PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 26 – Introduction to Optics

Spring 2016 Semester

Matthew Jones

Introduction to Optics

- If we can identify any system that is described by the wave equation, then we know everything!

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

- Except that…
  - The geometry might be complicated
  - Boundary conditions might not be simple
  - Initial conditions might not be known
  - The solution is the superposition of incident waves and all possible reflected waves
- So we develop new ways of looking at this problem and call it optics.

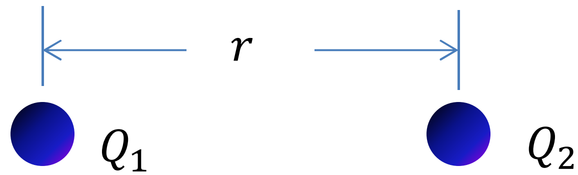
Electromagnetism

- Geometric optics overlooks the wave nature of light.
  - Light inconsistent with longitudinal waves in an ethereal medium
  - Still an excellent approximation when feature sizes are large compared with the wavelength of light
- But geometric optics could not explain
  - Polarization
  - Diffraction
  - Interference
- A unified picture was provided by Maxwell c. 1864

Forces on Charges


Charles-Augustin de Coulomb (1736 - 1806)

- Coulomb's law of electrostatic force:



- The magnitude of the attractive/repulsive force is

$$\vec{F} = k\,\frac{|Q_1||Q_2|}{r^2}\,\hat{r}$$

where

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ N \cdot m^2 \cdot C^{-2}$$

and therefore

$$\epsilon_0 = 8.85 \times 10^{-12}\ C^2 \cdot N^{-1} \cdot m^{-2}$$

(This constant is called the "permittivity of free space")

Electric Field

- An electric charge changes the properties of the space around it.
  - It is the source of an “electric field”.
  - It could be defined as the “force per unit charge”:

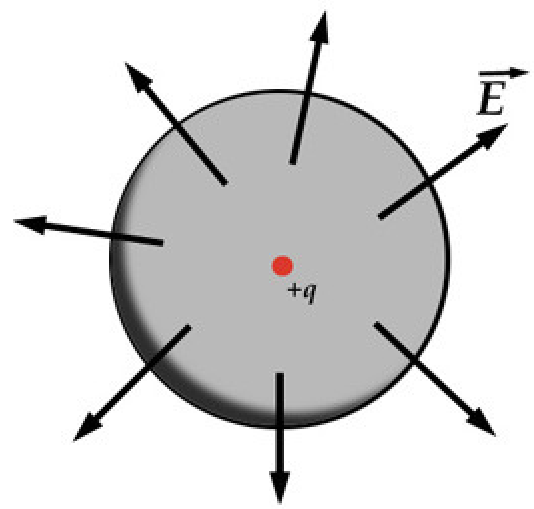
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad \Rightarrow \quad \vec{F} = q\vec{E}$$

  - Quantum field theory provides a deeper description…
- Gauss’s Law:

1

$$\int_S \widehat{n}\cdot\vec{E}\,dA = \frac{Q_{inside}}{\epsilon_0}$$

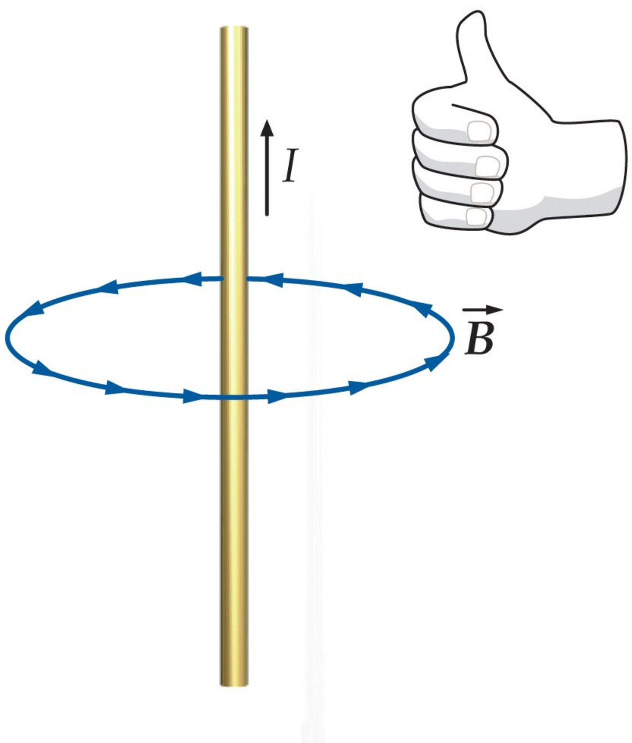
Electric “flux” through surface S



Magnetic Field

- Moving charges (ie, electric current) produce a magnetic field:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{\ell} \times \hat{r}}{r^2}$$



A moving charge in a magnetic field experiences a force:

$$\vec{F} = q \, \vec{v} \times \vec{B}$$

Lorentz force law:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Gauss's Law for Magnetism

- Electric charges produce electric fields:

$$\int_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$$

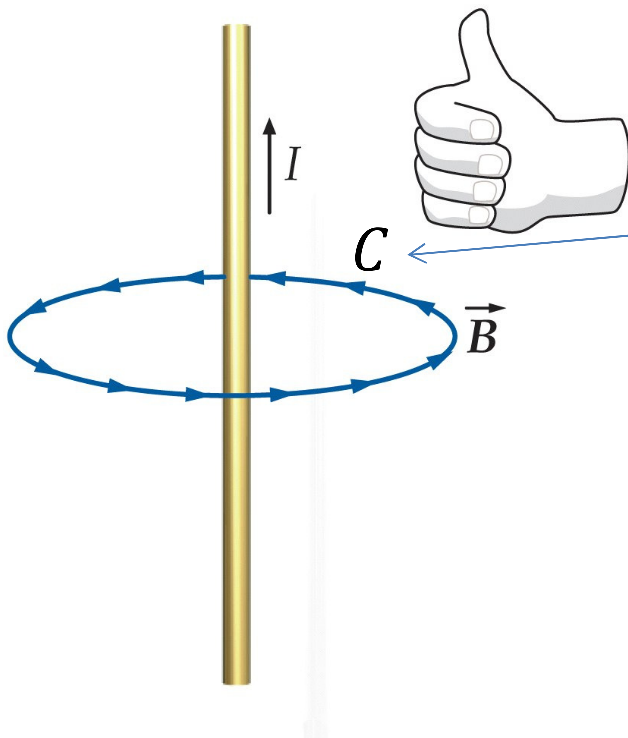
- But there are no "magnetic charges":

2

$$\underbrace{\int_S \hat{n} \cdot \vec{B}\, dA}_{\text{Magnetic "flux" through surface S}} = 0$$

Ampere's Law

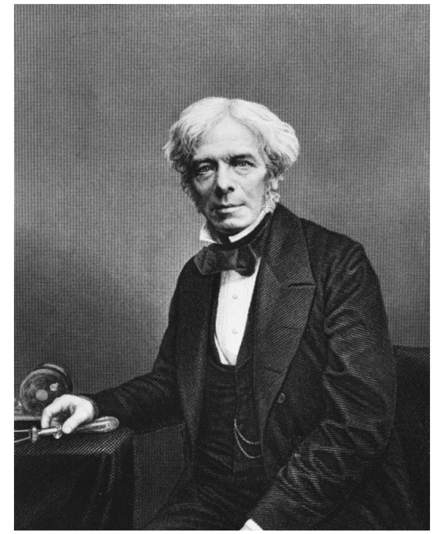
- An electric current produces a magnetic field that curls around it:



$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_C$$

(Close, but not quite the whole story)

Faraday's Law of Magnetic Induction



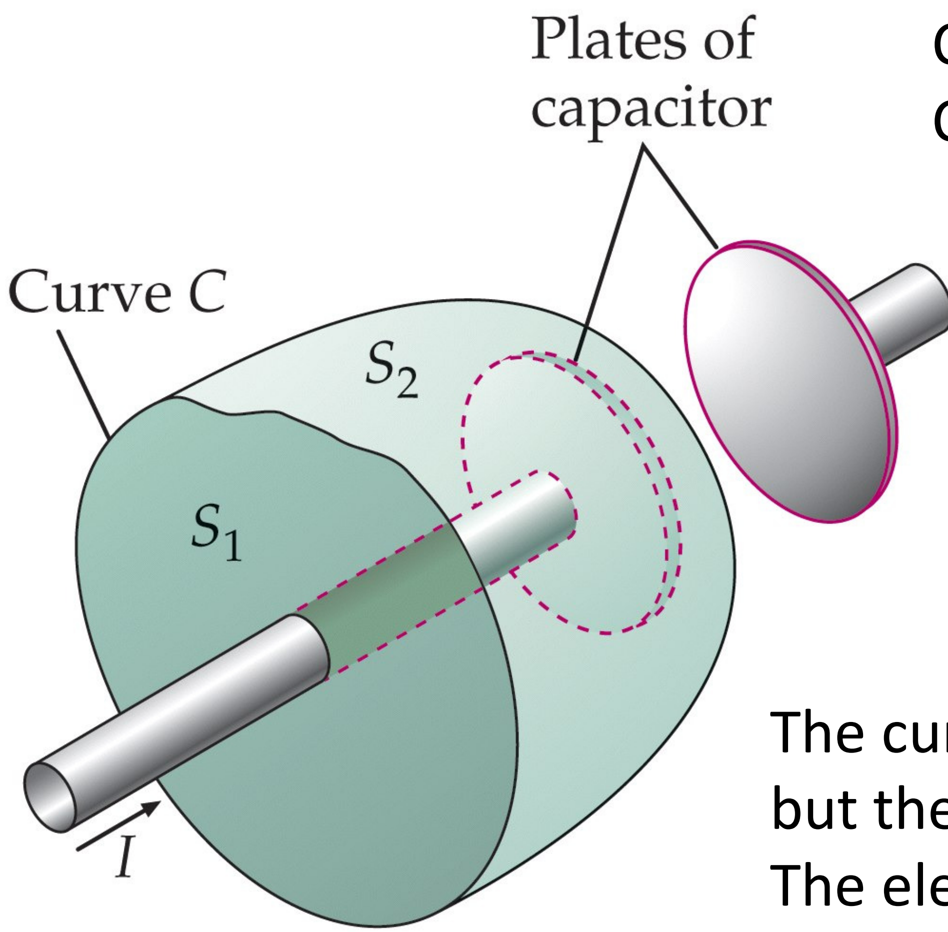
- Magnetic flux: $\phi_m = \int_S \vec{B} \cdot \hat{n}\, dA$
- Faraday observed that a changing magnetic flux through a wire loop induced a current
  - It transferred energy to the charge carriers in the wire

$$\mathcal{E} = -\frac{d\phi_m}{dt}$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \hat{n} \cdot \vec{B}\, dA$$

3

The Problem with Ampere's Law



Current through S_1 is I
Current through S_2 is 0!

The current through S_2 is zero, but the electric flux is not zero. The electric flux changes as charge flows onto the capacitor.

Maxwell's Displacement Current

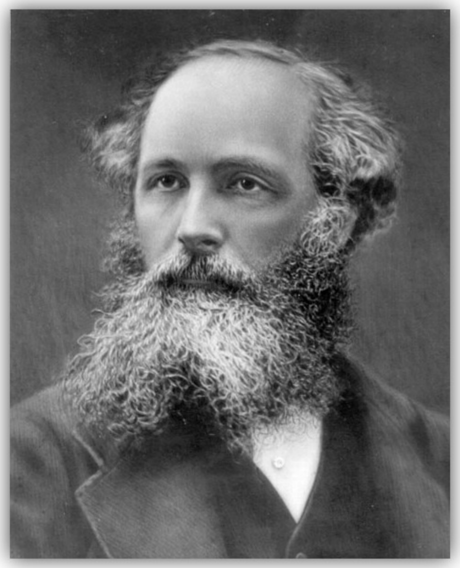
- We can think of the changing electric flux through S_2 as if it were a current:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \epsilon_0 \frac{d}{dt}\int_{S_2} \vec{E} \cdot \hat{n}\, dA$$

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \boldsymbol{\mu_0}(\boldsymbol{I} + \boldsymbol{I_d}) = \boldsymbol{\mu_0}\mathbf{I} + \boldsymbol{\mu_0\epsilon_0}\frac{\boldsymbol{d}}{\boldsymbol{dt}}\int_{S_2} \vec{\boldsymbol{E}} \cdot \hat{\boldsymbol{n}}\, \boldsymbol{dA}$$

4

Maxwell's Equations (1864)



$$\oint_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$$ ①

$$\oint_S \hat{n} \cdot \vec{B}\, dA = 0$$ ②

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt}$$ ③

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_e}{dt}$$ ④

Maxwell's Equations in Free Space

In "free space" where there are no electric charges or sources of current, Maxwell's equations are quite symmetric:

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{E}}\, d\boldsymbol{A} = \boldsymbol{0}$$

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{B}}\, d\boldsymbol{A} = \boldsymbol{0}$$

$$\oint_C \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\boldsymbol{\phi}_m}{dt}$$

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \boldsymbol{\mu}_0 \boldsymbol{\varepsilon}_0 \frac{d\boldsymbol{\phi}_e}{dt}$$

Maxwell's Equations in Free Space

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \varepsilon_0 \frac{d\phi_e}{dt}$$

A changing magnetic flux induces an electric field.

A changing electric flux induces a magnetic field.

Will this process continue indefinitely?

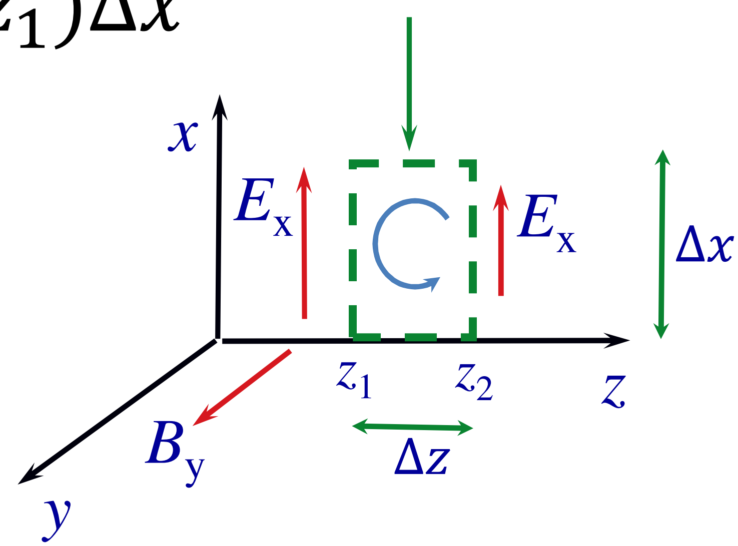
Light is an Electromagnetic Wave

- Faraday's Law:

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt} = -\frac{\partial B_y}{\partial t} \Delta x \Delta z$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = E_x(z_2)\Delta x - E_x(z_1)\Delta x$$

$$\approx \frac{\partial E_x}{\partial z} \Delta z \Delta x$$



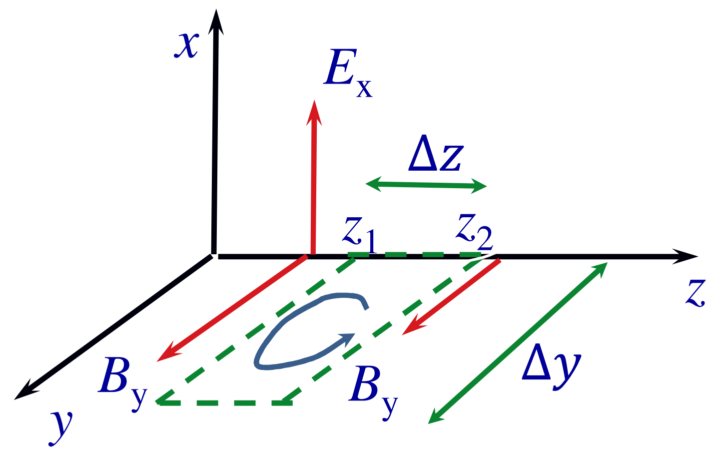
Light is an Electromagnetic Wave

- Ampere's law:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\phi_e}{dt} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t} \Delta y \Delta z$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = B_y(z_1)\Delta y - B_y(z_2)\Delta y$$

$$\approx -\frac{\partial B_y}{\partial z} \Delta z \Delta y$$



Putting these together…

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$-\frac{\partial B_y}{\partial z} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}$$

Differentiate the first with respect to z:

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial^2 B_y}{\partial z \partial t}$$

Differentiate the second with respect to t:

$$-\frac{\partial^2 B_y}{\partial z \partial t} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Velocity of Electromagnetic Waves

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Speed of wave propagation is

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$= \frac{1}{\sqrt{(4\pi \times 10^{-7} N/A^2)(8.854 \times 10^{-12}\ C^2/N \cdot m)}}$$

$$\mathbf{= 2.998 \times 10^8\ m/s}$$

(speed of light)

Light is an Electromagnetic Wave

- Maxwell showed that these equations contain the wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

where $y(x,t)$ is a function of position and time.

- General solution:

$$y(x,t) = f_1(x - vt) + f_2(x + vt)$$

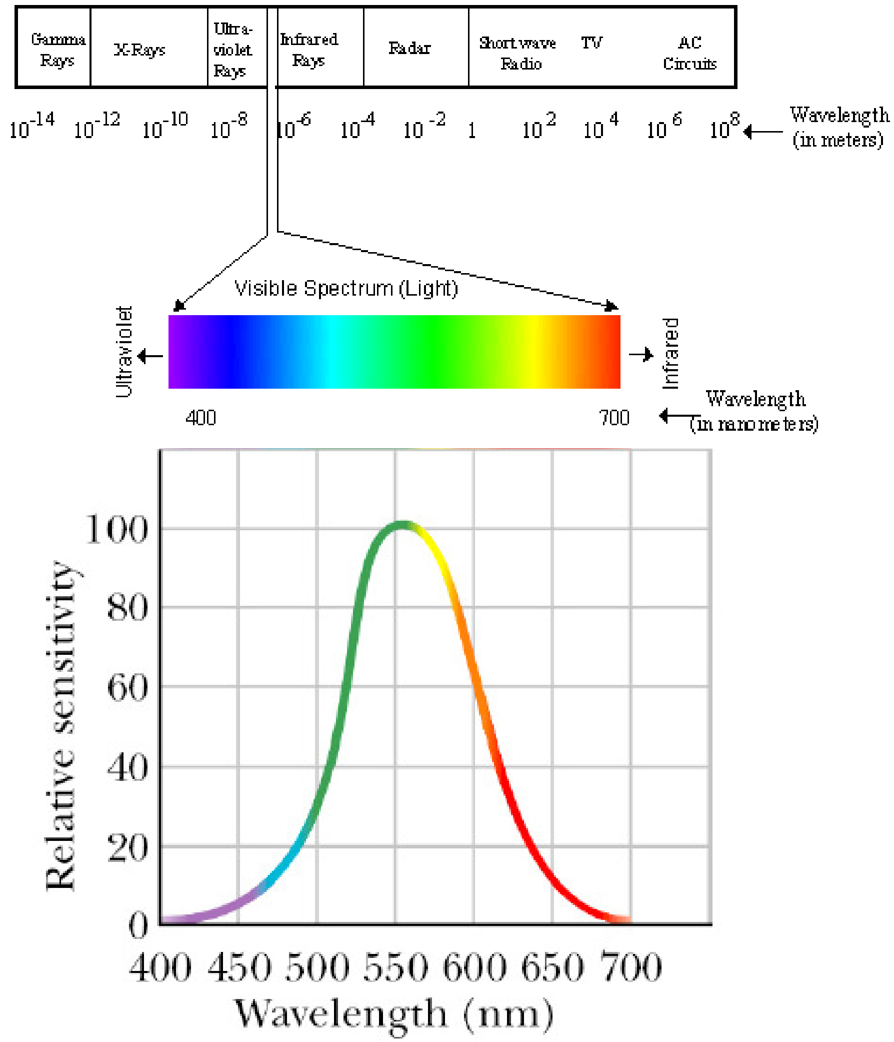
- Harmonic solutions:

$$y(x,t) = y_0 \sin(kx \pm \omega t)$$

- Wavelength, $\lambda = 2\pi/k$ and frequency $f = \omega/2\pi$.
- Velocity, $v = \lambda/f = \omega/k$.

The electromagnetic spectrum

- In 1850, the only known forms electromagnetic waves were ultraviolet, visible and infrared.
- The human eye is only sensitive to a very narrow range of wavelengths:



Discovery of Radio Waves

ELECTRIC WAVES

BEING

RESEARCHES ON THE PROPAGATION OF ELECTRIC ACTION WITH FINITE VELOCITY THROUGH SPACE

BY

DR. HEINRICH HERTZ

PROFESSOR OF PHYSICS IN THE UNIVERSITY OF BONN

AUTHORISED ENGLISH TRANSLATION

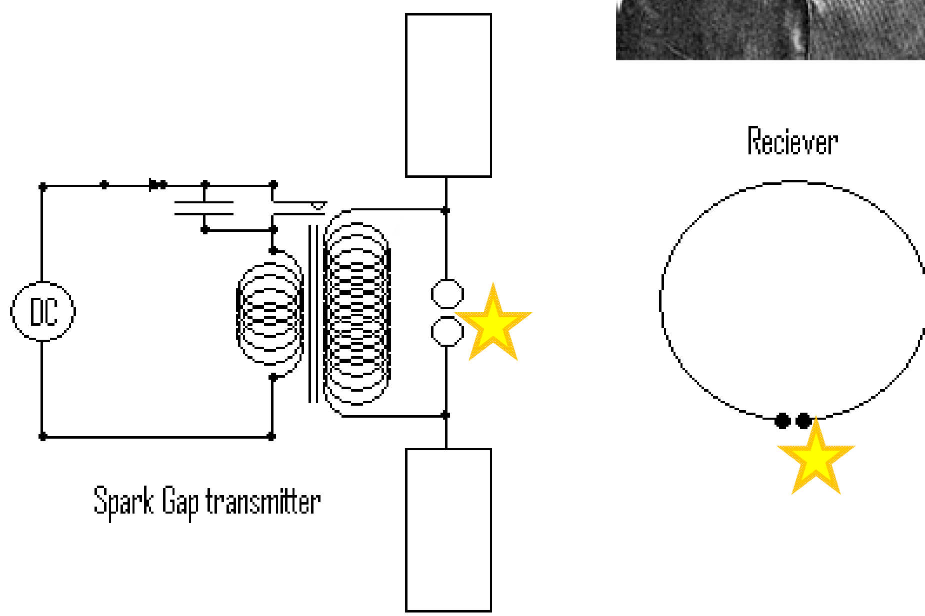
BY D. E. JONES, B.Sc.

DIRECTOR OF TECHNICAL EDUCATION TO THE STAFFORDSHIRE COUNTY COUNCIL
LATELY PROFESSOR OF PHYSICS IN THE UNIVERSITY COLLEGE OF WALES, ABERYSTWYTH

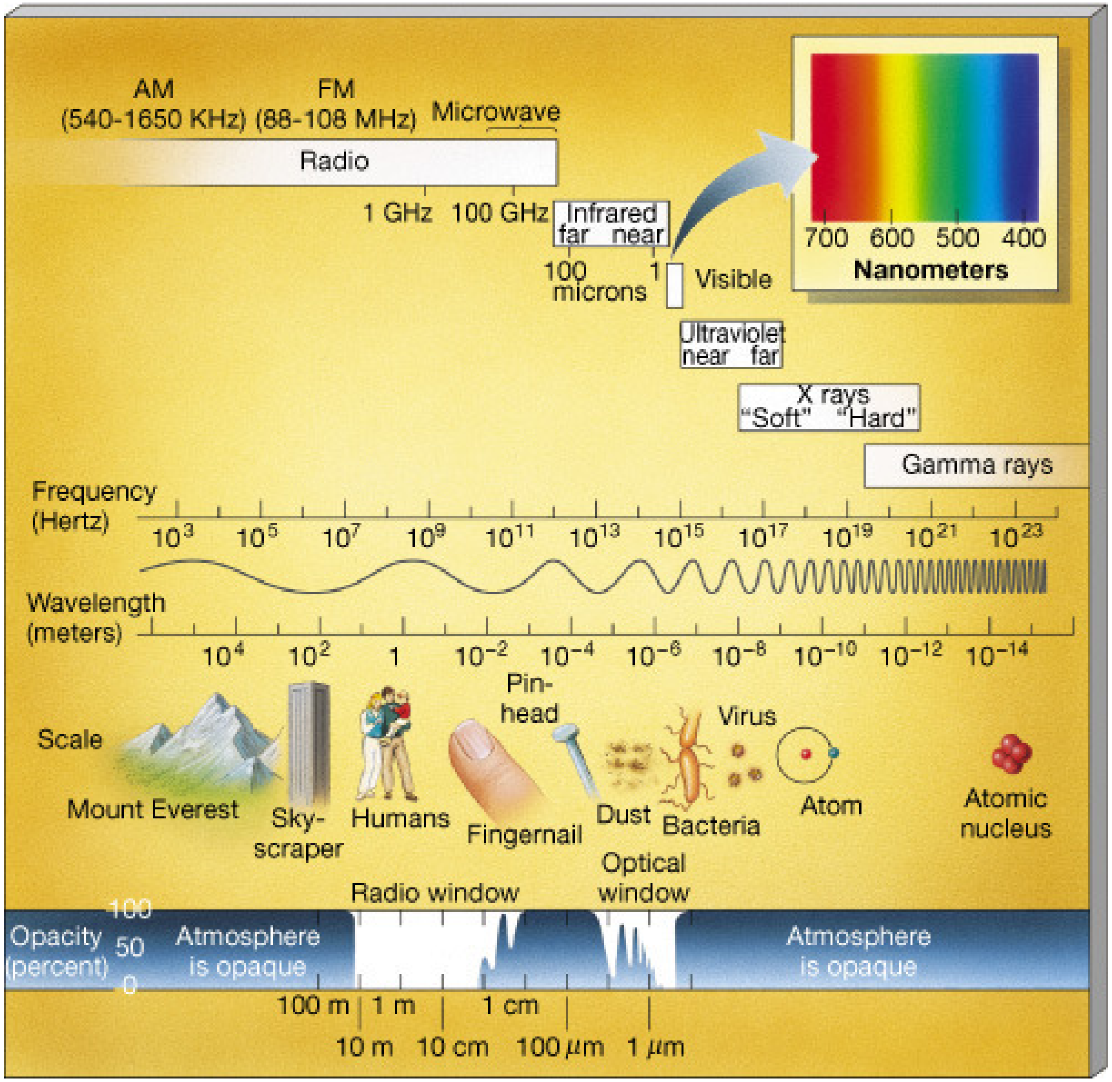
WITH A PREFACE BY LORD KELVIN, LL.D., D.C.L.

PRESIDENT OF THE ROYAL SOCIETY, PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF GLASGOW, AND FELLOW OF ST. PETER'S COLLEGE, CAMBRIDGE

London
MACMILLAN AND CO.
AND NEW YORK
1893



The Electromagnetic Spectrum



Electromagnetic Waves

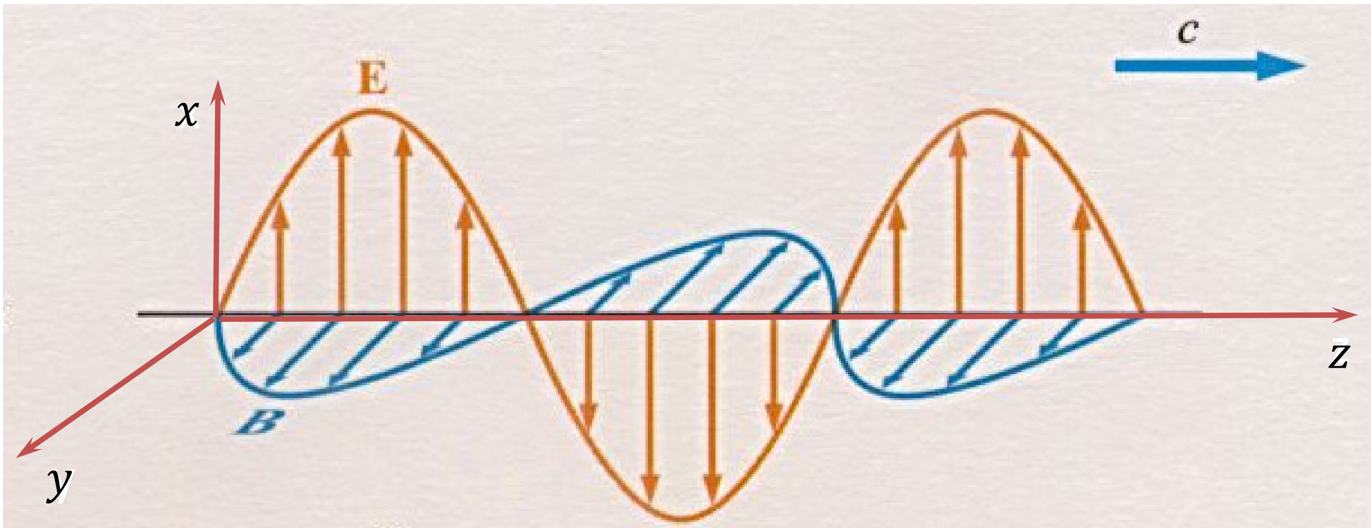
$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

- A solution is $E_x(z,t) = E_0 \sin(kz - \omega t)$ where $\omega = kc = 2\pi c/\lambda$
- What is the magnetic field?

$$\frac{\partial B_y}{\partial t} = -\frac{\partial E_x}{\partial z} = -kE_0 \cos(kz - \omega t)$$

$$B_y(x,t) = \frac{k}{\omega} E_0 \sin(kx - \omega t)$$

Electromagnetic Waves



- $\vec{E}$, $\vec{B}$ and $\vec{v}$ are mutually perpendicular.
- In general, the direction is

$$\hat{s} = \hat{E} \times \hat{B}$$

Energy in Electromagnetic Waves

Energy stored in electric and magnetic fields:

$$u_e = \frac{1}{2}\epsilon_0 E^2 \qquad\qquad u_m = \frac{1}{2\mu_0}B^2$$

For an electromagnetic wave, $B = E/c = E\sqrt{\mu_0 \epsilon_0}$

$$u_m = \frac{1}{2\mu_0}B^2 = \frac{1}{2}\epsilon_0 E^2 = u_e$$

The total energy density is

$$u = u_m + u_e = \epsilon_0 E^2$$

Intensity of Electromagnetic Waves

- Intensity is defined as the average power transmitted per unit area.

Intensity = Energy density × wave velocity

$$I = \epsilon_0 c \langle E^2 \rangle = \frac{\langle E^2 \rangle}{\mu_0 c}$$

$$\mu_0 c = 377\ \Omega \equiv Z_0$$

(Impedance of free space)

Poynting Vector

- We can construct a vector from the intensity and the direction $\hat{s} = \hat{E} \times \hat{B}$:

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\langle \vec{S} \rangle = \frac{\langle E^2 \rangle}{Z_0} = I$$

- This represents the flow of power in the direction $\hat{s}$
- Average electric field: $E_{rms} = E_0/\sqrt{2}$

$$\langle \vec{S} \rangle = \frac{(E_0)^2}{2Z_0}$$

- Units: Watts/m^2