PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 34 – Polarization of Light

Spring 2016 Semester

Matthew Jones

Polarization

$$\vec{E}_x(z,t) = E_{0x}\hat{\imath}\cos(kz - \omega t)$$
$$\vec{E}_y(z,t) = E_{0y}\hat{\jmath}\cos(kz - \omega t + \xi)$$

- Unpolarized light: *Random* E_{0x}, E_{0y}, ξ
- Linear polarization: $\xi = 0, \pm\pi$
- Circular polarization: $E_{0x} = E_{0y}$ and $\xi = \pm\frac{\pi}{2}$
- Elliptical polarization: *everything else*
- Polarization changed by
  - Absorption
  - Reflection
  - Propagation through birefringent materials

Polarization

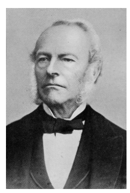
Two problems to be considered today:

1. How to measure the polarization state of an unknown beam of coherent light.
2. What is the resulting polarization after an initially polarized beam passes through a series of optical elements?

Measuring Polarization

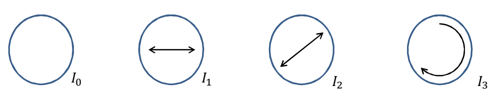
- Polarization is influenced by
  - Production mechanism
  - Propagation through birefringent material
- Measuring polarization tells us about each of these.
- Example: *polarization of light from stars*
  - First observed in 1949
  - Thermal (blackbody) radiation expected to be unpolarized
  - Interstellar medium is full of electrons and ionized gas
  - Interstellar magnetic fields polarize this material
  - Circular birefringence (L and R states propagate with different speeds)

Stokes Parameters



George Gabrial Stokes
1819-1903

- Stokes considered a set of four polarizing filters
  - The choice is not unique…
- Each filter transmits exactly half the intensity of unpolarized light



Unpolarized: filters out ½ the intensity of any incident light.

Linear: transmits only horizontal component

Linear: transmits only light polarized at 45°

Circular: transmits only R-polarized light

Stokes Parameters

- The Stokes parameters are defined as:

$$S_0 = 2I_0$$
$$S_1 = 2I_1 - 2I_0$$
$$S_2 = 2I_2 - 2I_0$$
$$S_3 = 2I_3 - 2I_0$$

- Usually normalize the incident intensity to 1.
- Unpolarized light:
  - half the light intensity is transmitted through each filter…

$S_0 = 1$ and $S_1 = S_2 = S_3 = 0$

Stokes Parameters

$$S_0 = 2I_0$$
$$S_1 = 2I_1 - 2I_0$$
$$S_2 = 2I_2 - 2I_0$$
$$S_3 = 2I_3 - 2I_0$$

- Horizontal polarization:
  - Half the light passes through the first filter
  - All the light passes through the second filter
  - Half the light passes through the third filter
  - Half the light passes through the fourth filter

$$S_0 = 1, S_1 = 1, S_2 = 0, S_3 = 0$$

Stokes Parameters

$$S_0 = 2I_0$$
$$S_1 = 2I_1 - 2I_0$$
$$S_2 = 2I_2 - 2I_0$$
$$S_3 = 2I_3 - 2I_0$$

- Vertical polarization:
  - Half the light passes through the first filter
  - No light passes through the second filter
  - Half the light passes through the third filter
  - Half the light passes through the fourth filter

$$S_0 = 1, S_1 = -1, S_2 = 0, S_3 = 0$$

Stokes Parameters

$$S_0 = 2I_0$$
$$S_1 = 2I_1 - 2I_0$$
$$S_2 = 2I_2 - 2I_0$$
$$S_3 = 2I_3 - 2I_0$$

- Polarized at 45°:

$$S_0 = 1, S_1 = 0, S_2 = 1, S_3 = 0$$

- Polarized at -45°:

$$S_0 = 1, S_1 = 0, S_2 = -1, S_3 = 0$$

- Right circular polarization:

$$S_0 = 1, S_1 = 0, S_2 = 0, S_3 = 1$$

- Left circular polarization:

$$S_0 = 1, S_1 = 0, S_2 = 0, S_3 = 1$$

Stokes Parameters

- Interpretation:

$$\vec{E}_x(z,t) = E_{0x}\hat{\imath}\cos(kz - \omega t)$$
$$\vec{E}_y(z,t) = E_{0y}\hat{\jmath}\cos(kz - \omega t + \xi)$$

- Averaged over a suitable interval:

$$S_0 = \langle E_{0x}^2 \rangle + \langle E_{0y}^2 \rangle$$
$$S_1 = \langle E_{0x}^2 \rangle - \langle E_{0y}^2 \rangle$$
$$S_2 = \langle 2E_{0x}E_{0y}\cos\xi \rangle$$
$$S_3 = \langle 2E_{0x}E_{0y}\sin\xi \rangle$$

Stokes Parameters

$$S_0 = \langle E_{0x}^2 \rangle + \langle E_{0y}^2 \rangle$$
$$S_1 = \langle E_{0x}^2 \rangle - \langle E_{0y}^2 \rangle$$
$$S_2 = \langle 2E_{0x}E_{0y}\cos\xi \rangle$$
$$S_3 = \langle 2E_{0x}E_{0y}\sin\xi \rangle$$

- Try it out:
  - Right circular polarization: $E_{0x} = E_{0y}$, $\xi = \frac{\pi}{2}$
  - Then, $S_1 = 0$, $S_2 = 0$, $S_3 = S_0$
  - When we normalize the intensity so that $S_0 = 1$,
    $S_0 = 1, S_1 = 0, S_2 = 0, S_3 = 1$

Stokes Parameters

- The "degree of polarization" is the fraction of incident light that is polarized:

$$V = \frac{I_p}{I_p + I_n}$$

  - A mixture (by intensity) of 40% polarized and 60% unpolarized light would have $V = 40\%$.
- The degree of polarization is given in terms of the Stokes parameters:

$$V = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$

Stokes Parameters

- Application: *what is the net polarization that results from a mixture of light with several polarized components?*
- Procedure:
  - Calculate Stokes parameters for each component
  - Add the Stokes parameters, weighted by the fractions (by intensity)
  - Calculate the degree of polarization
  - Interpret qualitative type of polarization

Stokes Parameters

- Example: Two components
  - 40% has vertical linear polarization
  - 60% has right circular polarization
- Calculate Stokes parameters:

$$S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = 0.4 \times \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + 0.6 \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.4 \\ 0 \\ 0.6 \end{bmatrix}$$

- Degree of polarization:

$$V = \sqrt{(0.4)^2 + (0.6^2)} = 0.72$$

The Jones Calculus

- Proposed by Richard Clark Jones (probably no relation) in 1941
- Only applicable to beams of coherent light
- Electric field vectors:

$$\vec{E}_x(z,t) = E_{0x}\hat{\imath}\cos(kz - \omega t + \varphi_x)$$
$$\vec{E}_y(z,t) = E_{0y}\hat{\jmath}\cos\left(kz - \omega t + \varphi_y\right)$$

- Jones vector:

$$\tilde{E} = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix}$$

The Jones Calculus

- It is convenient to pick $\varphi_x = 0$ and normalize the Jones vector so that $|\tilde{E}| = 1$

$$\tilde{E} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0y} e^{i\varphi_y} \end{bmatrix} \to \vec{E} = \frac{1}{\sqrt{E_{0x}^2 + E_{0y}^2}} \begin{bmatrix} E_{0x} \\ E_{0y} e^{i\xi} \end{bmatrix}$$

- Example:
  - Horizontal linear polarization: $\vec{E}_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
  - Vertical linear polarization: $\vec{E}_y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
  - Linear polarization at 45°: $\vec{E}_{45°} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

The Jones Calculus

- Circular polarization:

$$\vec{E}_R = \frac{E_0}{2} [\hat{\imath} \cos(kz - \omega t) + \hat{\jmath} \sin(kz - \omega t)]$$
$$\vec{E}_L = \frac{E_0}{2} [\hat{\imath} \cos(kz - \omega t) - \hat{\jmath} \sin(kz - \omega t)]$$

- Linear representation:

$$\vec{E}_x(z,t) = E_{0x} \hat{\imath} \cos(kz - \omega t)$$
$$\vec{E}_y(z,t) = E_{0y} \hat{\jmath} \cos(kz - \omega t + \xi)$$

  - What value of ξ gives $\cos(kz - \omega t + \xi) = \sin(kz - \omega t)$?
  - That would be $\xi = -\pi/2$

The Jones Calculus

- Right circular polarization:

$$\vec{E}_R = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ e^{-i\pi/2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

- Left circular polarization:

$$\vec{E}_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ e^{+i\pi/2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

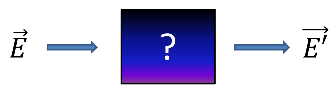
- Adding the Jones vectors adds the electric fields, not the intensities:

$$\vec{E}_R + \vec{E}_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix} = \frac{2}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Horizontal linear polarization

The Jones Calculus

- When light propagates through an optical element, its polarization can change:



- $\vec{E'}$ and $\vec{E}$ are related by a 2x2 matrix (the Jones matrix):

$$\vec{E'} = A\,\vec{E}$$

- If light passes through several optical elements, then

$$\vec{E'} = A_n \cdots A_2 A_1 \vec{E}$$

(Remember to write the matrices in reverse order)

The Jones Calculus

Examples:

- Transmission through an optically inactive material:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Rotation of the plane of linear polarization (eg, propagation through a sugar solution)

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

  – When $\alpha = \frac{\pi}{2}$, $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
  – If $\vec{E}_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ then $A\,\vec{E}_x = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \vec{E}_y$

The Jones Calculus

- Propagation through a quarter wave plate:

$$\vec{E} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \vec{E'} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

- What matrix achieves this?
  – The x-component is unchanged
  – The y-component is multiplied by $e^{-i\pi/2}$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\pi/2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$

- Note that an overall phase can be chosen for convenience and factored out
  – For example, in Hecht, Table 8.6: $A = e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
  – Important not to mix inconsistent sets of definitions!

Mueller Matrices

- We can use the same approach to describe the change in the Stokes parameters as light propagates through different optical elements

$$S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \rightarrow S' = MS$$

M is a 4x4 matrix: "the Mueller matrix"

Mueller Matrices

- Example: horizontal linear polarizer
  - Incident unpolarized light

    $S_0 = 1, S_1 = 0, S_2 = 0, S_3 = 0$
  - Emerging linear polarization

    $S_0 = \frac{1}{2}, S_1 = \frac{1}{2}, S_2 = 0, S_3 = 0$
  - Mueller matrix:

$$M = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Mueller Matrices

- What is the polarization state of light that initially had right-circular polarization but passed through a horizontal polarizer?

$$S = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$S' = MS = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Mueller Matrices

- Example: linear polarizer with transmission axis at 45°:
  - Incident unpolarized light:
    $S_0 = 1, S_1 = 0, S_2 = 0, S_3 = 0$
  - Emerging linear polarization:
    $S_0 = 1, S_1 = 0, S_2 = 1, S_3 = 0$
  - Mueller matrix:
    $$M = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
- This works in this specific case. You would need to check that it also works for other types of incident polarized light.

Jones Calculus/Mueller Matrices

- Some similarities:
  - Polarization state represented as a vector
  - Optical elements represented by matrices
- Differences:
  - Jones calculus applies only to coherent light
  - Jones calculus quantifies the phase evolution of the electric field components
  - Can be used to analyze interference
  - Stokes parameters only describe the irradiance (intensity) of light
  - Stokes parameters/Mueller matrices only apply to incoherent light – they do not take into account phase information or interference effects