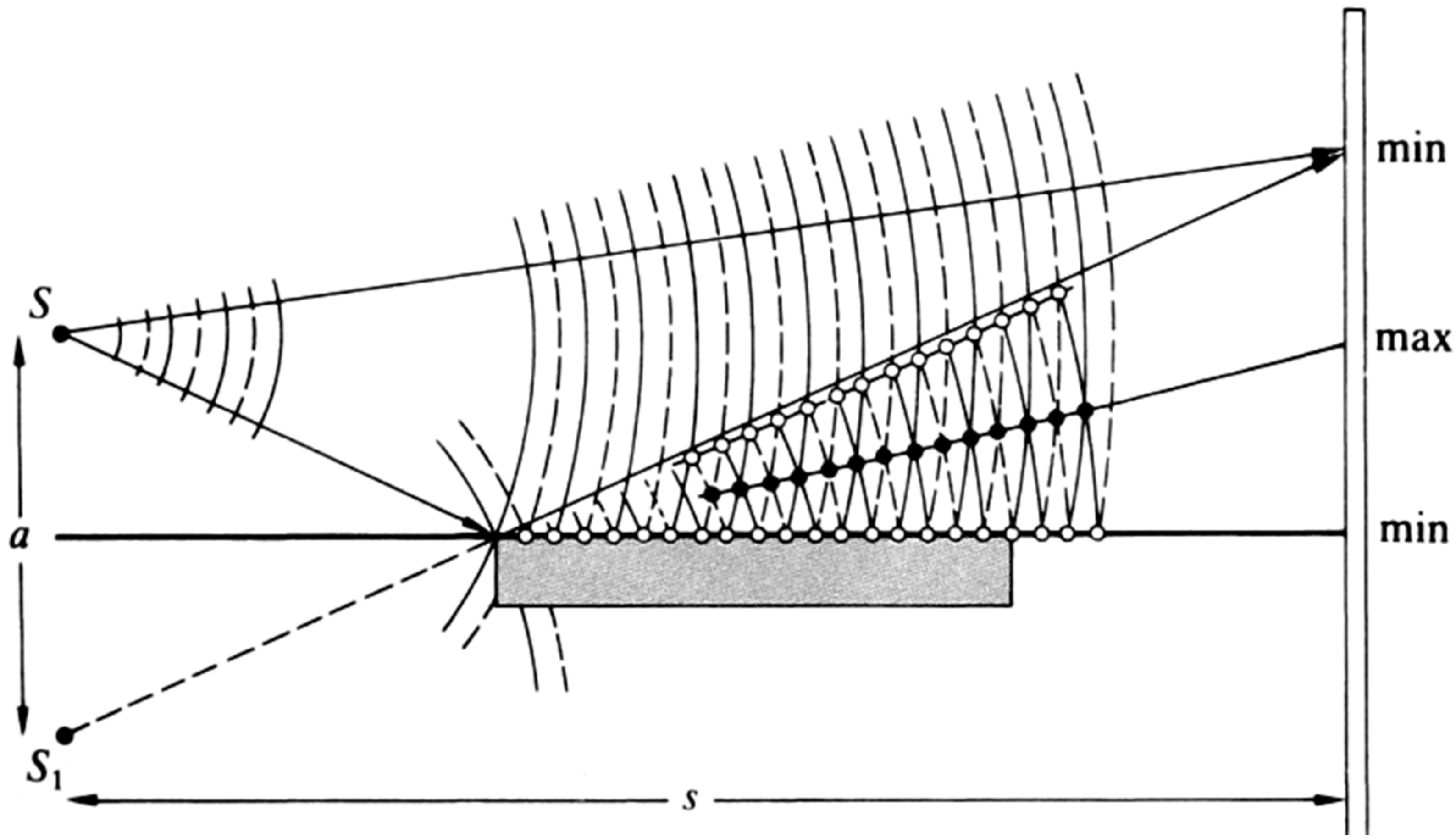
Lasers have very long coherence lengths

White light is coherent only over short distances: $l_c \sim 3\lambda$

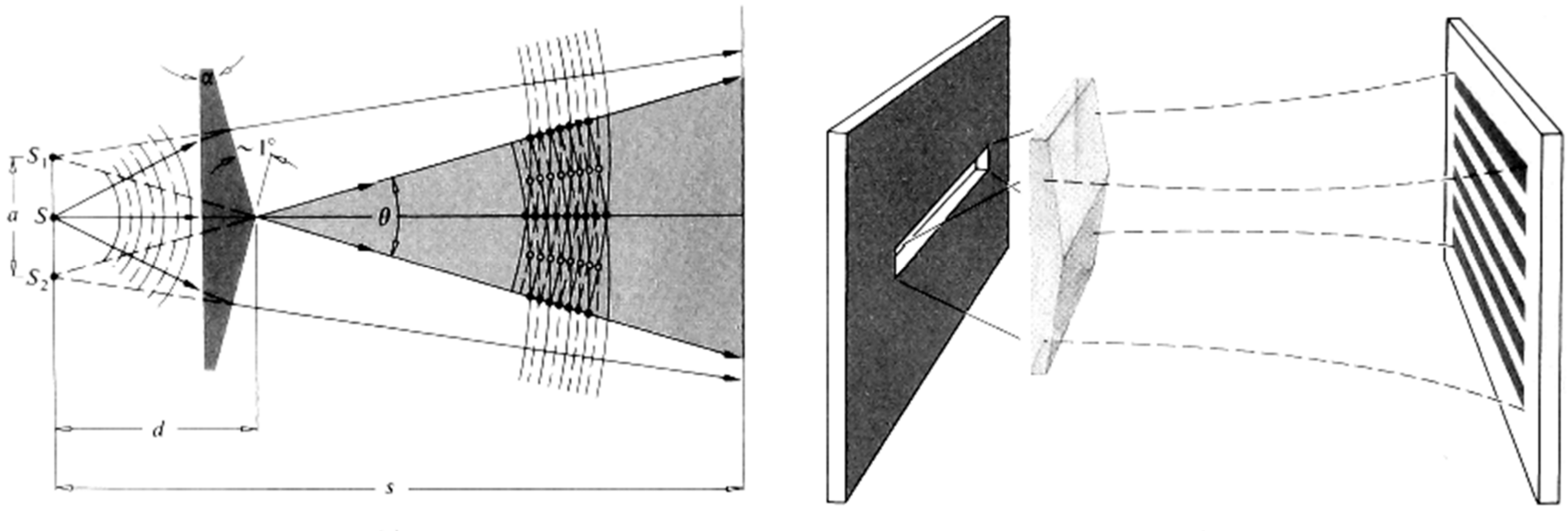
Other Interference Experiments: Fresnel's Double Mirror Interferometer



Other Interference Experiments: Lloyd's Mirror Interferometer



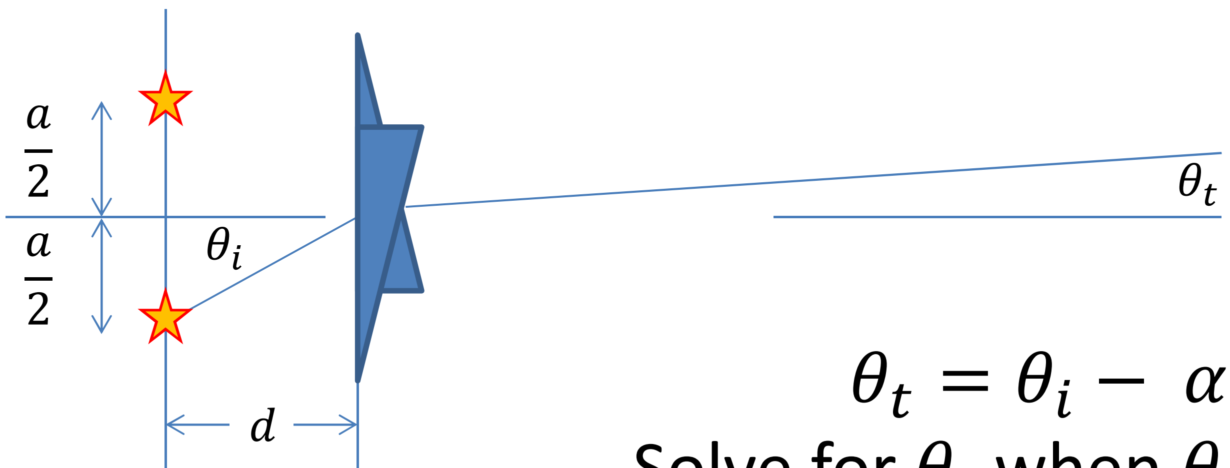
Other Interference Experiments: Fresnel's Double Prism Interferometer



- The general approach with many interference problems is to figure out how a particular system is equivalent to a double-slit experiment.

Fresnel's Double Prism Interferometer

- First, what is the spacing between the two equivalent light sources?
  - Where is the image of the light source?

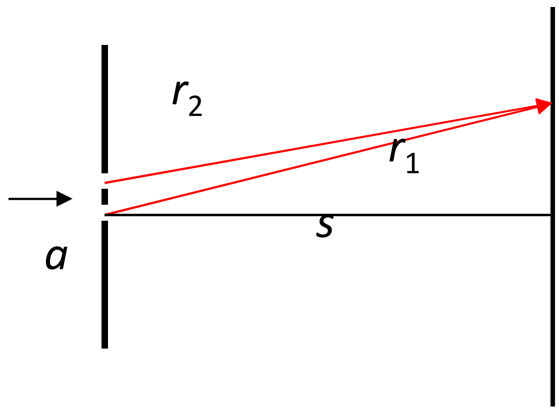


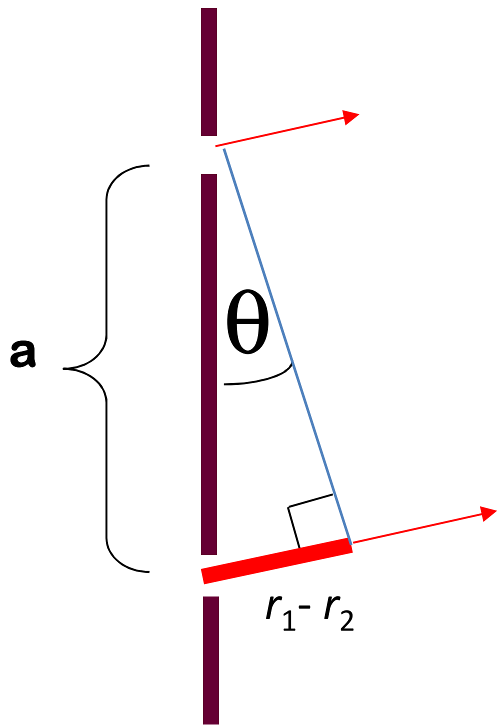
$$\theta_t = \theta_i - \alpha(n-1)$$

Solve for θ_i when $\theta_t = 0°$…

$$\frac{a}{2} = d\,\theta = d\alpha(n-1)$$

Young's Double Slit Experiment





Far from the source, $s \gg a$,

$$I = 4I_0 \cos^2 \frac{\delta}{2}$$

$$= 4I_0 \cos^2 \left(\frac{k(r_1 - r_2)}{2} \right)$$

$$r_1 - r_2 = a \sin\theta \approx a \tan\theta \approx \frac{ay}{s}$$

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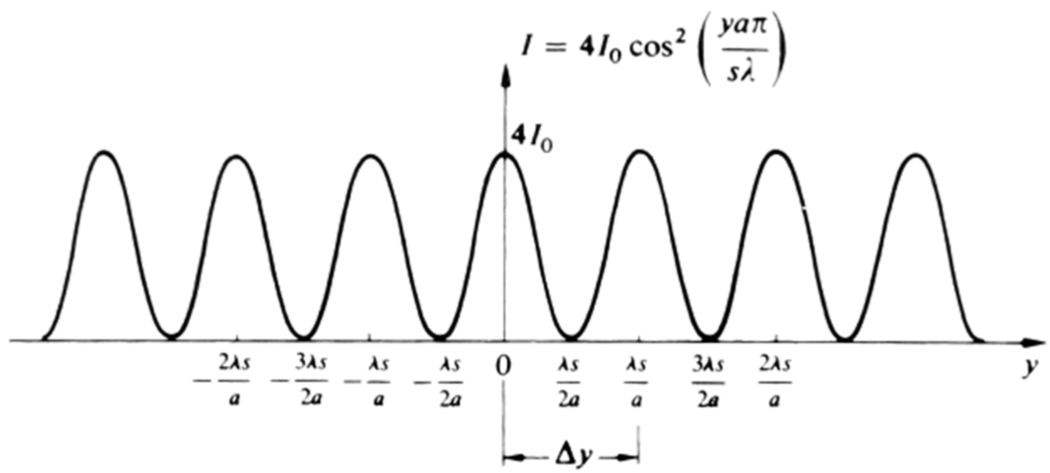
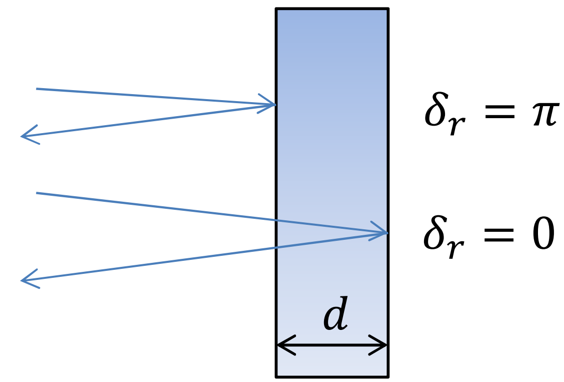


Interference From Thin Films



- Important result:

$$\left(\frac{E_r}{E_i}\right)_\perp = \frac{n_1 - n_2}{n_1 + n_2}$$

  – external reflection introduces a phase shift of π

- Wavelength in a material with index of refraction n:

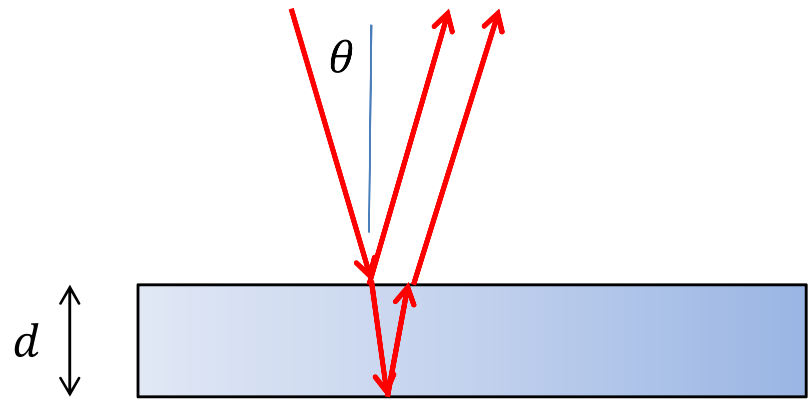
$$\lambda = \lambda_0 / n$$

- Number of wavelengths in thickness $2d$:

$$N = \frac{2dn}{\lambda_0}$$

- Phase difference: $\delta = 2\pi\left(N + \frac{1}{2}\right)$

Interference from Thin Films



- Phase difference for normal incidence:

$$\delta = 2\pi\left(\frac{2nd}{\lambda_0} + \frac{1}{2}\right)$$

- Phase difference when angle of incidence is θ:

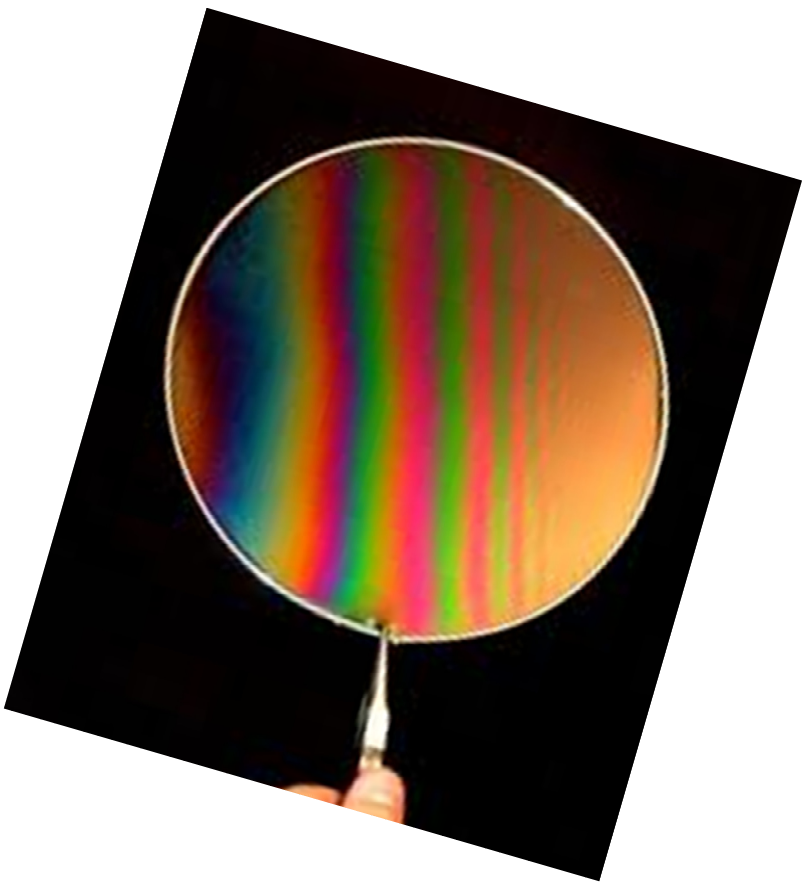
$$\delta = 2\pi\left(\frac{2nd}{\lambda_0 \cos\theta} + \frac{1}{2}\right)$$

- For monochromatic light, bright fringes have $\delta = 2\pi m$ and are located at

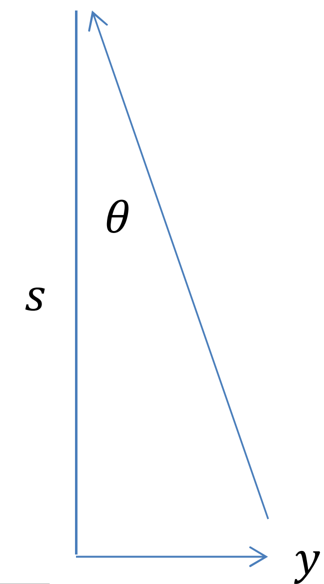
$$\cos\theta = \frac{nd}{\pi\lambda_0\left(m - \frac{1}{2}\right)}$$

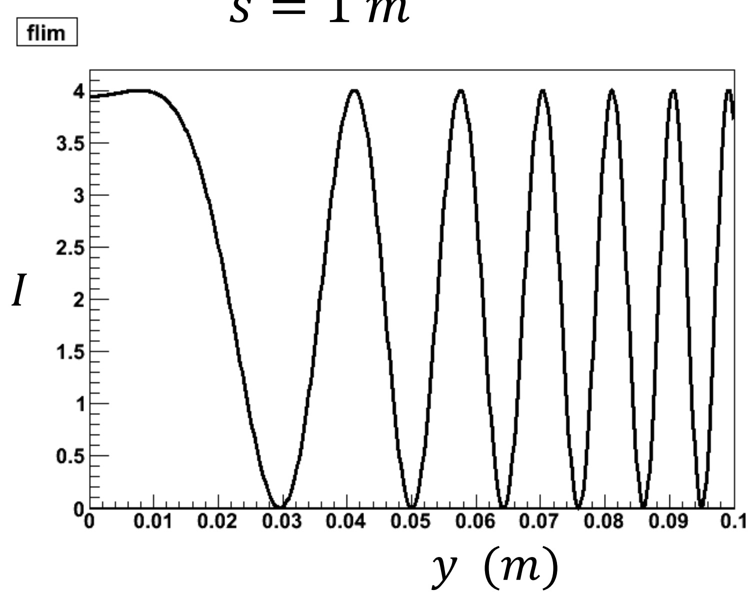
Interference from Thin Films

$$\delta = 2\pi\left(\frac{2nd}{\lambda_0 \cos\theta} + \frac{1}{2}\right)$$



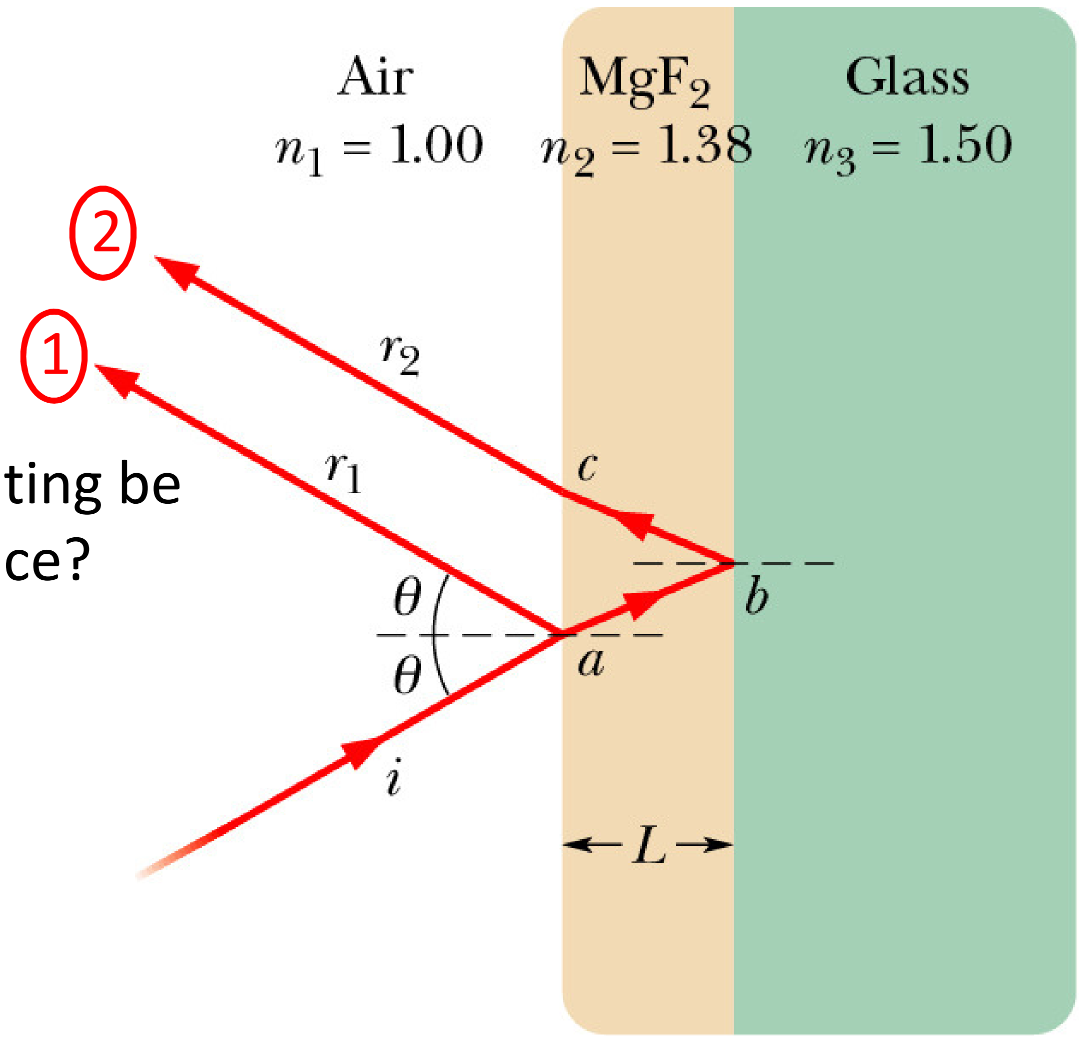
$\lambda_0 = 650\ nm$ (red light)
$d = 0.3\ mm$
$n = 1.333$
$s = 1\ m$





Coating a Glass Lens to Suppress Reflections:

180^0 phase change at both *a* and *b* since reflection is off a more optically dense medium



How thick should the coating be for destructive interference?

$$2t = \lambda'/_2$$

$$t = \lambda'/_4 = \lambda/_{4n_2}$$

What frequency to use?

Visible light: 400-700 nm

Coating a Glass Lens to Suppress Reflections:

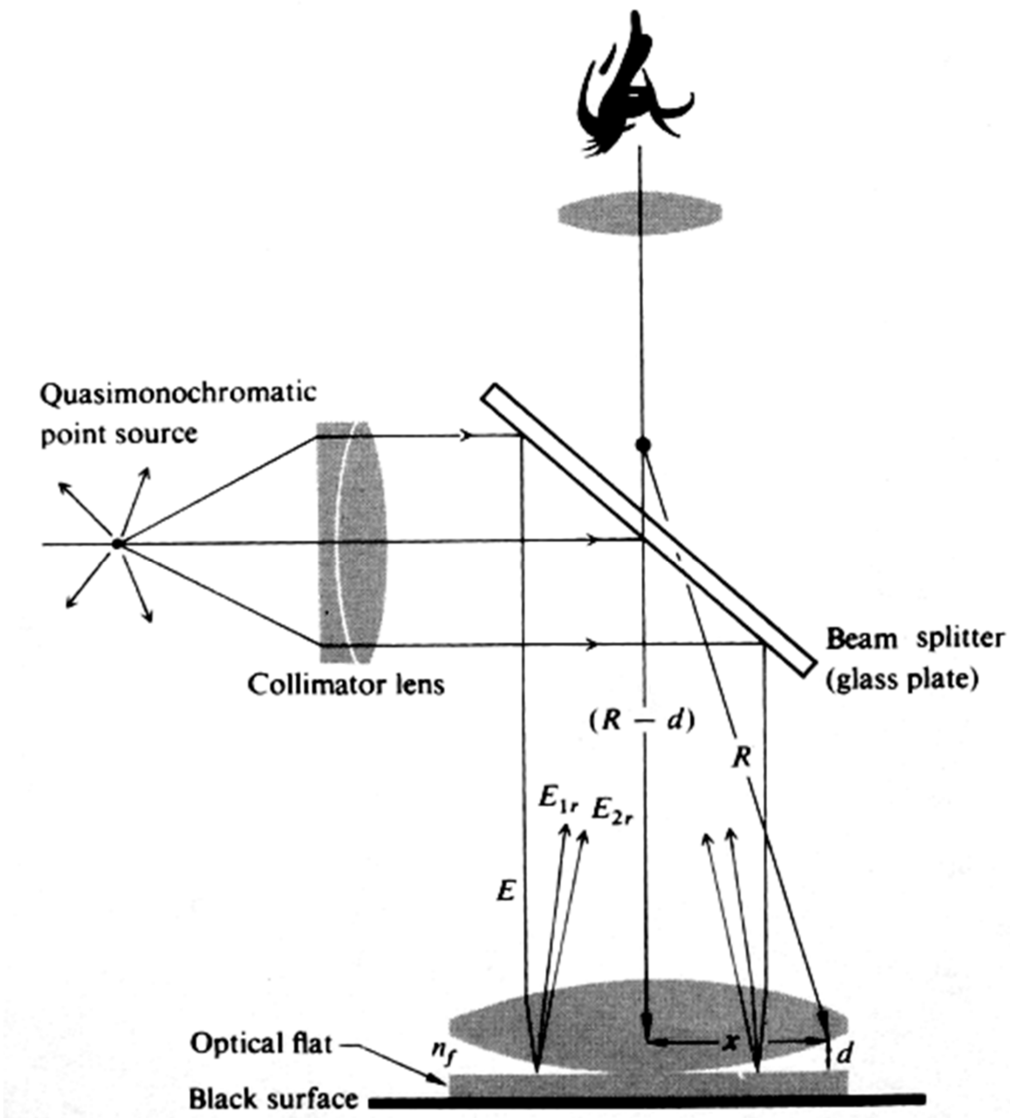
For λ= 550 nm and least thickness (m=1)

$$t = \frac{\lambda}{4n}$$

$$= \frac{550 \text{ nm}}{4 \times 1.38} = 99.6 \text{ nm}$$

- Note that the thickness needs to be different for different wavelengths.
- If the light reflected off the front and back surfaces interferes destructively, then all the energy must be transmitted

Newton's Rings



Why is center dark?

$$x^2 + (R-d)^2 = R^2$$

$$\downarrow$$

$$x^2 = 2Rd$$

maxima: $2d = (m+\frac{1}{2})\lambda$

$$x^2 = \left(m + \frac{1}{2}\right)R\lambda$$