

Physics 42200 Waves & Oscillations

Spring 2016 Semester
Matthew Jones

Preliminary Information

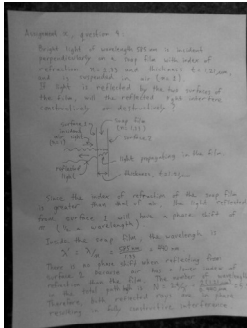
- Course web page:
http://www.physics.purdue.edu/~mjones/phys42200_Spring2016
- Contact information:
 - E-mail: mjones@physics.purdue.edu
 - Office: PHYS 378
 - Phone: 765-496-2464
 - Office hours: Tuesday/Thursday 1 pm-2 pm or by appointment
- Textbooks:
 - A. P. French, Vibrations and Waves (required)
 - E. Hecht, Optics – 4th Edition (highly recommended)
- Lecture:
 - Not guaranteed to cover *all* the material you are responsible for on the homework or exams.
 - *May* cover material you are not responsible for but which will benefit your general education on the subject matter.
 - Read the text first, ask questions in class.

Preliminary Information

- Grading policy:
 - 40% homework
 - 30% midterm evening exam
 - 30% comprehensive final exam
- Homework policy:
 - Assigned on Friday, due the following Friday.
 - At most, one assignment will be dropped when computing final homework grade
 - Corollary #1: You can freely skip one assignment at your discretion
 - Corollary #2: No late homework will be accepted

Homework solutions must include complete, legible explanations of your work. It must be easy for the reader to follow your reasoning. If it takes longer for the grader to figure out your reasoning than it took you to write out your solution then something is wrong!

Assignments



← **Good!**



BAD!
(But I've seen worse)

Assignments

- General advice:
 - In general, we don't care if you can get the right answer or not. You need to explain *why* it is correct.
 - Explain using words and algebra you arrive at a solution. Then substitute in numerical data, if applicable.
 - The algebra should be readable as part of a good English sentence: state what idea or information the formula is used to describe.
 - Symbols that are used without definitions or explanations are usually meaningless.
 - Diagrams can be extremely helpful. Make them large enough to convey the information they need to communicate.
 - Explain your logic! A bunch of formulas are not sufficient to explain the solution.
 - Re-iterate principles learned in class.
 - Your solution should read like an example found in a good text book.
 - Your solution should be useful for studying or reference long after it has been graded.

More Advice about Assignments

- This year, more weight is given to the assignments when calculating the final grade
- I have instructed the graders to pay close attention to the clarity of the solutions:
 - You will probably only get 1/10 on a question if you just write down the correct answer.
 - If you don't understand the material, it will be hard to explain how you arrive at a correct solution.
 - If you can explain your reasoning (and the logic is correct) then you probably understand the material quite well.
 - This is good practice for any future technical writing you will do.
- There are lots of examples on the web from the past two years. We will study *some* of these in class.

Oscillations and Waves

- Why study oscillations and waves?
 - A large fraction of all physical situations involve periodic or oscillatory behavior
 - Motion of the planets
 - Stable mechanical systems
 - Electrical systems
 - Fundamental forces
 - Periodic motion in continuous media
 - Wave propagation
 - Electromagnetic radiation (light/optics)
 - Quantum mechanics
 - Most fundamental description of nature
 - Quantum Mechanics + Special Relativity \rightarrow Quantum Field Theory
 - Matter particles are quantized “waves” in an underlying “field”.

Early Studies of Oscillations



Robert Hooke
(1635-1703)



Christian Huygens
(1623-1697)



Isaac Newton
(1642-1727)

Hooke's Law

“Ut tensio, sic vis!”

As the extension, thus the force!

Extension is proportional to force ($F \propto x$).

In static equilibrium, the net force must vanish.

There are two forces:

1. The force applied to the spring
2. The reaction force applied *by the spring*:

$$F(x) = -k x$$

How to Read

$$F(x) = -k x$$

- We are describing a force...
- The force depends on a displacement, x ...
- The magnitude of the force is proportional to x ...
- The direction of the force is opposite the direction of the displacement...
- This defines the k as the constant of proportionality.

The formula by itself, out of context, conveys very little meaning.

Nevertheless, algebra is very useful for solving equations.

Mechanics

Newton's second law:

Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimitur.

The alteration of motion is ever proportional to the motive force impressed, and is made in the right line in which that force is impressed.

$$\sum \vec{F} = m\vec{a} = m \frac{d^2 \vec{x}}{dt^2}$$

Putting it together...

- Pick a suitable coordinate system so that $\vec{F}(x) = F(x)\hat{i}$
- When the only force is described by Hooke's law, $F(x) = -k x$

the resulting acceleration is given by Newton's law:

$$F(x) = -k x = m \frac{d^2 x}{dt^2} = m \ddot{x}$$

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

But what is the resulting motion?

More notation you should get used to...

Oscillating Systems

- Differential equation:

$$m \frac{d^2x}{dt^2} + kx = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\omega^2 = k/m$$

- Does a solution exist?
 - Can you find a function $x(t)$ that you can differentiate twice and substitute in to solve this equation?
- Is it unique?
 - Are there other (different) functions that are also solutions?

Oscillating Systems

- Try out this solution:

$$x(t) = \sin \omega t$$

- Differentiate it once:

$$\frac{dx}{dt} = \omega \cos \omega t$$

- Differentiate it again:

$$\frac{d^2x}{dt^2} = -\omega^2 \sin \omega t$$

- Substitute it in... does it solve the equation?

$$\frac{d^2x}{dt^2} + \omega^2 x = (-\omega^2 \sin \omega t) + (\omega^2 \sin \omega t) = 0$$

- Yes!

Oscillating Systems

- Try out another solution:

$$x(t) = \cos \omega t$$

- Differentiate it once:

$$\frac{dx}{dt} = -\omega \sin \omega t$$

- Differentiate it again:

$$\frac{d^2x}{dt^2} = -\omega^2 \cos \omega t$$

- Substitute it in... does it solve the equation?

$$\frac{d^2x}{dt^2} + \omega^2 x = (-\omega^2 \cos \omega t) + (\omega^2 \cos \omega t) = 0$$

- Yes!

Oscillating Systems

- In general, any function that is of the form

$$x(t) = A \sin \omega t + B \cos \omega t,$$

where **A** and **B** are real numbers, will be a solution.

- There are other ways to write this:

$$x(t) = A \cos(\omega t + \varphi)$$

- What if we aren't restricted to real numbers?

Oscillating Systems

- Complex numbers: $i^2 = -1$

- Try this solution:

$$x(t) = e^{i\omega t}$$

- Differentiate it once:

$$\frac{dx}{dt} = i\omega e^{i\omega t}$$

- Differentiate it again:

$$\frac{d^2x}{dt^2} = (i\omega)^2 e^{i\omega t} = -\omega^2 e^{i\omega t}$$

- Substitute it in... does it solve the equation?

$$\frac{d^2x}{dt^2} + \omega^2 x = (-\omega^2 e^{i\omega t}) + (\omega^2 e^{i\omega t}) = 0$$

- General solution: $x(t) = c e^{i\omega t}$, for any complex constant **c**.

Oscillating Systems

- Differential equation:

$$m \frac{d^2x}{dt^2} + kx = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

- Solutions:

$$x(t) = A \cos(\omega t + \varphi)$$

$$x(t) = A \sin \omega t + B \cos \omega t$$

- The solutions are periodic with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{k/m}$$

Other Dynamical Variables

- The position, x , is used to describe the “state” of the system:
 - If you know x and \dot{x} at one time, then you can predict what they will be at any other time.
- Other systems can be described using other dynamical variables:
 - Angles for rotating mechanical systems
 - Voltage or current in electrical systems
 - Electric or magnetic fields (*eg*, light!)

Other Oscillating Systems

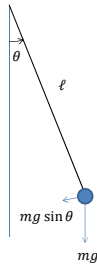
- A pendulum:

$$m\ell\ddot{\theta} = -mg \sin \theta \approx -mg\theta$$

$$\ddot{\theta} + \omega^2 \theta = 0,$$

$$\omega = \sqrt{g/\ell}$$

$$\theta(t) = A \cos(\omega t + \varphi)$$



Other Oscillating Systems

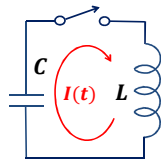
- An electric circuit:

$$\frac{1}{C}I + L \frac{d^2 I}{dt^2} = 0$$

$$\ddot{I} + \omega^2 I = 0,$$

$$\omega = \sqrt{1/LC}$$

$$I(t) = A \cos(\omega t + \varphi)$$



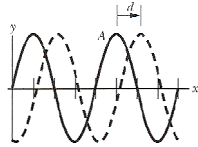
Summary so far...

$$\frac{d^2x}{dt^2} + \omega^2x = 0$$

- The first half of the course is only concerned with the solutions to this type of differential equation.
- How to solve it is not very interesting.
- Of more importance is what the solutions tell us about the behavior of various physical systems.

Other Oscillating Systems

- Waves:
 - Amplitude is a function of both x and t



- Wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

- A solution can be written:

$$y(x, t) = A \cos(kx - \omega t + \varphi)$$

$$v = \omega/k$$

Is light a wave?

Descriptions of Light



Christian Huygens
(1623-1697)

Promoted the "wave theory" of light.



Isaac Newton
(1642-1727)

Described light as particles that moved in straight lines.

Electromagnetic Radiation

By 1864, Maxwell had introduced four equations that described all known electromagnetic phenomena.

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\phi_e}{dt}$$



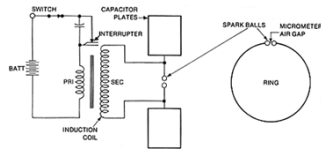
"An changing electric field induces a magnetic field and a changing magnetic field induces an electric field..."

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.998 \times 10^8 \text{ m/s}$$

Observation of Electromagnetic Radiation



Heinrich Hertz
(1857-1894)



"It's of no use whatsoever ... this is just an experiment that proves Maestro Maxwell was right—we just have these mysterious electromagnetic waves that we cannot see with the naked eye. But they are there."

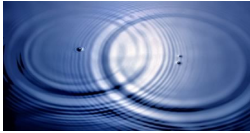
Asked about the ramifications of his discoveries, Hertz replied, "Nothing, I guess."

Modern Physics

- Maxwell's Equations could not explain:
 - Spectrum of light from blackbody radiation
 - Photoelectric effect
 - Speed of light is the same in all reference frames
- Quantum Mechanics:
 - Planck proposed that the energy of light was quantized, $E = h\nu$
 - Einstein explained the photoelectric effect: light was composed of particles with $E = h\nu$
 - Special theory of relativity: speed of light is the same in all reference frames.

Quantum Field Theory

- All particles are quantized excitations of a *field*.
 - Photons are quantized excitations of the electromagnetic field.
 - Electrons are quantized excitations of the “electron field”.
 - Explains particle creation and annihilation.



- A natural mathematical consequence: the Higgs field.
- This seems to be what was directly observed in 2012...
- But in this course we will only consider classical waves.
