Purdue University Department of Physics

Physics 42200

Waves & Oscillations

Lecture 33 – Polarization of Light

Spring 2016 Semester

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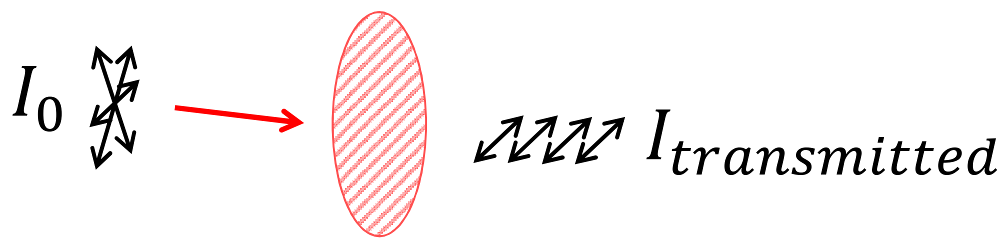
Types of Polarization

- Light propagating through different materials:
  - One polarization component can be absorbed more than the other
  - One polarization component can propagate with a different speed
- What are the properties of the light that emerges?
- Principle of superposition:

$$\vec{E} = \vec{E}_{\parallel} + \vec{E}_{\perp}$$

Linear Polarization

- Electric field component along one axis is completely absorbed



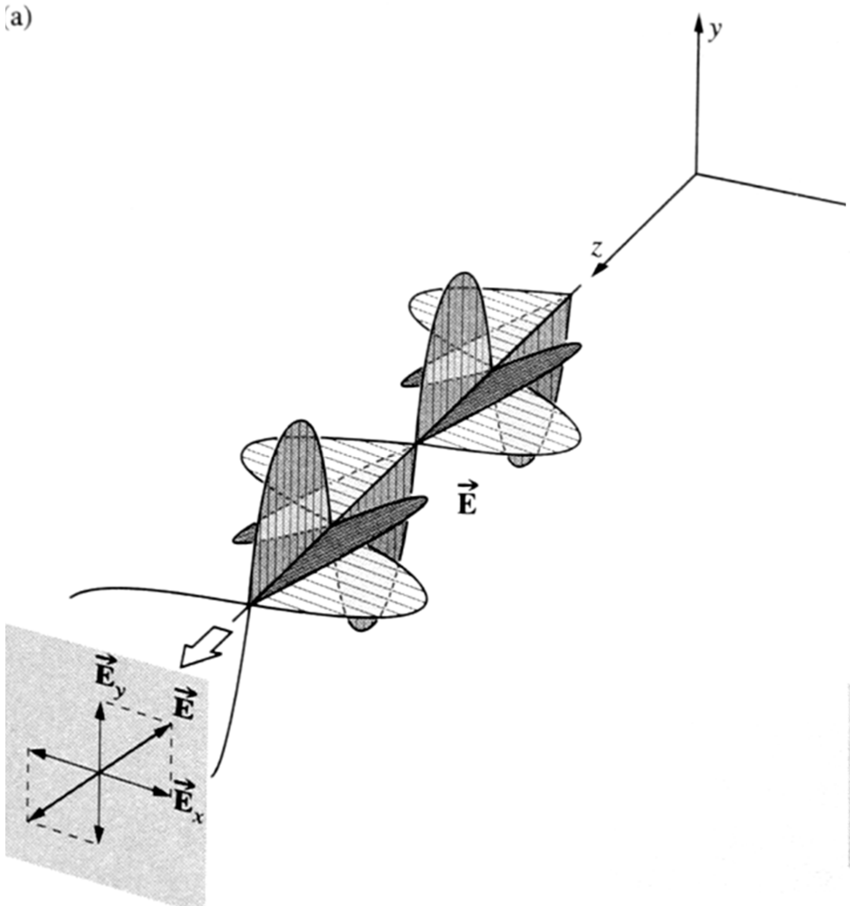
- Components of the transmitted $\vec{E}$-field are in phase, $\vec{E}$ is aligned with the polarizing axis:

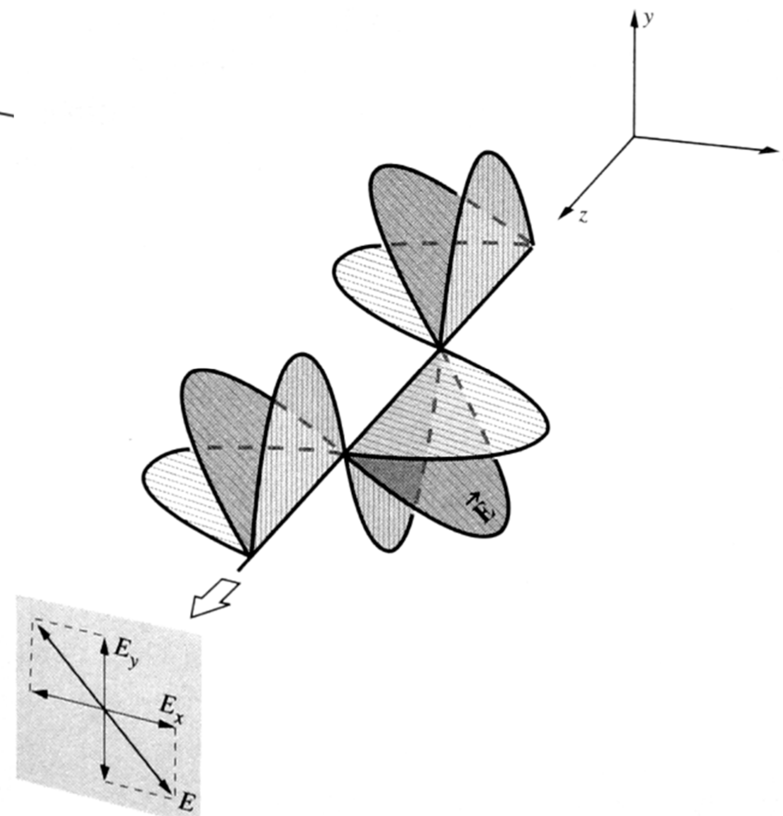
$$\vec{E}(z,t) = (E_x\hat{\imath} + E_y\hat{\jmath})\cos(kz - \omega t)$$

- Phases that differ by $\pm\pi$, $\pm 3\pi$, … are still linearly polarized:

$$\text{eg., } \vec{E}(z,t) = (E_x\hat{\imath} - E_y\hat{\jmath})\cos(kz - \omega t)$$

Linear Polarization





- Both components are still in phase
  - Nodes occur at common points on the z-axis

Circular Polarization

- What if the two components had the same amplitude but a different phase?

$$\vec{E}_x(z,t) = E_0\hat{\imath}\cos(kz - \omega t)$$
$$\vec{E}_y(z,t) = E_0\hat{\jmath}\cos(kz - \omega t + \xi)$$

- In particular, what if $\xi = \pm\frac{\pi}{2}$

$$\vec{E}_y(z,t) = E_0\hat{\jmath}\sin(kz - \omega)$$

- Resultant electric field:

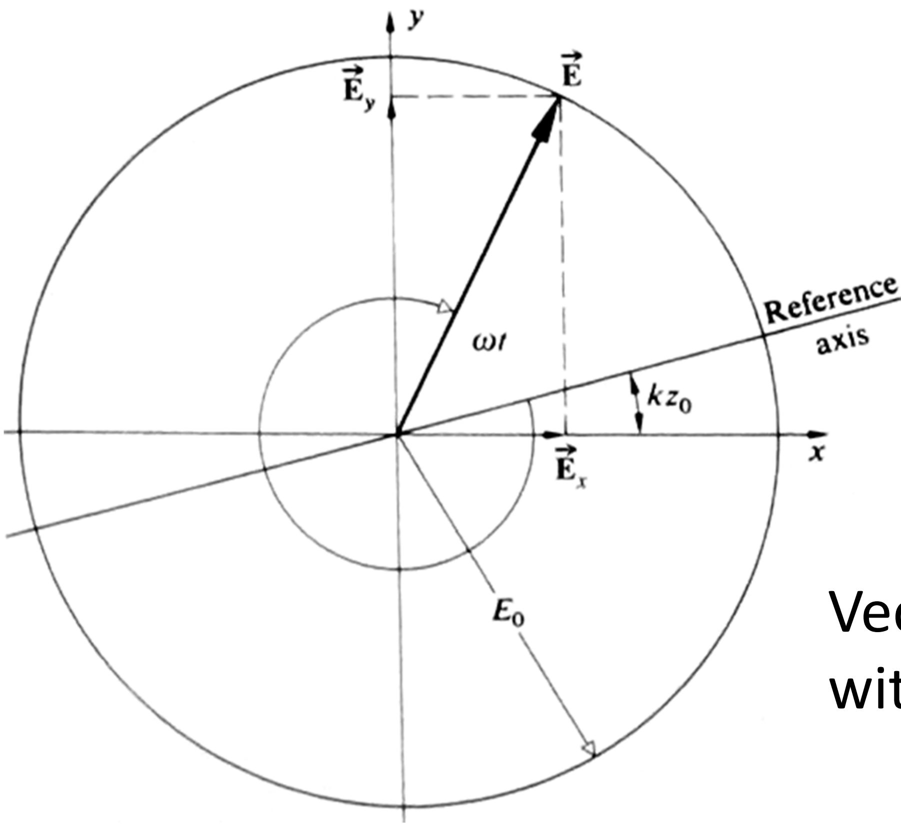
$$\vec{E}(z,t) = E_0(\hat{\imath}\cos(kz - \omega t) + \hat{\jmath}\sin(kz - \omega t))$$

Circular Polarization

$$\vec{E}(z,t) = E_0(\hat{\imath}\cos(kz - \omega t) + \hat{\jmath}\sin(kz - \omega t))$$

x and *y* components oscillate: $E_x = E_0 \cos(kz - \omega t)$

$E_y = E_0 \sin(kz - \omega t)$



Angle $\alpha = kz_0 - \omega t$

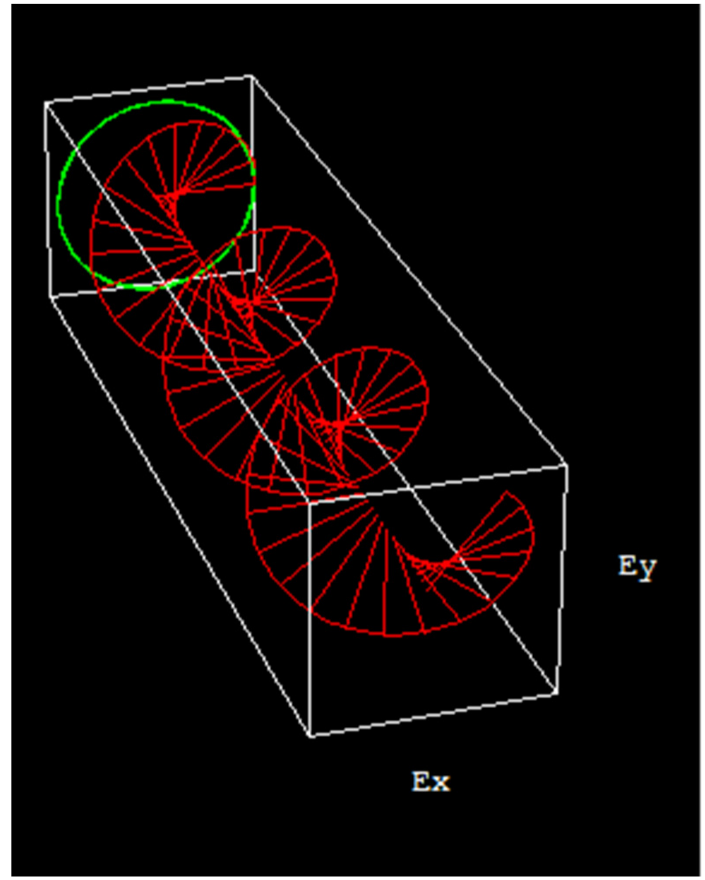
Vector *E* rotates in time
with angular frequency $-\omega$

Vector *E* rotates in space
with angular spatial speed *k*

Circular Polarization

$$\vec{E}(z,t) = E_0\left[\hat{\mathbf{i}}\cos(kz-\omega t) + \hat{\mathbf{j}}\sin(kz-\omega t)\right]$$

Right circularly polarized light:
E rotates clockwise as seen by observer



Vector makes full turn as wave advances one wavelength

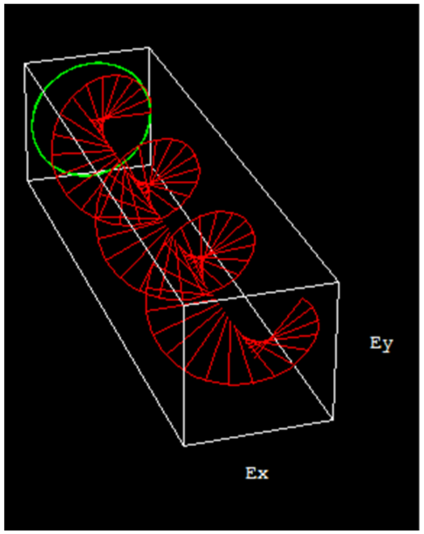
Left circularly polarized light:
E rotates counter clockwise

$$\vec{E}(z,t) = E_0\left[\hat{\mathbf{i}}\cos(kz-\omega t) - \hat{\mathbf{j}}\sin(kz-\omega t)\right]$$

What if we have a superposition of left and right circularly polarized light of equal amplitude?

$$\vec{E}(z,t) = 2E_0\hat{\mathbf{i}}\cos(kz-\omega t)$$ - linearly polarized light

Circular Polarization and Angular Momentum



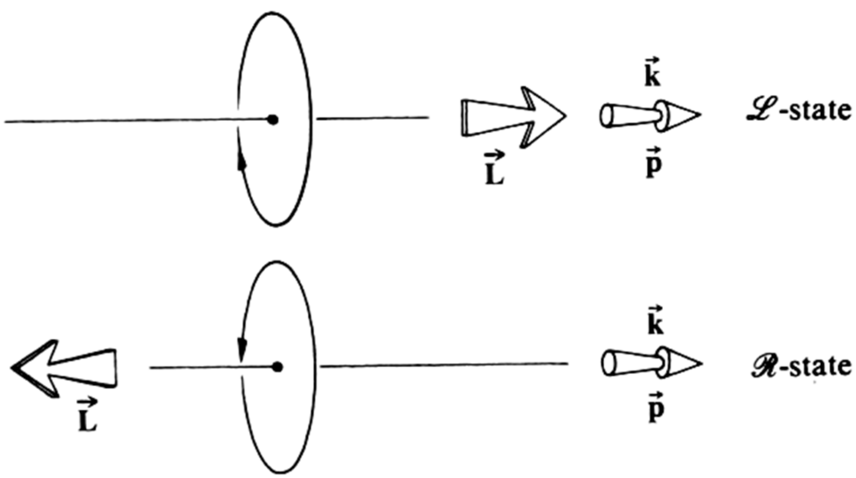
What would happen with an electron under circularly polarized light?
The rotating electric field would push it in a circle…

Angular velocity ω - angular momentum L

Light is absorbed, and if it was circularly polarized:

$$L = \frac{E}{\omega}$$

angular momentum → L; energy of light → E; frequency of light → ω



Photons and Angular Momentum

$$L = \frac{E}{\omega}$$

Photon has energy: $E = h\nu = \frac{h}{2\pi}\omega = \hbar\omega$

Angular momentum of a photon is independent of its energy: $L = \pm\hbar$

Photon has a *spin*, $+\hbar$ - *L*-state
$-\hbar$ - *R*-state

Whenever a photon is absorbed or emitted by a charged particle, along with the change in its energy it will undergo a change in its angular momentum

First measured in 1935 by Richard Beth

Linearly polarized light: photons exist in either spin state with equal probability

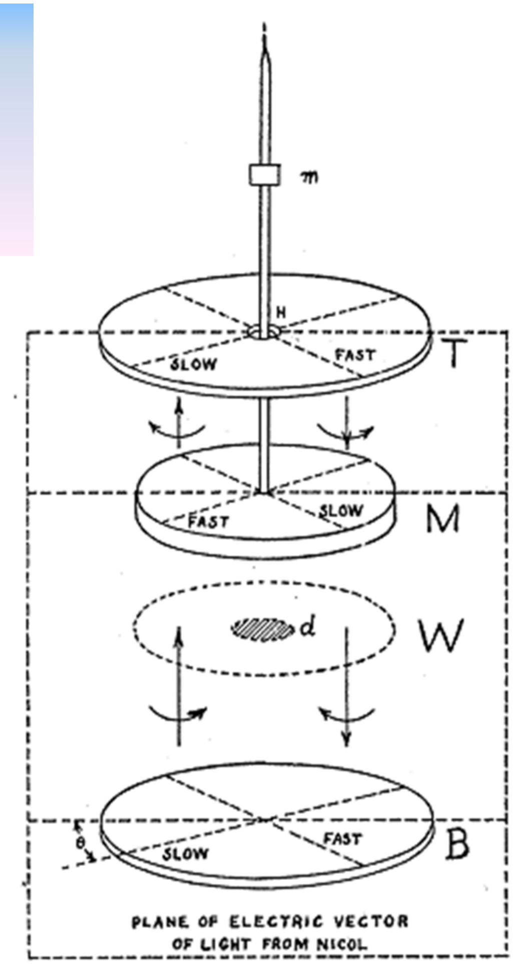
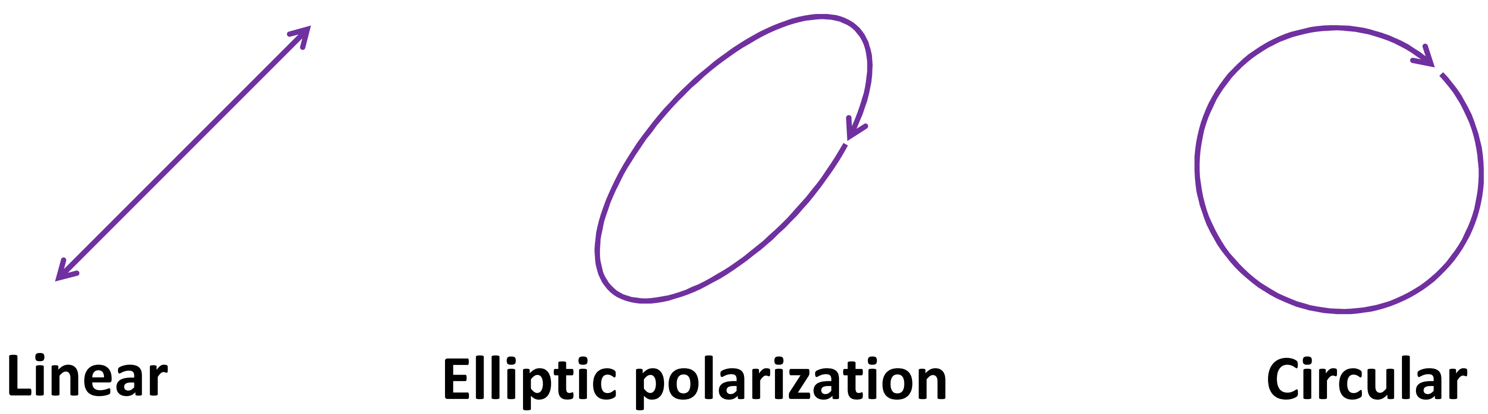


FIG. 3. Wave plate arrangement.

Elliptic Polarization

$$\vec{E}_x(z,t) = E_0\hat{\imath}\cos(kz - \omega t)$$
$$\vec{E}_y(z,t) = E_0\hat{\jmath}\cos(kz - \omega t + \xi)$$

- $\xi = 0, \pm\pi$ corresponds to linear polarization
- $\xi = \pm\frac{\pi}{2}$ corresponds to circular polarization
- What about other values of ξ?

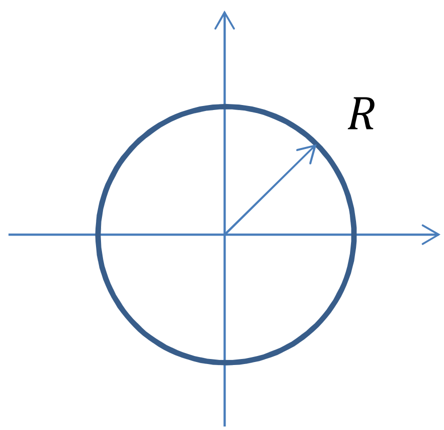


Geometry Lesson

- Equation for a circle:

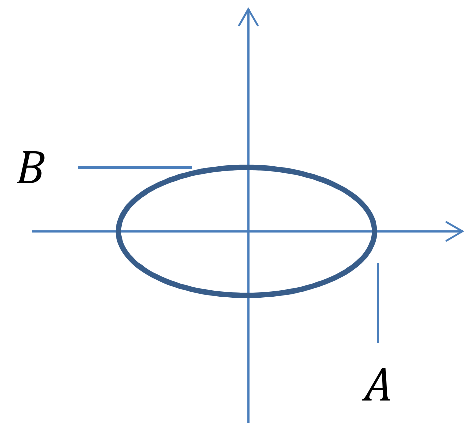
$$x^2 + y^2 = R^2$$

$$\left(\frac{x}{R}\right)^2 + \left(\frac{y}{R}\right)^2 = 1$$



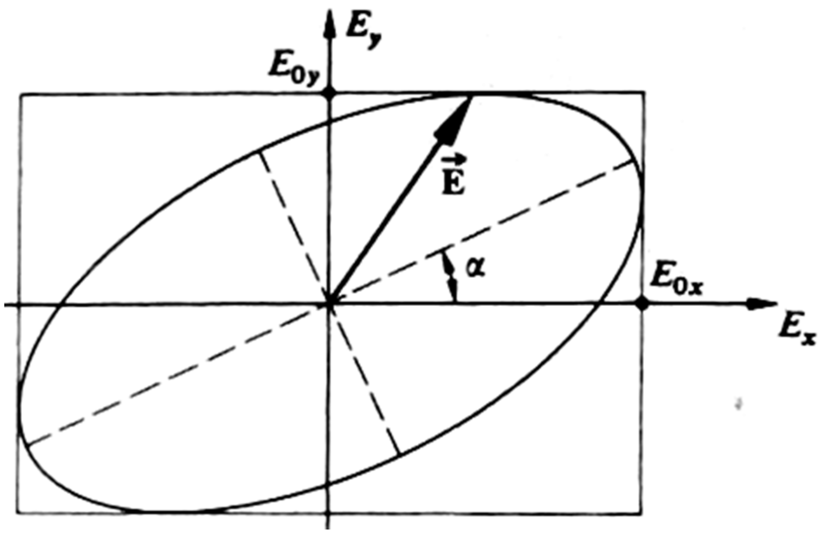
- Equation for an ellipse:

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 = 1$$



Geometry Lesson

- Equation for an ellipse oriented at an angle α with respect to the x-axis:



$$\tan 2\alpha = \frac{2AB\cos\xi}{A^2 - B^2}$$

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 - 2\left(\frac{x}{A}\right)\left(\frac{y}{B}\right)\cos\xi = \sin^2\xi$$

Most General Case

$$\vec{E}_x(z,t) = E_{0x}\hat{\imath}\cos(kz-\omega t)$$
$$\vec{E}_y(z,t) = E_{0y}\hat{\jmath}\cos(kz-\omega t+\xi)$$

- Magnitudes of x- and y-components are different
- Phase difference between x- and y-components
- Algebra:

$$E_x = E_{0x}\cos(kz-\omega t)$$
$$E_y = E_{0y}\cos(kz-\omega t+\xi)$$

$$\frac{E_y}{E_{0y}} = \cos(kz-\omega t)\cos\xi - \sin(kz-\omega t)\sin\xi$$
$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\xi = -\sin(kz-\omega t)\sin\xi$$

$$\left(\frac{E_x}{E_{0x}}\right)^2 = \cos^2(kz-\omega t) = 1-\sin^2(kz-\omega t)$$
$$\sin^2(kz-\omega t) = 1-\left(\frac{E_x}{E_{0x}}\right)^2$$

Most General Case

$$\left(\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}}\cos\xi\right)^2 = \sin^2(kz - \omega t)\sin^2\xi$$

$$= \left[1 - \left(\frac{E_x}{E_{0x}}\right)^2\right]\sin^2\xi$$

$$\left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_y}{E_{0y}}\right)\left(\frac{E_x}{E_{0x}}\right)\cos\xi + \left(\frac{E_x}{E_{0x}}\right)^2(\sin^2\xi + \cos^2\xi) = \sin^2\xi$$

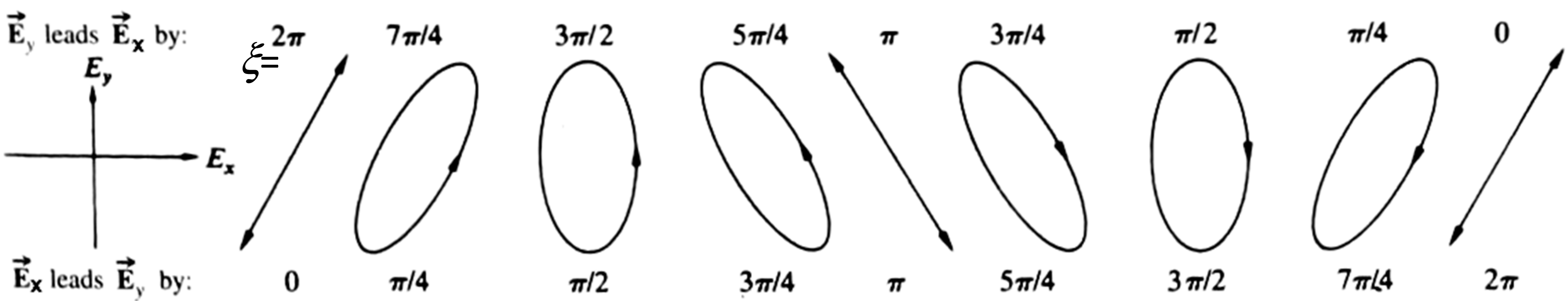
$$\boldsymbol{\left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_y}{E_{0y}}\right)\left(\frac{E_x}{E_{0x}}\right)\cos\xi + \left(\frac{E_x}{E_{0x}}\right)^2 = \sin^2\xi}$$

This is the equation for an ellipse.

Elliptic Polarization

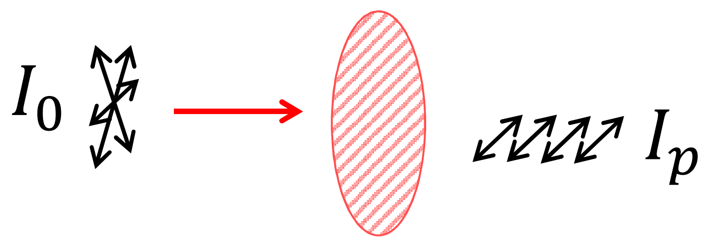
$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_y}{E_{0y}}\right)\left(\frac{E_x}{E_{0x}}\right)^2 \cos\xi = \sin^2\xi$$

When $E_y \approx 2E_x$:



Changing Polarization

- How can we turn unpolarized light into linearly polarized light?



  - Pass it through a polarizer
- How do we get equal magnitudes of the x- and y-components?
  - Rotate the polarizer so that it is $45°$ with respect to the x-axis.

Changing Polarization

- How do we turn linearly polarized light into light with circular polarization?

$$\vec{E}(z,t) = E_0(\hat{\imath}\cos(kz - \omega t) + \hat{\jmath}\sin(kz - \omega t))$$

  – Shift the phase of one component by $\pm\pi/2$
  – But how?
- Remember the birefringent crystal?
  – Light polarized along the optic axis travels at a different speed compared with light polarized perpendicular to the optic axis
  – Make the thickness such that light emerges with the desired phase shift.
  – This device is called a "quarter-wave plate" because it shifts one component by one quarter of a wavelength.

Quarter Wave Plates

- Consider a birefringent crystal with ordinary and extraordinary indices of refraction n_o and n_e
- How thick should the crystal be to retard one component by $\Delta\lambda/\lambda = 2m\pi + \pi/2 = (4m+1)\pi/2$?
- Ordinary component: $E_o(z,t) = E\cos(k_o z - \omega t)$
- Extraordinary component: $E_e(z,t) = E\cos(k_e z - \omega t)$

$$k_o = \frac{\omega}{v_o} = \frac{\omega n_o}{c} = 2\pi\frac{n_o}{\lambda_0}$$

$$k_e = \frac{\omega}{v_e} = \frac{\omega n_e}{c} = 2\pi\frac{n_e}{\lambda_0}$$

- After propagating a distance, d, we want

$$(k_o - k_e)d = (4m+1)\pi/2$$

$$(n_o - n_e)d = \lambda_0(4m+1)/4$$

$$\boldsymbol{d = \frac{\lambda_0}{n_o - n_e}(4m+1)/4}$$

Quarter Wave Plates

- Consider calcite: (n_e - n_o) = -0.172
- The thickness at normal incidence depends on the wavelength of light.
  - Suppose $\lambda_0 = 560\ nm$ (green) and pick $m = 1$

$$\boldsymbol{d = \frac{\lambda_0}{n_o - n_e}(4m + 1)/4}$$

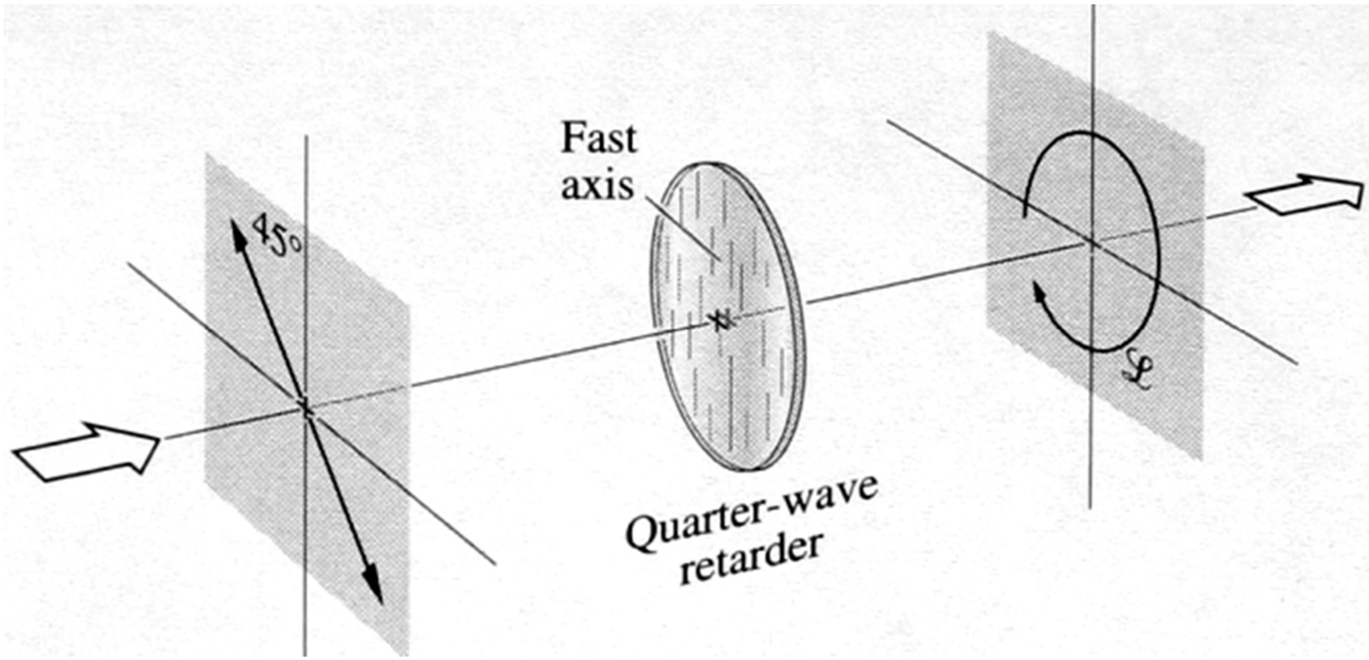
$$= \left(\frac{560\ nm}{0.172}\right)\frac{1}{4} = 814\ nm$$

- This is very thin… if the thickness were to be $500\ \mu m$ then $m \approx 150$

$$d = \left(\frac{560\ nm}{0.172}\right)\frac{1}{2}(4 \times 150 + 1) = 489.186\ \mu m$$

  - Less fragile but the path length would be significantly different if light was not at precisely normal incidence.

Quarter Wave Plate

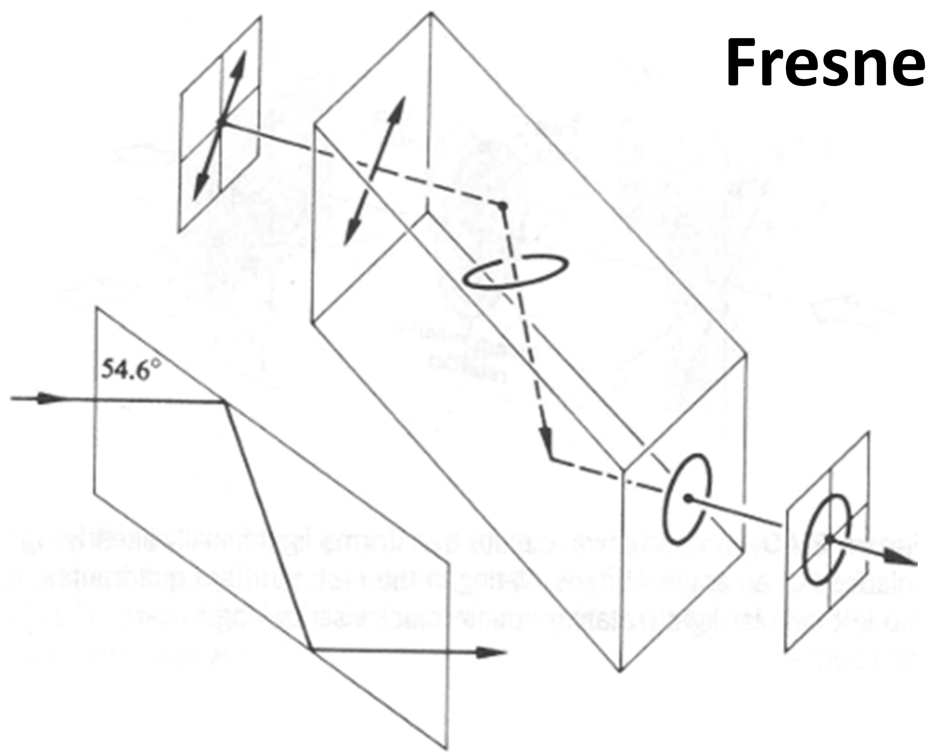


- Additional quarter-wave plates introduce additional phase shifts:
  - Two quarter-wave plates produces linear polarization rotated by $90°$ with respect to the polarization axis of the incident light.

Reflective Retarders

Total internal reflection: phase shift between the two components.
Glass - n=1.51, and 45^o shift occurs at incidence angle 54.6^o.

Fresnel rhomb

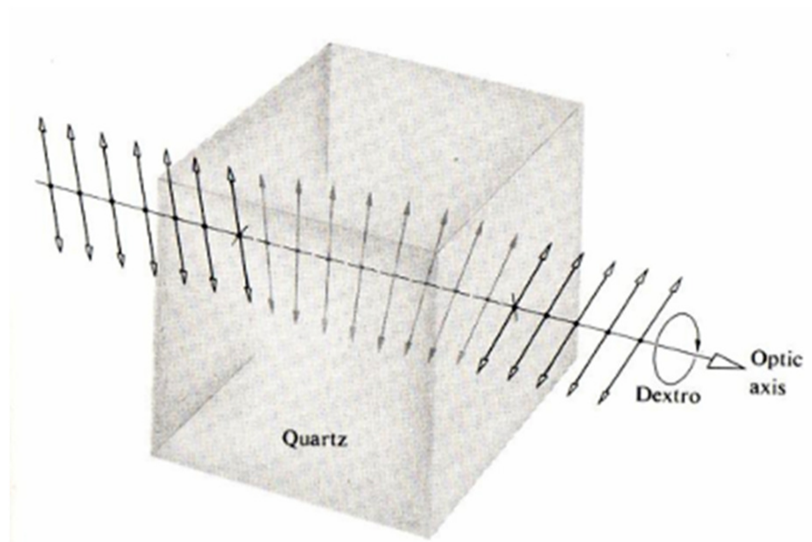


Unlike devices that use birefringent materials this is achromatic.
(works for all wavelengths)

Optical Activity

Discovered in 1811, Dominique F. J. Arago

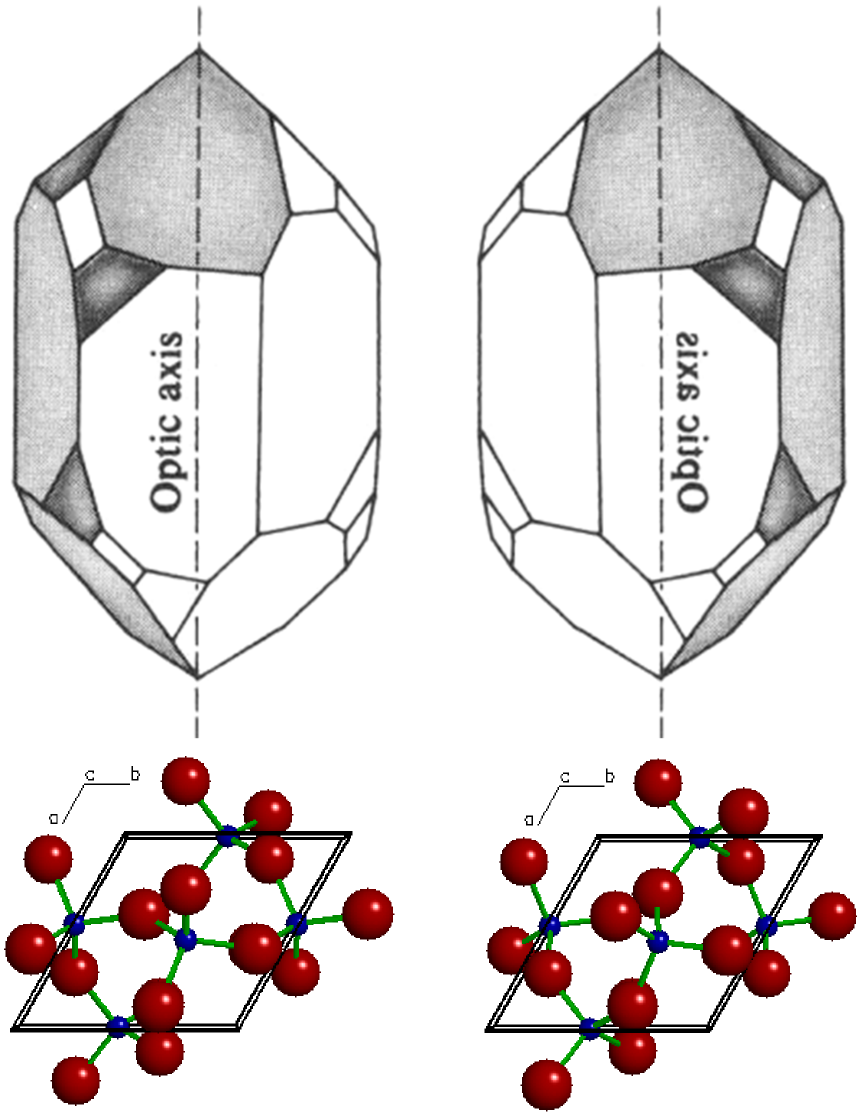
Optically active material: any substance that cause a plane of polarization to appear to rotate



Dextrorotatory (d-rotatory) - polarization rotates right (CW)
Levorotatory (l-rotatory) - polarization rotates left (CCW)

Latin: *dextro* - right
levo - left

Optical Activity



Many materials have molecules or crystal structures that are non mirror-symmetric.

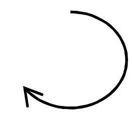
These usually interact with left- and right-circular polarized light in different ways.

Circular birefringence: different indices of refraction for L- or R-polarized light.

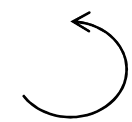
Optical Activity

- Fresnel described linearly polarized light as the superposition of L- and R-polarization:

$$\vec{E}_R = \frac{E_0}{2}[\hat{\imath}\cos(k_R z - \omega t) + \hat{\jmath}\sin(k_R z - \omega t)]$$



$$\vec{E}_L = \frac{E_0}{2}[\hat{\imath}\cos(k_L z - \omega t) - \hat{\jmath}\sin(k_L z - \omega t)]$$



- Add them together:

$$\vec{E} = E_0 \cos\left(\frac{k_R + k_L}{2} z - \omega t\right)\left[\hat{\imath}\cos\left(\frac{k_R - k_L}{2} z\right) + \hat{\jmath}\sin\left(\frac{k_R - k_L}{2} z\right)\right]$$

- Linear polarization: direction at a fixed point in z is constant.

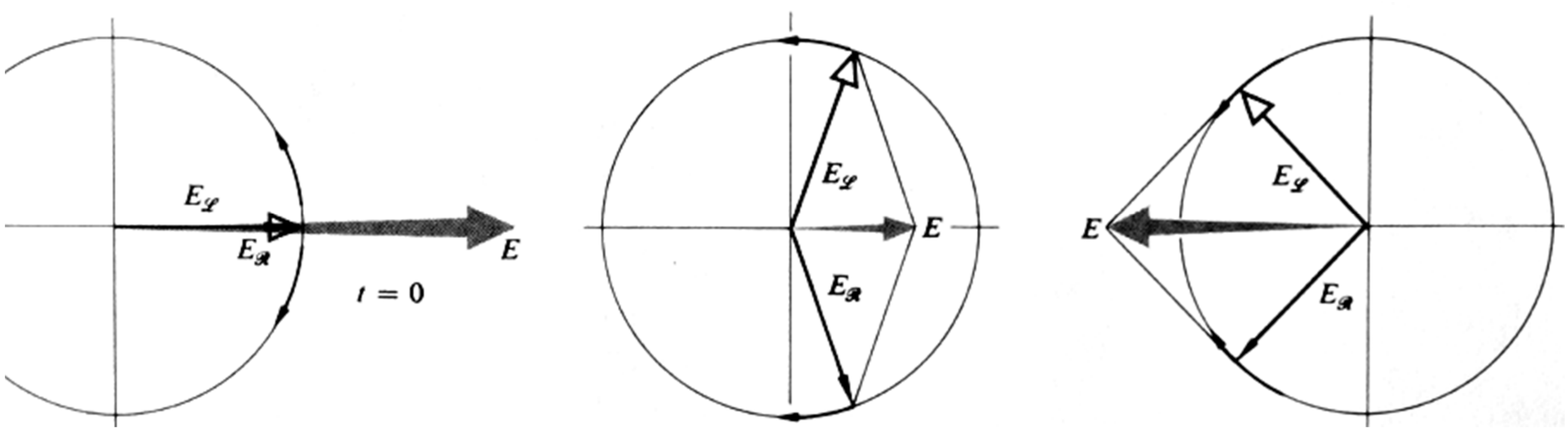
Optical Activity

$$\vec{E}_R = \frac{E_0}{2}\left[\hat{\imath}\cos(k_R z - \omega t) + \hat{\jmath}\sin(k_R z - \omega t)\right]$$

$$\vec{E}_L = \frac{E_0}{2}\left[\hat{\imath}\cos(k_L z - \omega t) - \hat{\jmath}\sin(k_L z - \omega t)\right]$$

$$\vec{E} = E_0\cos\left(\frac{k_R + k_L}{2}z - \omega t\right)\left[\hat{\imath}\cos\left(\frac{k_R - k_L}{2}z\right) + \hat{\jmath}\sin\left(\frac{k_R - k_L}{2}z\right)\right]$$

Time dependence of the $\vec{E}$ vector at $z = 0$: $\vec{E} = E_0\hat{\imath}\cos(\omega t)$

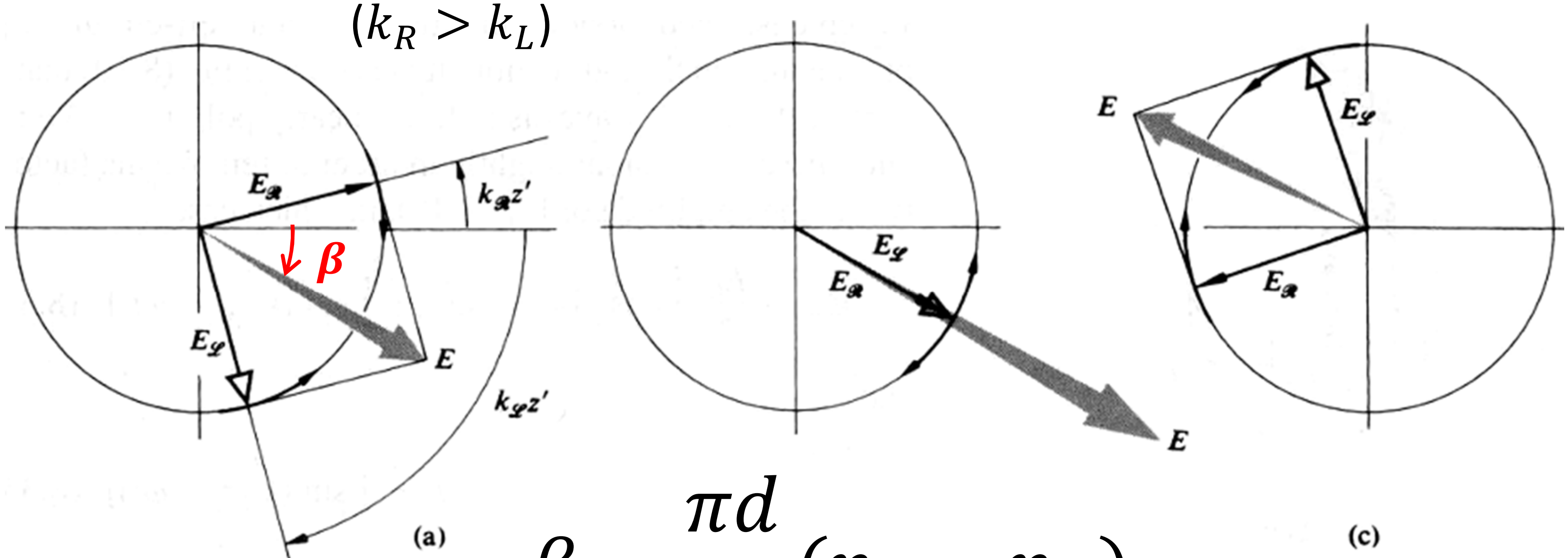


Optical Activity

$$\vec{E} = E_0 \cos\left(\frac{k_R + k_L}{2} z - \omega t\right)\left[\hat{\imath}\cos\left(\frac{k_R - k_L}{2} z\right) + \hat{\jmath}\sin\left(\frac{k_R - k_L}{2} z\right)\right]$$

- After propagating a distance $z = d$ through an optically active material, $\vec{E} = E_0[\hat{\imath}\cos\beta + \hat{\jmath}\sin\beta]\cos(\omega t)$

$(k_R > k_L)$



$$\beta = \frac{\pi d}{\lambda_0}(n_L - n_R)$$

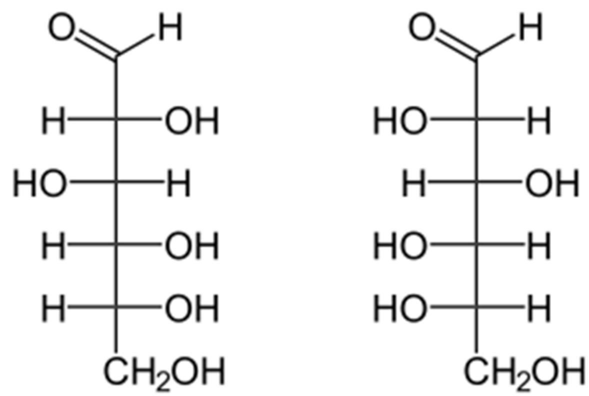
Optical Activity

- Application in organic chemistry:
- Experimental setup:
  - Sample concentration: 1 gram per millileter
  - Path length: 10 cm
  - Wavelength is 589 nm (Sodium “D-line”)
  - The “*specific rotation*” is the angle α that linearly polarized light is rotated
- Examples:

Substance	Specific rotation
Sucrose	$+66.47°$
Lactose	$+52.3°$
Levulose (D-Fructose)	$-92.4°$
Dextrose (D-Glucose)	$+52.5°$
L-Glucose	$-52°$

Stereoisomers

- Same chemical formula but mirror-symmetric structure:



D-Glucose L-Glucose

- Measurements of optical rotation provide information about sample purity and composition.
- Useful for analytic chemistry, regardless of the underlying physics.