PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200
Waves & Oscillations

Lecture 32 – Electromagnetic Waves

Spring 2016 Semester

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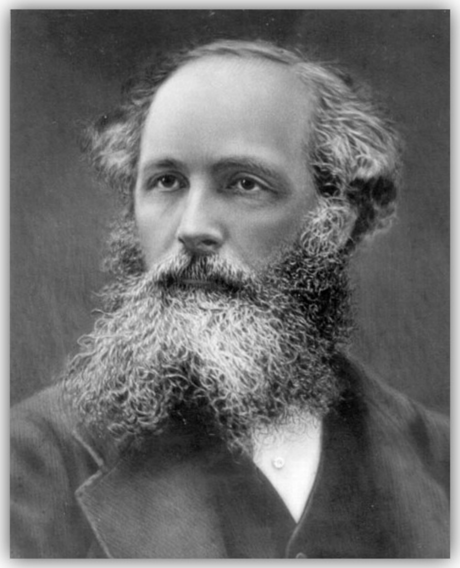
Electromagnetism

- Geometric optics overlooks the wave nature of light.
  - Light inconsistent with longitudinal waves in an ethereal medium
  - Still an excellent approximation when feature sizes are large compared with the wavelength of light
- But geometric optics could not explain
  - Polarization
  - Diffraction
  - Interference
- A unified picture was provided by Maxwell c. 1864

Maxwell's Equations (1864)



$$\oint_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0} \quad (1)$$

$$\oint_S \hat{n} \cdot \vec{B}\, dA = 0 \quad (2)$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt} \quad (3)$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_e}{dt} \quad (4)$$

Maxwell's Equations in Free Space

In "free space" where there are no electric charges or sources of current, Maxwell's equations are quite symmetric:

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{E}}\, d\boldsymbol{A} = \boldsymbol{0}$$

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{B}}\, d\boldsymbol{A} = \boldsymbol{0}$$

$$\oint_C \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\boldsymbol{\phi}_m}{dt}$$

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \boldsymbol{\mu}_0 \boldsymbol{\varepsilon}_0 \frac{d\boldsymbol{\phi}_e}{dt}$$

Velocity of Electromagnetic Waves

$$\frac{\partial^2 y}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Speed of wave propagation is

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$= \frac{1}{\sqrt{(4\pi \times 10^{-7} N/A^2)(8.854 \times 10^{-12}\ C^2/N \cdot m)}}$$

$$= \mathbf{2.998 \times 10^{8}\ m/s}$$

(speed of light)

Electromagnetic Waves

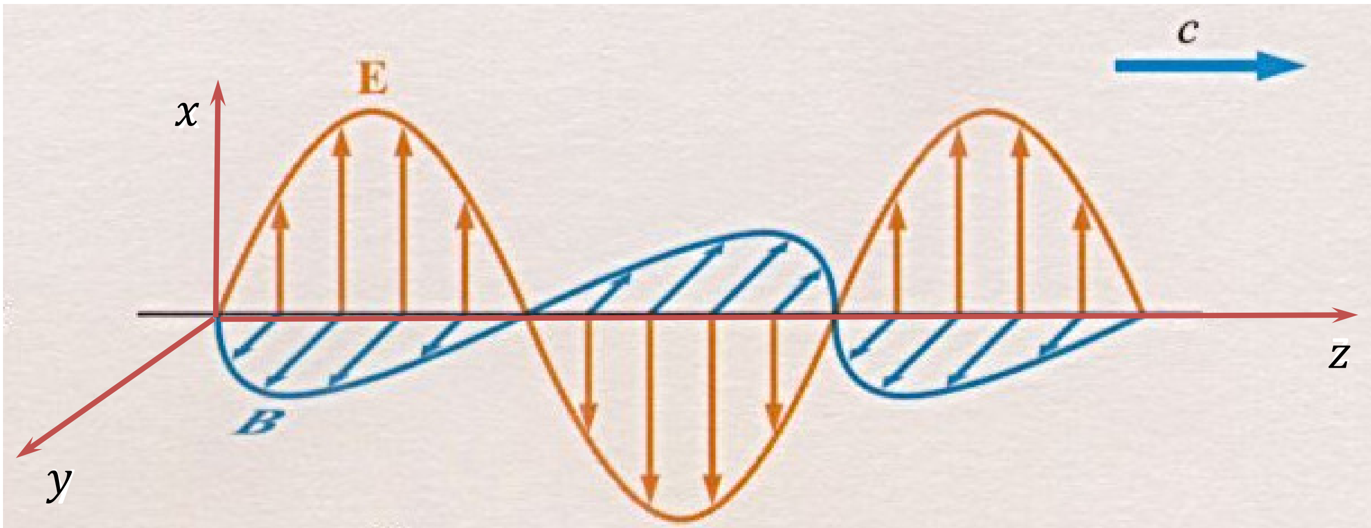
$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

- A solution is $E_x(z,t) = E_0 \sin(kz - \omega t)$ where $\omega = kc = 2\pi c/\lambda$
- What is the magnetic field?

$$\frac{\partial B_y}{\partial t} = -\frac{\partial E_x}{\partial z} = -kE_0 \cos(kz - \omega t)$$

$$B_y(x,t) = \frac{k}{\omega} E_0 \sin(kx - \omega t)$$

Electromagnetic Waves



- $\vec{E}$, $\vec{B}$ and $\vec{v}$ are mutually perpendicular.
- In general, the direction is

$$\hat{s} = \hat{E} \times \hat{B}$$

Energy in Electromagnetic Waves

Energy density of electric and magnetic fields:

$$u_e = \frac{1}{2}\epsilon_0 E^2 \qquad\qquad u_m = \frac{1}{2\mu_0}B^2$$

For an electromagnetic wave, $B = E/c = E\sqrt{\mu_0\epsilon_0}$

$$u_m = \frac{1}{2\mu_0}B^2 = \frac{1}{2}\epsilon_0 E^2 = u_e$$

The total energy density is

$$u = u_m + u_e = \epsilon_0 E^2$$

Intensity of Electromagnetic Waves

- Intensity is defined as the average power transmitted per unit area.

Intensity = Energy density × wave velocity

$$I = \epsilon_0 c \langle E^2 \rangle = \frac{\langle E^2 \rangle}{\mu_0 c}$$

$$\mu_0 c = 377\ \Omega \equiv Z_0$$

(Impedance of free space)

Polarization

- Light is an oscillating electromagnetic field
- The electric field has a direction

$$\vec{E}(x,t) = \vec{E}_0 \cos\left(\vec{k} \cdot \vec{r} - \omega t\right)$$

- No need to specify the magnetic field direction:

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{H} = (\hat{k} \times \vec{E})/Z \qquad \text{where } Z = \sqrt{\mu/\varepsilon}$$

- $\vec{H}$ refers to the magnetic field due to the light, not including any induced magnetic fields in the presence of matter.
- *Coherent* light has the same phase over macroscopic distances and time
- *Polarized* light has the electric field aligned over macroscopic distances and time

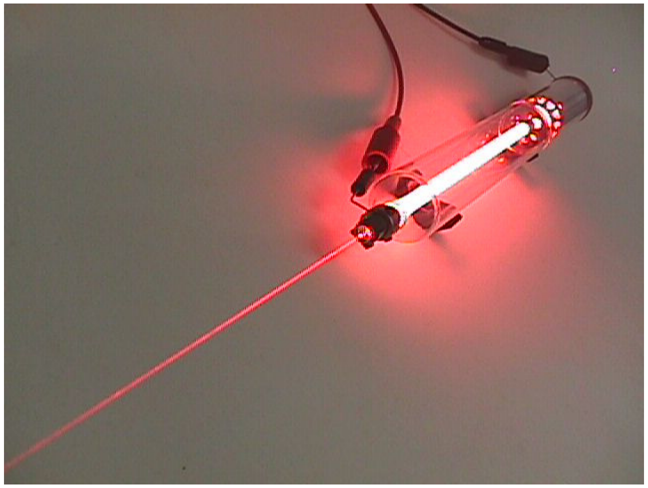
Polarization

Sources of un-polarized light

- Hot atoms transfer kinetic energy to electrons randomly
- Electrons randomly de-excite, emitting incoherent light – uncorrelated in phase and polarization

Polarization

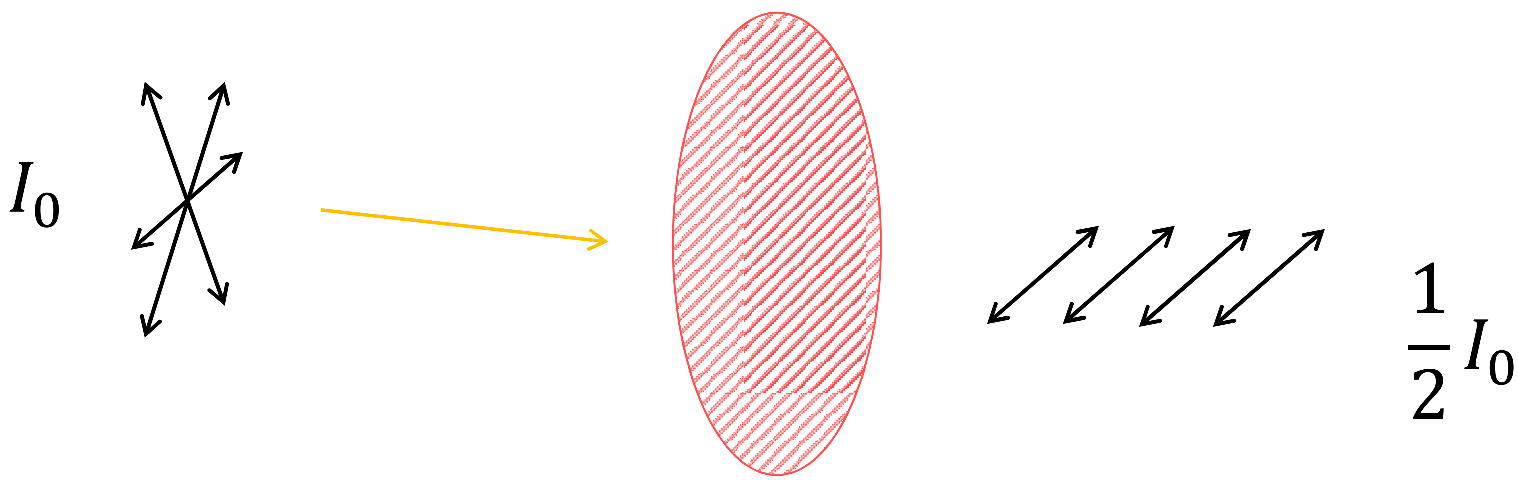
Sources of polarized light



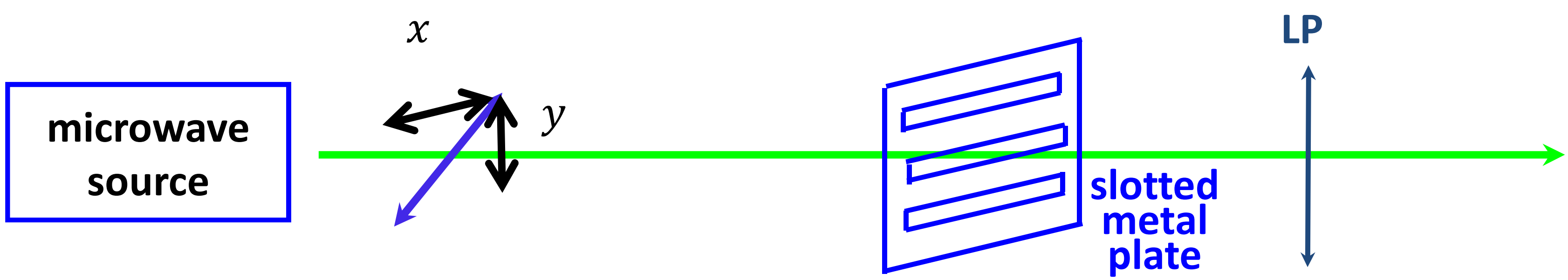
- Lasers produce light by stimulated emission
  - A photon causes an excited atom to emit another photon
  - The photon is emitted in phase and with the same polarization
- The resulting beam is highly coherent and polarized

Polarization by Absorption

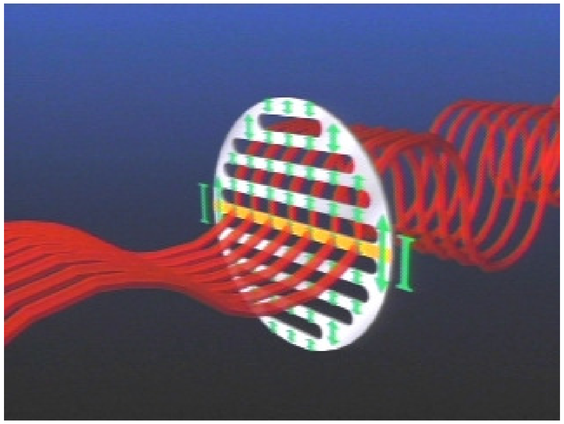
- A polarizer absorbs the component with $\vec{E}$ oriented along a particular axis.
- The light that emerges is linearly polarized along the perpendicular axis.
- If the light is initially un-polarized, half the light is absorbed.



Example with Microwaves



- The electric field in the x-direction induces currents in the metal plate and loses energy:
  - Horizontally polarized microwaves are absorbed
- No current can flow in the y-direction because of the slots
  - Vertically polarized microwaves are transmitted



Polarization by Reflection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- Reflected light is preferentially polarized
- The other component must be transmitted
- Transmission and reflection coefficients must depend on the polarization

Boundary Conditions

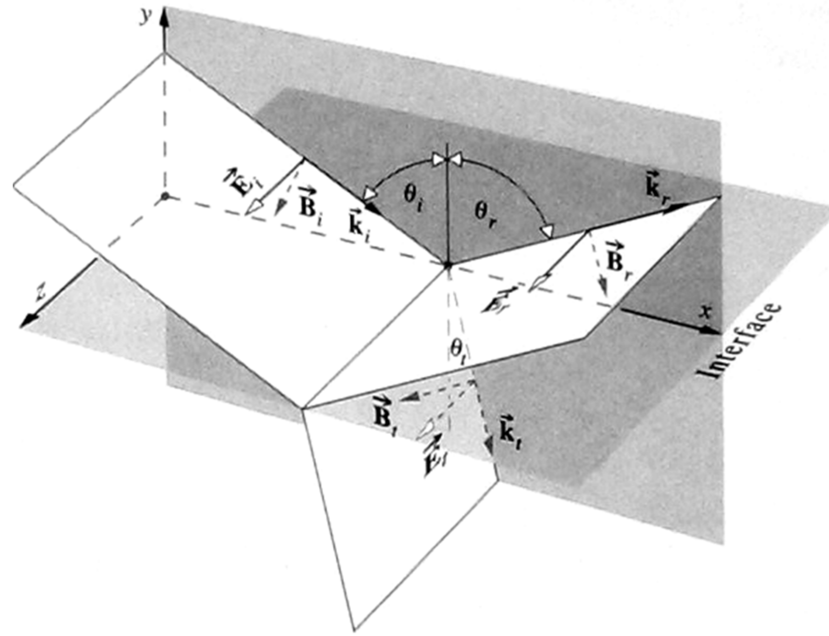
- In the presence of matter, the components electric and magnetic fields perpendicular to a surface change abruptly
- The component parallel to a surface is the same on both sides.
- Summary:
  - Perpendicular to surface

$$\varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$$
$$\mu_1 H_{1\perp} = \mu_2 H_{2\perp}$$

  - Parallel to surface

$$E_{1\parallel} = E_{2\parallel}$$
$$H_{1\parallel} = H_{2\parallel}$$

Reflection From a Surface



First case:

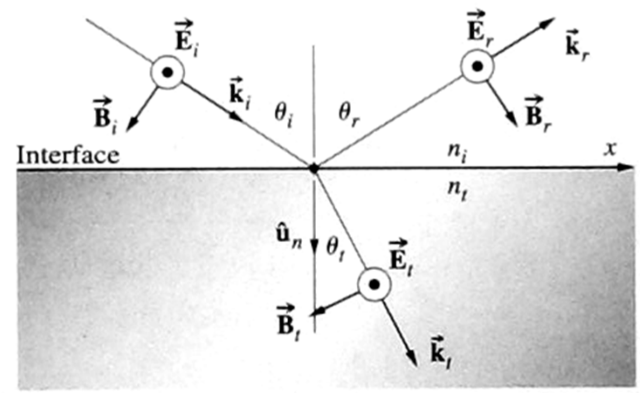
$\vec{E}$ is parallel to the surface…

$$E_i + E_r = E_t$$

$\vec{H}$ has components parallel and perpendicular to the surface

$$H_{\parallel i} + H_{\parallel r} = H_{\parallel t}$$

But $H_{\parallel} = \vec{H} \cdot \hat{\tau}$…



Reflection From a Surface

$\vec{E}$ is perpendicular to $\hat{n}$

$$\hat{\tau} = \frac{\vec{n} \times \hat{E}}{|\vec{E}|}$$

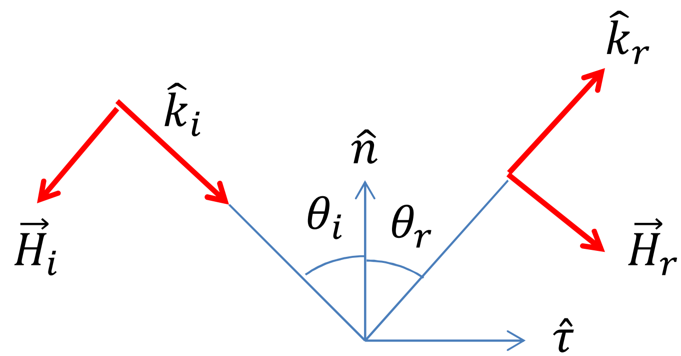
$\vec{H}$ can be written

$$\vec{H} = \frac{\hat{k} \times \vec{E}}{Z}$$

So we can write

$$\vec{H}_i \cdot \hat{\tau} = \frac{1}{ZE_i}\left(\hat{k} \times \vec{E}_i\right) \cdot \left(\vec{n} \times \hat{E}_i\right)$$

$$= \frac{E_i}{Z}\left(\hat{k} \cdot \hat{n}\right) = -\frac{E_i}{Z}\cos\theta_i$$



$\vec{E}$ is out of the page

Likewise,

$$\vec{H}_r \cdot \hat{\tau} = \frac{E_r}{Z}\cos\theta_r$$

$\vec{\boldsymbol{E}}$ perpendicular to $\hat{\boldsymbol{n}}$

- Boundary condition for $\vec{H}$:

$$H_{\parallel i} + H_{\parallel r} = H_{\parallel t}$$

- Previous results:

$$H_{i\parallel} = -\frac{E_i}{Z_1}\cos\theta_i \qquad H_{r\parallel} = \frac{E_r}{Z_1}\cos\theta_r = \frac{E_r}{Z_1}\cos\theta_i$$

$$\frac{-E_i\cos\theta_i + E_r\cos\theta_i}{Z_1} = \frac{-E_t\cos\theta_t}{Z_2}$$

$$E_i + E_r = E_t$$

- Two equations in two unknowns…

$\vec{E}$ perpendicular to $\hat{n}$

$$\frac{-E_i \cos\theta_i + E_r \cos\theta_i}{Z_1} = \frac{-E_t \cos\theta_t}{Z_2}$$

$$\frac{E_i \cos\theta_t + E_r \cos\theta_t}{Z_2} = \frac{E_t \cos\theta_t}{Z_2}$$

- Solve for E_r/E_i:

$$\boldsymbol{\frac{E_r}{E_i} = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}}$$

- Solve for E_t/E_i:

$$\boldsymbol{\frac{E_t}{E_i} = \frac{2Z_2 \cos\theta_i}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}}$$

Reflection From A Surface

$\vec{H}$ parallel to surface

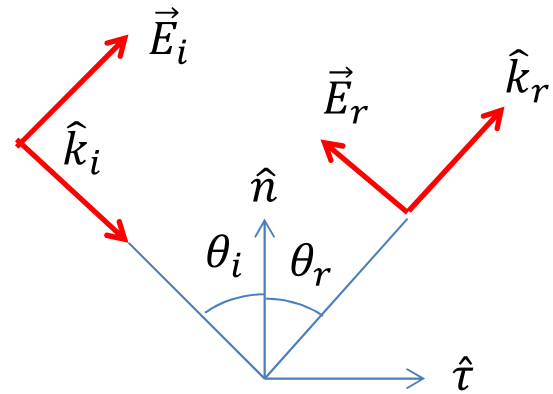
$$H_i + H_r = H_t$$

$$E_{\parallel i} + E_{\parallel r} = E_{\parallel t}$$

$$E_i \cos\theta_i - E_r \cos\theta_r = E_t \cos\theta_t$$

$$\frac{E_i}{Z_1} + \frac{E_r}{Z_1} = \frac{E_t}{Z_2}$$

- Two equations in two unknowns…



$\vec{H}$ is out of the page

$\vec{\boldsymbol{H}}$ perpendicular to $\hat{\boldsymbol{n}}$

$$\frac{E_i\cos\theta_i - E_r\cos\theta_i}{Z_2} = \frac{E_t\cos\theta_t}{Z_2}$$

$$\frac{E_i\cos\theta_t + E_r\cos\theta_t}{Z_1} = \frac{E_t\cos\theta_t}{Z_2}$$

- Solve for E_r/E_i:

$$\boldsymbol{\frac{E_r}{E_i} = \frac{Z_1\cos\theta_i - Z_2\cos\theta_t}{Z_1\, cos\,\theta_i + Z_2\, cos\,\theta_t}}$$

- Solve for E_t/E_i:

$$\boldsymbol{\frac{E_t}{E_i} = \frac{2Z_1\, cos\,\theta_i}{Z_1\, cos\,\theta_i + Z_2\, cos\,\theta_t}}$$

Fresnel's Equations

- In most dielectric media, $\mu_1 = \mu_2$ and therefore

$$\frac{Z_1}{Z_2} = \sqrt{\frac{\mu_1 \varepsilon_2}{\mu_2 \varepsilon_1}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1} = \frac{\sin\theta_i}{\sin\theta_t}$$

- After some trigonometry…

$$\left(\frac{E_r}{E_i}\right)_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad \left(\frac{E_r}{E_i}\right)_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\left(\frac{E_t}{E_i}\right)_\perp = \frac{(n_1/n_2)\sin(2\theta_i)}{\sin(\theta_i+\theta_t)} \quad \left(\frac{E_t}{E_i}\right)_\parallel = \frac{2\cos(\theta_i)\sin(\theta_t)}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$$

For $\vec{E}$ perpendicular and parallel to ***plane of incidence***.
(not the same as perpendicular or parallel to the surface)

Normal Incidence

- At normal incidence, $\cos\theta = 1$ there really is no component parallel to the plane of incidence.
- In this case

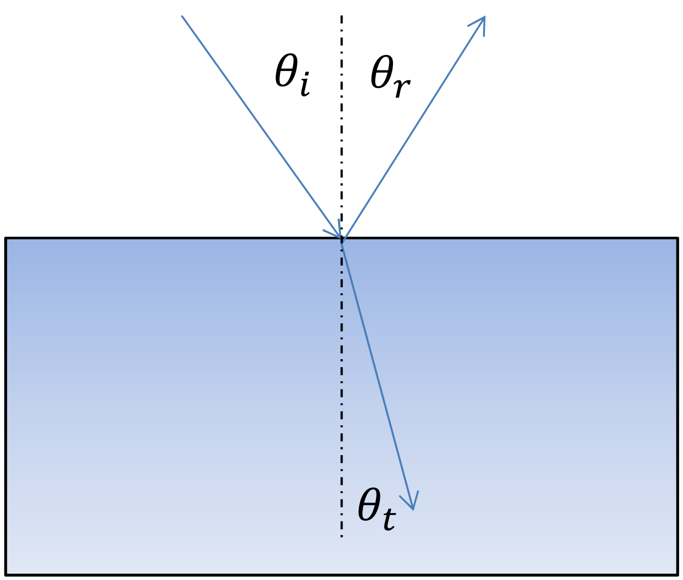
$$\left(\frac{E_r}{E_i}\right)_\perp = \frac{n_1 - n_2}{n_1 + n_2}$$

$$\left(\frac{E_t}{E_i}\right)_\perp = \frac{2n_2}{n_1 + n_2}$$

- This what we arrived at previously using only the difference in the speed of light in the two materials.

Application of Fresnel's Equations

- Unpolarized light in air ($n = 1$) is incident on a surface with index of refraction $n' = 1.5$ at an angle $\theta_i = 30°$
- What are the magnitudes of the electric field components of the reflected light?



First calculate the angles of reflection and refraction…

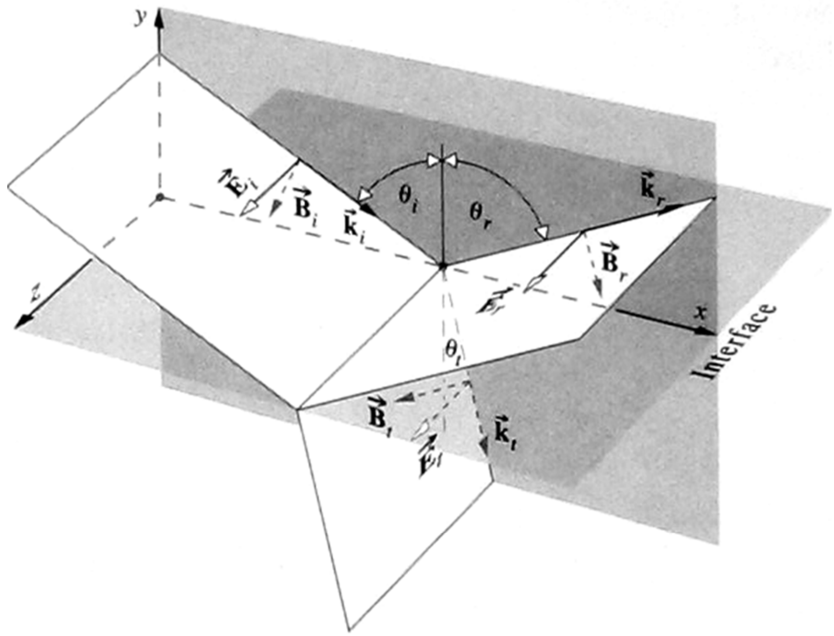
$$\boldsymbol{\theta_r = \theta_i = 30°}$$

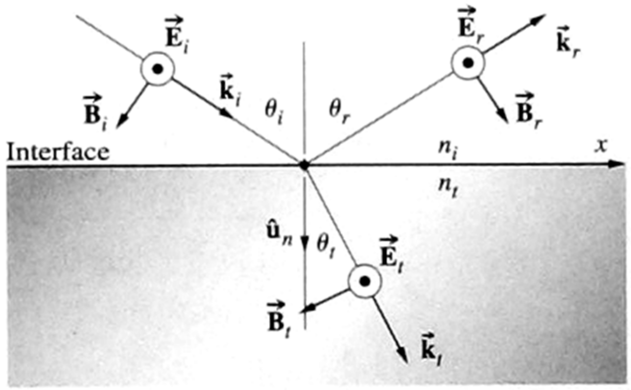
$$\sin\theta_i = n' \sin\theta_t$$

$$\sin\theta_t = \frac{\sin 30°}{1.5} = 0.333$$

$$\boldsymbol{\theta_t = 19.5°}$$

Application of Fresnel’s Equations





- Component of $\vec{E}$ parallel to the surface is perpendicular to the plane of incidence

$$\left(\frac{E_r}{E_i}\right)_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$= -\frac{\sin(30° - 19.5°)}{\sin(30° + 19.5°)}$$

$$= -0.240$$

Application of Fresnel's Equations

- Component of $\vec{E}$ perpendicular to the surface is parallel to the plane of incidence

$$\left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$= \frac{\tan(30° - 19.5°)}{\tan(30° + 19.5°)}$$

$$= 0.158$$

- Reflected light is preferentially polarized parallel to the surface.

- Component of $\vec{E}$ perpendicular to the surface is parallel to the plane of incidence

$$\left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$= \frac{\tan(30° - 19.5°)}{\tan(30° + 19.5°)}$$

$$= 0.158$$

- Reflected light is preferentially polarized parallel to the surface.

Reflected Components

- Since $\theta_t < \theta_i$ the component perpendicular to the plane of incidence is always negative:

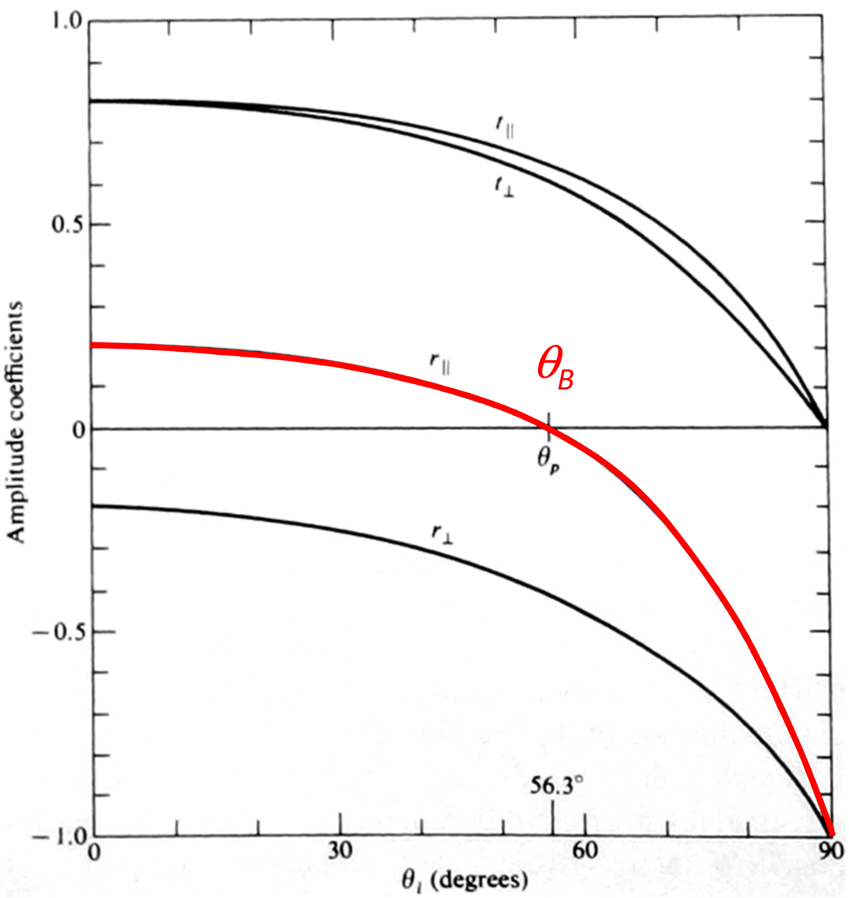
$$\left(\frac{E_r}{E_i}\right)_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

- The component parallel to the plane of incidence could be positive or negative:

$$\left(\frac{E_r}{E_i}\right)_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

- What happens when $\theta_i + \theta_t = 90°$?
  - Can this happen? Sure… check when $\theta_i \to 90°$.

Brewster's Angle



$\tan\theta \to \infty$ as $\theta \to 90°$ while $\tan(\theta_i - \theta_t)$ remains finite.

Therefore,

$$\left(\frac{E_r}{E_i}\right)_{\parallel} \to 0$$

Brewster's angle, θ_B is the angle of incidence for which $r_{\parallel} \to 0$.

Brewster's Angle

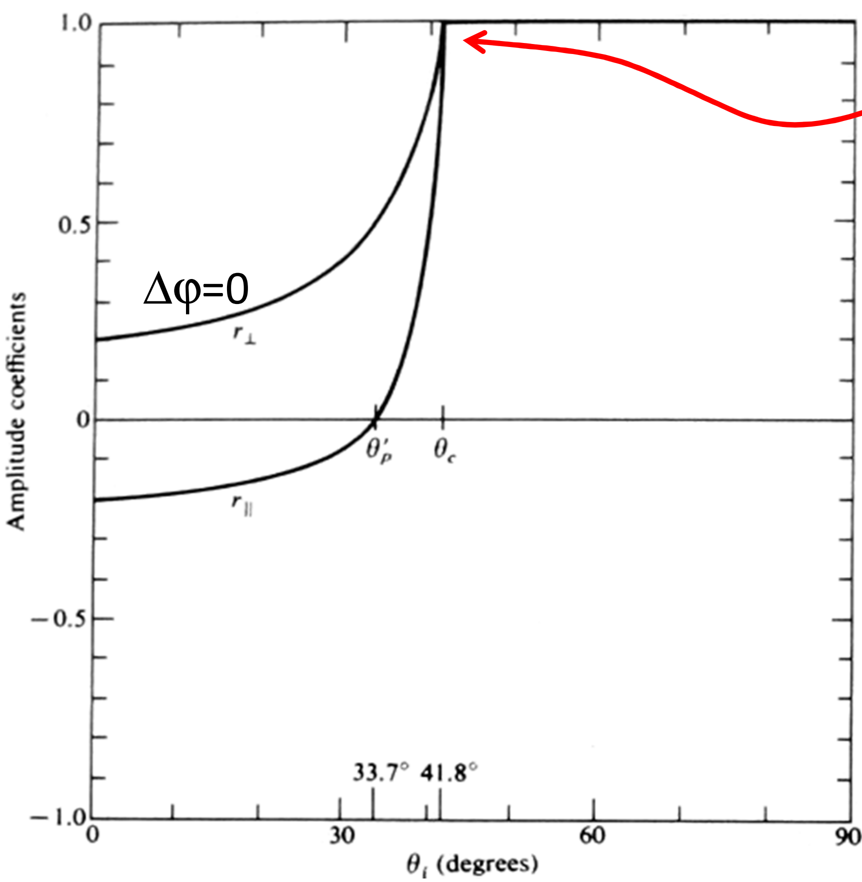
- Brewster's angle can be calculated from the relation $\theta_i + \theta_t = 90°$
- We can always calculate θ_t using Snell's law
- Good assignment question:

  Show that $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$

- Light reflected from a surface at an angle θ_B will be linearly polarized parallel to the surface (perpendicular to the plane of incidence)

Total Internal Reflection

- Consider the other case when $n_i > n_t$, for example, glass to air:



At some incidence angle (critical angle θ_c) everything is reflected (and nothing transmitted).

It can be shown that for any angle larger than θ_c no waves are transmitted into media: **total internal reflection**.

No phase shift upon reflection.

Reflected Intensity

- Remember that the *intensity* (*irradiance*) is related to the energy carried by light:

$$I = \epsilon v \langle E^2 \rangle_T$$

(averaged over some time $T \gg 1/f$)

- Reflectance is defined as

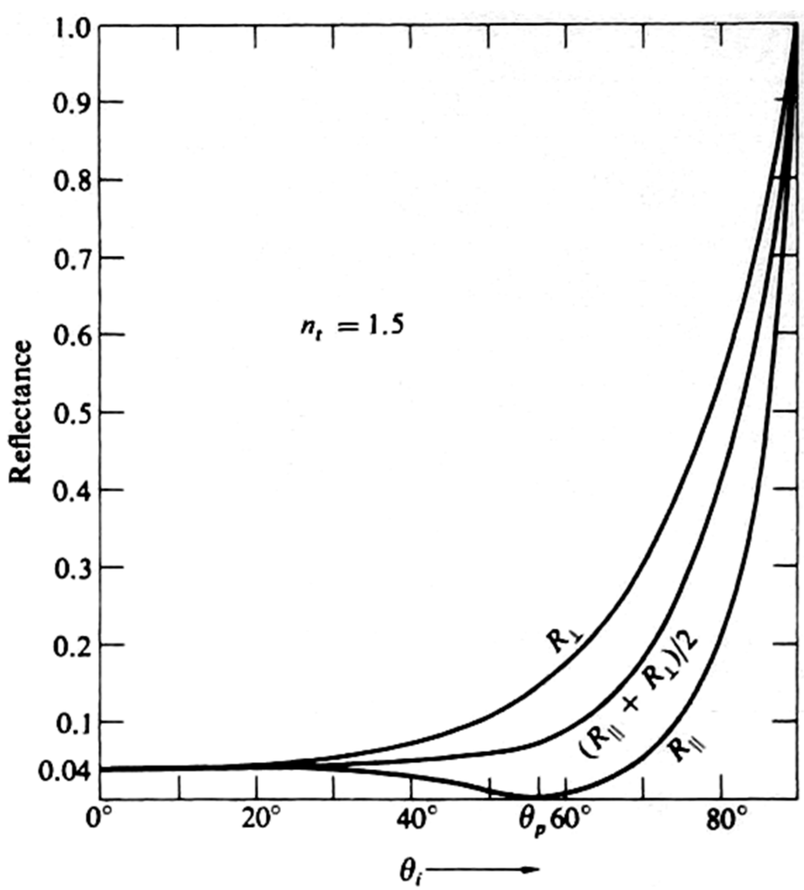
$$R_\perp = (r_\perp)^2 = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)}$$

$$R_\parallel = (r_\parallel)^2 = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)}$$

- Unpolarized reflectance:

$$R = \frac{1}{2}(R_\perp + R_\parallel)$$

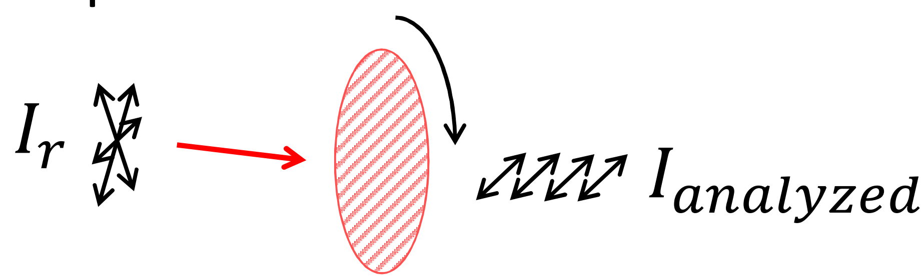
Reflected Intensity



- How polarized is the reflected light?
- Degree of polarization:

$$V = I_p / I_{total}$$

- Measured using an analyzing polarizer



$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$