PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 28 – Geometric Optics

Spring 2016 Semester

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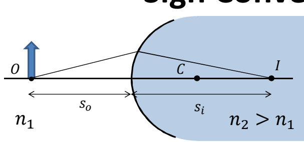
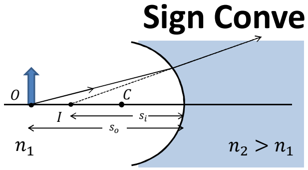
Sign Conventions



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

- Convex surface:
  - s_o is positive for objects on the incident-light side
  - s_i is positive for images on the refracted-light side
  - R is positive if C is on the refracted-light side

Sign Conventions



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

(same formula)

- Concave surface:
  - s_o is positive for objects on the incident-light side
  - s_i is negative for images on the incident-light side
  - R is negative if C is on the incident-light side

Magnification

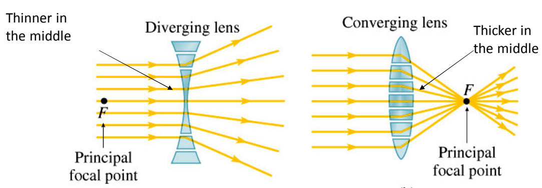
- Using these sign conventions, the magnification is

$$m = -\frac{n_1 s_i}{n_2 s_o}$$

- Ratio of image height to object height
- Sign indicates whether the image is inverted

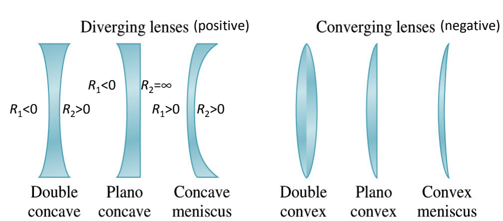
Thin Lenses

- The previous examples were for one spherical surface.
- Two spherical surfaces make a thin lens

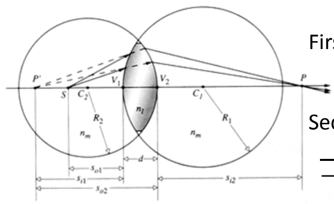


Thin Lens Classification

- A flat surface corresponds to $R \rightarrow \infty$
- All possible combinations of two surfaces:



Thin Lens Equation



First surface:

$$\frac{n_m}{s_{o1}} + \frac{n_l}{s_{i1}} = \frac{n_l - n_m}{R_1}$$

Second surface:

$$\frac{n_l}{-s_{i1} + d} + \frac{n_m}{s_{i2}} = \frac{n_m - n_l}{R_2}$$

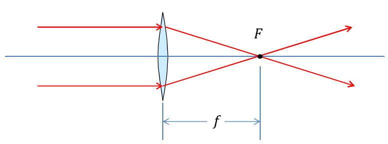
Add these equations and simplify using $n_m = 1$ and $d \to 0$:

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(Thin lens equation)

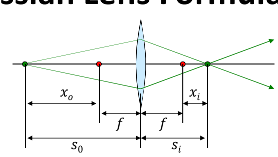
Gaussian Lens Formula

- Recall that the focal point was the place to which parallel rays were made to converge



- Parallel rays from the object correspond to $s_o \to \infty$ and $s_i \to f$: $\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
- This lens equation: $\frac{1}{s_i} + \frac{1}{s_o} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$

Gaussian Lens Formula



- Gaussian lens formula:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

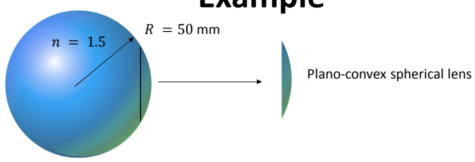
- Newtonian form:

$$x_o x_i = f^2$$

(follows from the Gaussian formula after about 5 lines of algebra)

- All you need to know about a lens is its focal length

Example



- What is the focal length of this lens?
  - Let $s_o \to \infty$, then $s_i \to f$

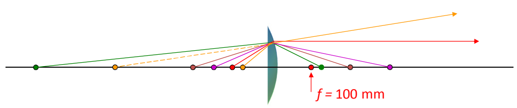
$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

  - The flat surface has $R_1 \to \infty$ and we know that $R_2 = -50$ mm

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{-50\ mm}\right) = \frac{1}{100\ mm}$$

$$\boldsymbol{f = 100\ mm}$$

Example

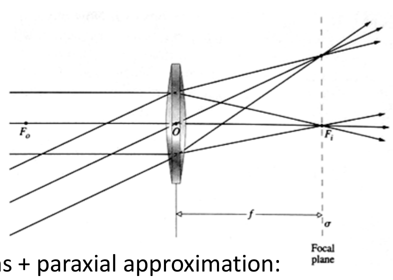


- Objects are placed at $s_i = 600\text{ mm}, 200\text{ mm}, 150\text{ mm}, 100\text{ mm}, 80\text{ mm}$
- Where are their images?

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \Rightarrow s_i = \frac{s_o f}{s_o - f}$$

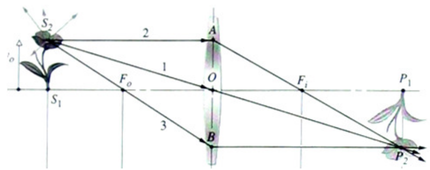
$$s_i = 120\text{ mm}, 200\text{ mm}, 300\text{ mm}, \infty, -400\text{ mm}$$

Focal Plane



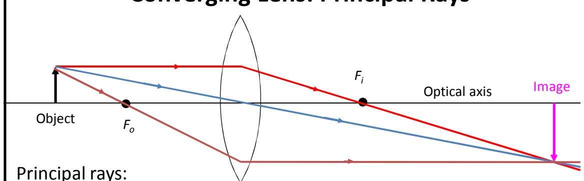
Thin lens + paraxial approximation:
- All rays that pass through the center, O, do not bend
- All rays converge to points in the focal plane (back focal plane)
- F_o lies in the front focal plane

Imaging with a Thin Lens



- For each point on the object we can draw three rays:
  1. A ray straight through the center of the lens
  2. A ray parallel to the central axis, then through the image focal point
  3. A ray through the object focal point, then parallel to the central axis.

Converging Lens: Principal Rays



Principal rays:

1) Rays parallel to principal axis pass through focal point F_i.
2) Rays through center of lens are not refracted.
3) Rays through F_o emerge parallel to principal axis.

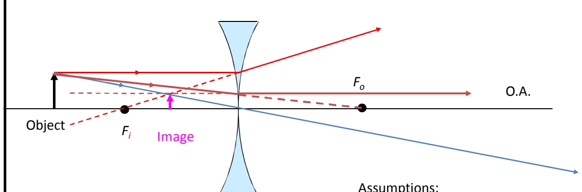
In this case image is real, inverted and enlarged

Assumptions:

- Monochromatic light
- Thin lens
- Paraxial rays (near the optical axis)

Since n is function of λ, in reality each color has different focal point: *chromatic aberration.* Contrast to mirrors: angle of incidence/reflection not a function of λ.

Diverging Lens: Forming Image



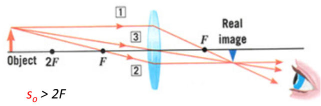
Assumptions:
- paraxial monochromatic rays
- thin lens

Principal rays:

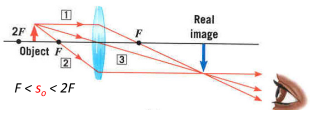
1) Rays parallel to principal axis appear to come from focal point F_i.
2) Rays through center of lens are not refracted.
3) Rays toward F_o emerge parallel to principal axis.

Image is virtual, upright and reduced.

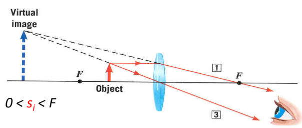
Converging Lens: Examples



This could be used in a camera. Big object on small film

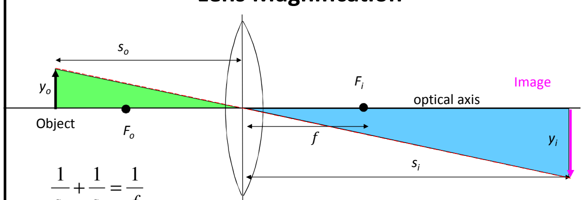


This could be used as a projector. Small slide(object) on big screen (image)



This is a magnifying glass

Lens Magnification



$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

Green and blue triangles are similar:

Magnification equation:

$$M_T \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

$_T$ = transverse

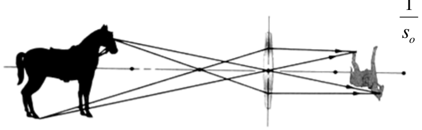
Example: *f=10 cm, s_o=15 cm*

$$\frac{1}{15cm}+\frac{1}{s_i}=\frac{1}{10cm} \quad \Rightarrow \quad s_i = 30\text{ cm}$$

$$M_T = -\frac{30cm}{15cm} = -2$$

Longitudinal Magnification

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$



The 3D image of the horse is distorted:
- transverse magnification changes along optical axis
- longitudinal magnification is not linear

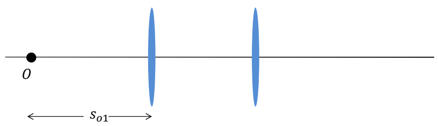
Longitudinal magnification:

$$M_L \equiv \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$$

Negative: a horse looking towards the lens forms an image that looks away from the lens

$$x_o x_i = f^2 \rightarrow x_i = f^2/x_o \rightarrow \frac{dx_i}{dx_o} = \frac{d}{dx_o}\left(f^2/x_o\right) = -\left(f^2/x_o^2\right)$$

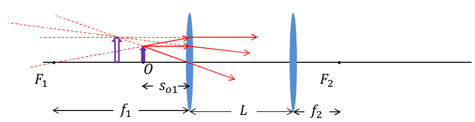
Two Lens Systems



- Calculate s_{i1} using $\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}}$
- Ignore the first lens, treat s_{i1} as the object distance for the second lens. Calculate s_{i2} using
$$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$$
- Overall magnification: $M = m_1 m_2 = \left(-\frac{s_{i1}}{s_{o1}}\right)\left(-\frac{s_{i2}}{s_{o2}}\right)$

Example: Two Lens System

An object is placed in front of two thin symmetrical coaxial lenses (lens 1 & lens 2) with focal lengths f_1=+24 cm & f_2=+9.0 cm, with a lens separation of L=10.0 cm. The object is 6.0 cm from lens 1. Where is the image of the object?



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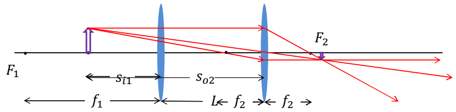

(not really to scale…)

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Lens 1: $\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}}$ ➡ $s_{i1} = -8\ cm$

Image 1 is virtual.

Lens 2: Treat image 1 as O_2 for lens 2. O_2 is outside the focal point of lens 2. So, image 2 will be real & inverted on the other side of lens 2.

$s_{o2} = L - s_{i1}$ ➡ $s_{i2} = 18.0\ cm$

$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$

Image 2 is real.

Magnification: $M_T = \left(-\frac{-8\ cm}{6\ cm}\right)\left(-\frac{18cm}{18cm}\right) = -1.33$