PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 26 – Introduction to Optics

Spring 2016 Semester

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Introduction to Optics

- If we can identify any system that is described by the wave equation, then we know everything!

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

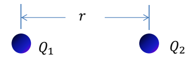
- Except that…
  - The geometry might be complicated
  - Boundary conditions might not be simple
  - Initial conditions might not be known
  - The solution is the superposition of incident waves and all possible reflected waves
- So we develop new ways of looking at this problem and call it optics.

Electromagnetism

- Geometric optics overlooks the wave nature of light.
  - Light inconsistent with longitudinal waves in an ethereal medium
  - Still an excellent approximation when feature sizes are large compared with the wavelength of light
- But geometric optics could not explain
  - Polarization
  - Diffraction
  - Interference
- A unified picture was provided by Maxwell c. 1864

Forces on Charges

- Coulomb's law of electrostatic force:



- The magnitude of the attractive/repulsive force is

$$\vec{F} = k\,\frac{|Q_1||Q_2|}{r^2}\,\hat{r}$$

where

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ N \cdot m^2 \cdot C^{-2}$$

and therefore

$$\epsilon_0 = 8.85 \times 10^{-12}\ C^2 \cdot N^{-1} \cdot m^{-2}$$

(This constant is called the "permittivity of free space")

Electric Field

- An electric charge changes the properties of the space around it.
  - It is the source of an "electric field".
  - It could be defined as the "force per unit charge":

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad \Rightarrow \quad \vec{F} = q\vec{E}$$

  - Quantum field theory provides a deeper description…
- Gauss's Law:

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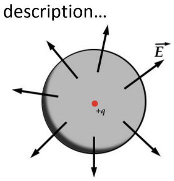
$$\int_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$$

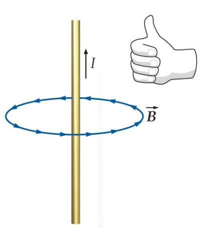
Electric "flux" through surface S



Magnetic Field

- Moving charges (ie, electric current) produce a magnetic field:

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$



A moving charge in a magnetic field experiences a force:

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

Lorentz force law:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Gauss's Law for Magnetism

- Electric charges produce electric fields:

$$\int_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$$

- But there are no "magnetic charges":

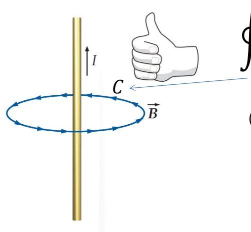
2

$$\int_S \hat{n} \cdot \vec{B}\, dA = 0$$

Magnetic "flux" through surface S

Ampere's Law

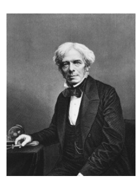
- An electric current produces a magnetic field that curls around it:



$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_C$$

(Close, but not quite the whole story)

Faraday's Law of Magnetic Induction



- Magnetic flux: $\phi_m = \int_S \vec{B} \cdot \hat{n}\, dA$
- Faraday observed that a changing magnetic flux through a wire loop induced a current
  - It transferred energy to the charge carriers in the wire

$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \hat{n} \cdot \vec{B} dA$$

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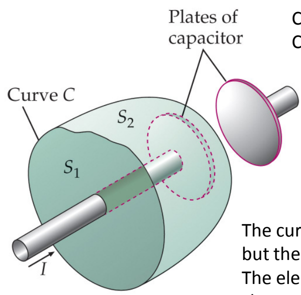
The Problem with Ampere's Law



Current through S_1 is I
Current through S_2 is 0!

The current through S_2 is zero, but the electric flux is not zero. The electric flux changes as charge flows onto the capacitor.

Maxwell's Displacement Current

- We can think of the changing electric flux through S_2 as if it were a current:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \epsilon_0 \frac{d}{dt} \int_{S_2} \vec{E} \cdot \hat{n}\, dA$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 (I + I_d) = \mu_0 \mathrm{I} + \mu_0 \epsilon_0 \frac{d}{dt} \int_{S_2} \vec{E} \cdot \hat{n}\, dA$$

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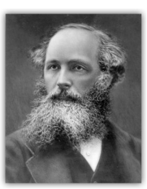
Maxwell's Equations (1864)



$$\oint_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0} \quad (1)$$

$$\oint_S \hat{n} \cdot \vec{B}\, dA = 0 \quad (2)$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt} \quad (3)$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_e}{dt} \quad (4)$$

Maxwell's Equations in Free Space

In "free space" where there are no electric charges or sources of current, Maxwell's equations are quite symmetric:

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{E}}\, \boldsymbol{dA} = \boldsymbol{0}$$

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{B}}\, \boldsymbol{dA} = \boldsymbol{0}$$

$$\oint_C \vec{\boldsymbol{E}} \cdot \boldsymbol{d}\vec{\ell} = -\frac{\boldsymbol{d\phi_m}}{\boldsymbol{dt}}$$

$$\oint_C \vec{\boldsymbol{B}} \cdot \boldsymbol{d}\vec{\ell} = \boldsymbol{\mu_0 \varepsilon_0} \frac{\boldsymbol{d\phi_e}}{\boldsymbol{dt}}$$

Maxwell's Equations in Free Space

$$\oint_C \vec{\boldsymbol{E}} \cdot \boldsymbol{d}\vec{\ell} = -\frac{\boldsymbol{d\phi_m}}{\boldsymbol{dt}}$$

$$\oint_C \vec{\boldsymbol{B}} \cdot \boldsymbol{d}\vec{\ell} = \boldsymbol{\mu_0 \varepsilon_0} \frac{\boldsymbol{d\phi_e}}{\boldsymbol{dt}}$$

A changing magnetic flux induces an electric field.
A changing electric flux induces a magnetic field.

Will this process continue indefinitely?

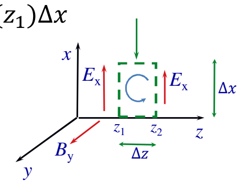
Light is an Electromagnetic Wave

- Faraday's Law:

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt} = -\frac{\partial B_y}{\partial t} \Delta x \Delta z$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = E_x(z_2)\Delta x - E_x(z_1)\Delta x$$

$$\approx \frac{\partial E_x}{\partial z} \Delta z \Delta x$$



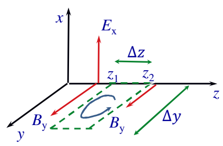
Light is an Electromagnetic Wave

- Ampere's law:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\phi_e}{dt} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t} \Delta y \Delta z$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = B_y(z_1)\Delta y - B_y(z_2)\Delta y$$

$$\approx -\frac{\partial B_y}{\partial z} \Delta z \Delta y$$



Putting these together…

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$-\frac{\partial B_y}{\partial z} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}$$

Differentiate the first with respect to z:

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial^2 B_y}{\partial z \partial t}$$

Differentiate the second with respect to t:

$$-\frac{\partial^2 B_y}{\partial z \partial t} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Velocity of Electromagnetic Waves

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Speed of wave propagation is

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$= \frac{1}{\sqrt{(4\pi \times 10^{-7} N/A^2)(8.854 \times 10^{-12}\ C^2/N \cdot m)}}$$

$$= \mathbf{2.998 \times 10^8\ m/s}$$

(speed of light)

Light is an Electromagnetic Wave

- Maxwell showed that these equations contain the wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

where $y(x,t)$ is a function of position and time.

- General solution:

$$y(x,t) = f_1(x - vt) + f_2(x + vt)$$

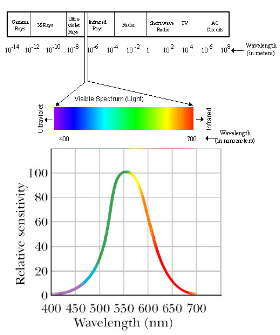
- Harmonic solutions:

$$y(x,t) = y_0 \sin(kx \pm \omega t)$$

- Wavelength, $\lambda = 2\pi/k$ and frequency $f = \omega/2\pi$.
- Velocity, $v = \lambda/f = \omega/k$.

The electromagnetic spectrum

- In 1850, the only known forms electromagnetic waves were ultraviolet, visible and infrared.
- The human eye is only sensitive to a very narrow range of wavelengths:



Discovery of Radio Waves

ELECTRIC WAVES

BEING

RESEARCHES ON THE PROPAGATION OF ELECTRIC ACTION WITH FINITE VELOCITY THROUGH SPACE

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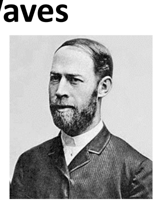
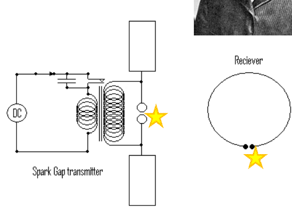
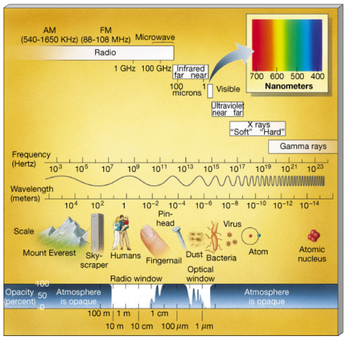




The Electromagnetic Spectrum



Electromagnetic Waves

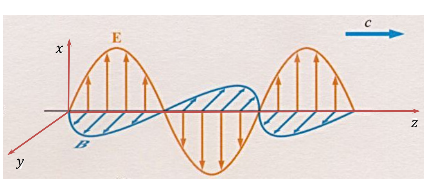
$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

- A solution is $E_x(z,t) = E_0 \sin(kz - \omega t)$ where $\omega = kc = 2\pi c/\lambda$
- What is the magnetic field?

$$\frac{\partial B_y}{\partial t} = -\frac{\partial E_x}{\partial z} = -kE_0 \cos(kz - \omega t)$$

$$B_y(x,t) = \frac{k}{\omega} E_0 \sin(kx - \omega t)$$

Electromagnetic Waves



- $\vec{E}$, $\vec{B}$ and $\vec{v}$ are mutually perpendicular.
- In general, the direction is

$$\hat{s} = \hat{E} \times \hat{B}$$

Energy in Electromagnetic Waves

Energy stored in electric and magnetic fields:

$$u_e = \frac{1}{2}\epsilon_0 E^2 \qquad u_m = \frac{1}{2\mu_0}B^2$$

For an electromagnetic wave, $B = E/c = E\sqrt{\mu_0\epsilon_0}$

$$u_m = \frac{1}{2\mu_0}B^2 = \frac{1}{2}\epsilon_0 E^2 = u_e$$

The total energy density is

$$u = u_m + u_e = \epsilon_0 E^2$$

Intensity of Electromagnetic Waves

- Intensity is defined as the average power transmitted per unit area.

  Intensity = Energy density × wave velocity

$$I = \epsilon_0 c\langle E^2\rangle = \frac{\langle E^2\rangle}{\mu_0 c}$$

$$\mu_0 c = 377\ \Omega \equiv Z_0$$

(Impedance of free space)

Poynting Vector

- We can construct a vector from the intensity and the direction $\hat{s} = \hat{E} \times \hat{B}$:

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

$$\langle\vec{S}\rangle = \frac{\langle E^2\rangle}{Z_0} = I$$

- This represents the flow of power in the direction $\hat{s}$
- Average electric field: $E_{rms} = E_0/\sqrt{2}$

$$\langle\vec{S}\rangle = \frac{(E_0)^2}{2Z_0}$$

- Units: Watts/m²