PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

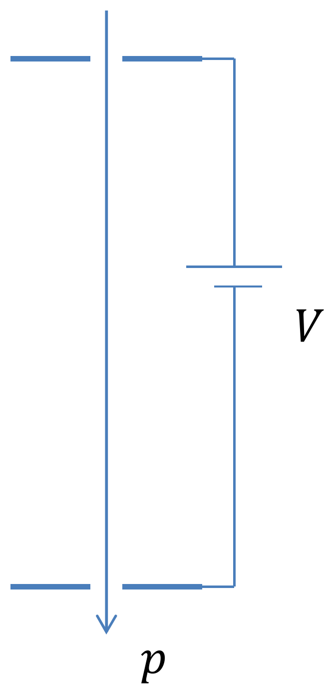
Lecture 9 – French, Chapter 4

Spring 2015 Semester

Matthew Jones

Resonance in Nuclear Physics

- A proton accelerated through a potential difference V gains kinetic energy $T = eV$:



Phys. Rev. 75, 246 (1949).

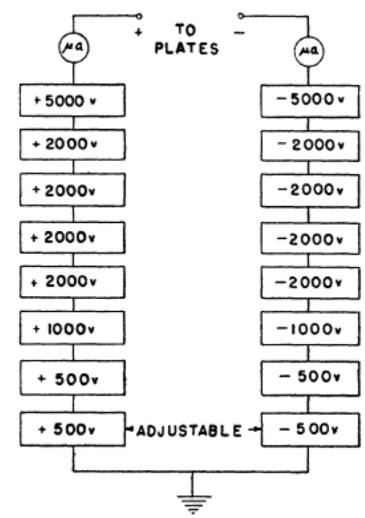


FIG. 1. Block diagram of battery stacks. Adjustable 500-volt boxes were set to any voltage below 500 volts by means of a potentiometer. The polarity of any of the boxes except the adjustable 500-volt box could be selected at will for comparison purposes.

* 5000 volt boxes used Eveready No. 493, 300 volt batteries. All other batteries were of the Burgess XX45, 67½ volt type except for several heavier duty batteries under continuous drain to provide continuous range of adjustments.

** The actual voltage is 504.08 Int. volts and is determined by the resistor divider ratio and the 1.50000 volt setting on the potentiometer.

Resonance in Nuclear Physics

- In quantum mechanics, energy and frequency are proportional:

$$E = \hbar\omega$$

- A given energy corresponds to a driving force with frequency ω.
- When a nucleus resonates at this frequency, the proton energy is easily absorbed.

Nuclear Resonance

ABSOLUTE VOLTAGE DETERMINATION 255

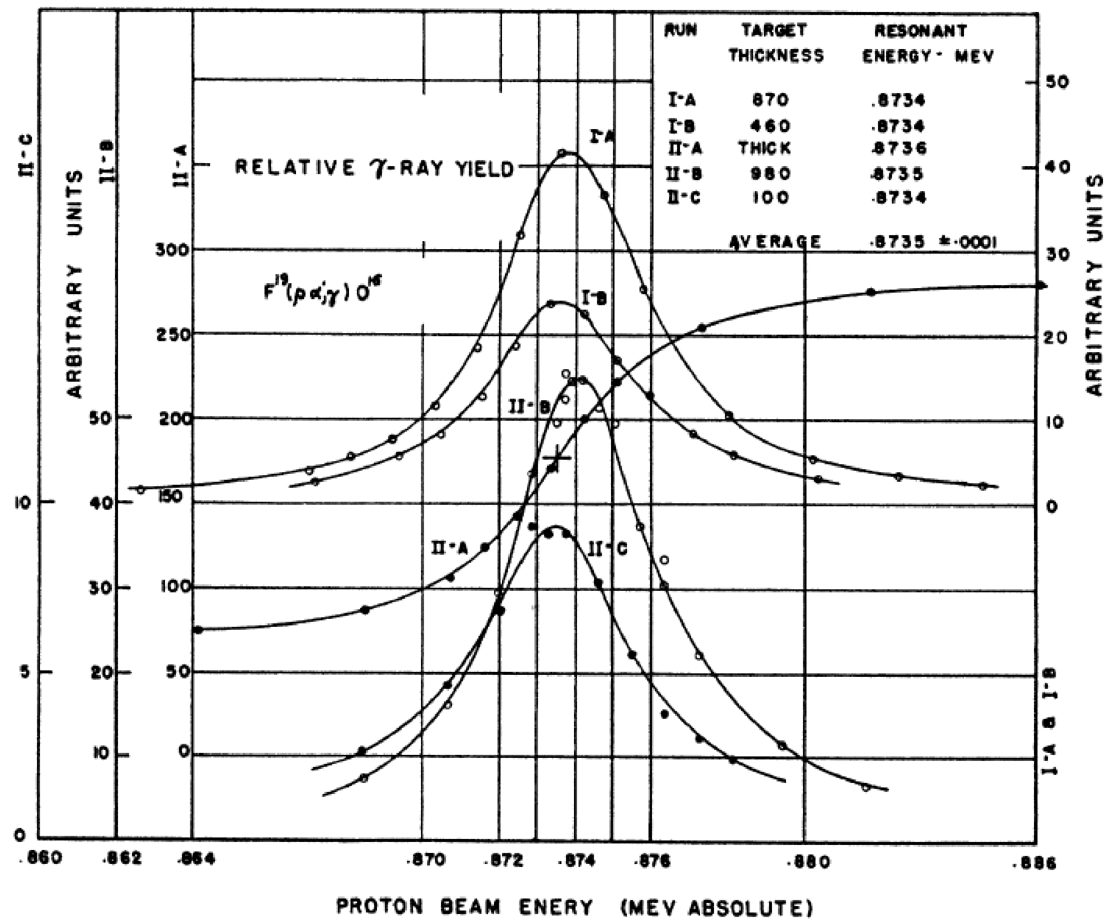
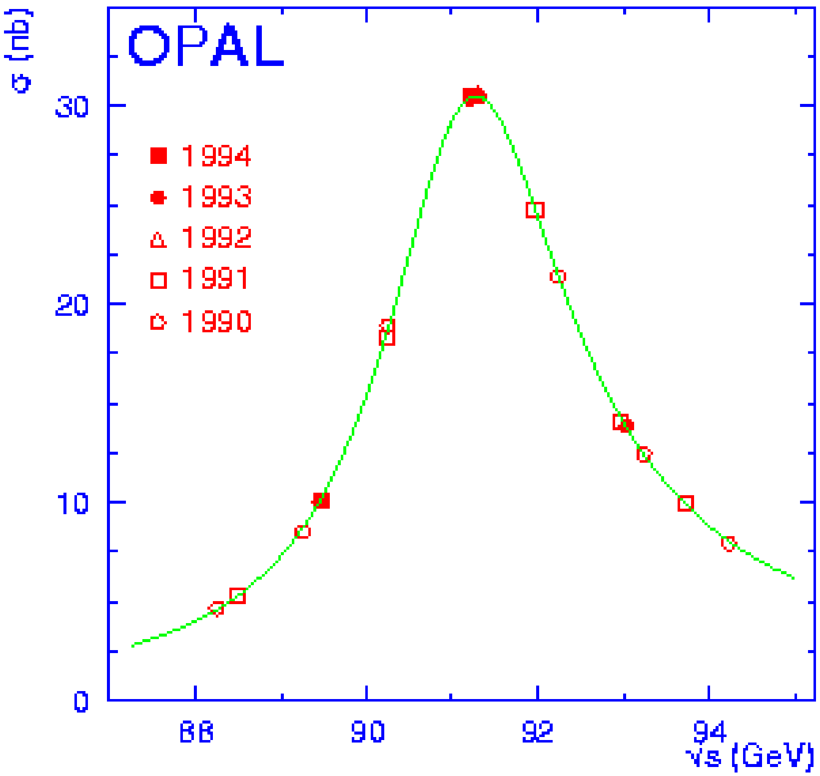
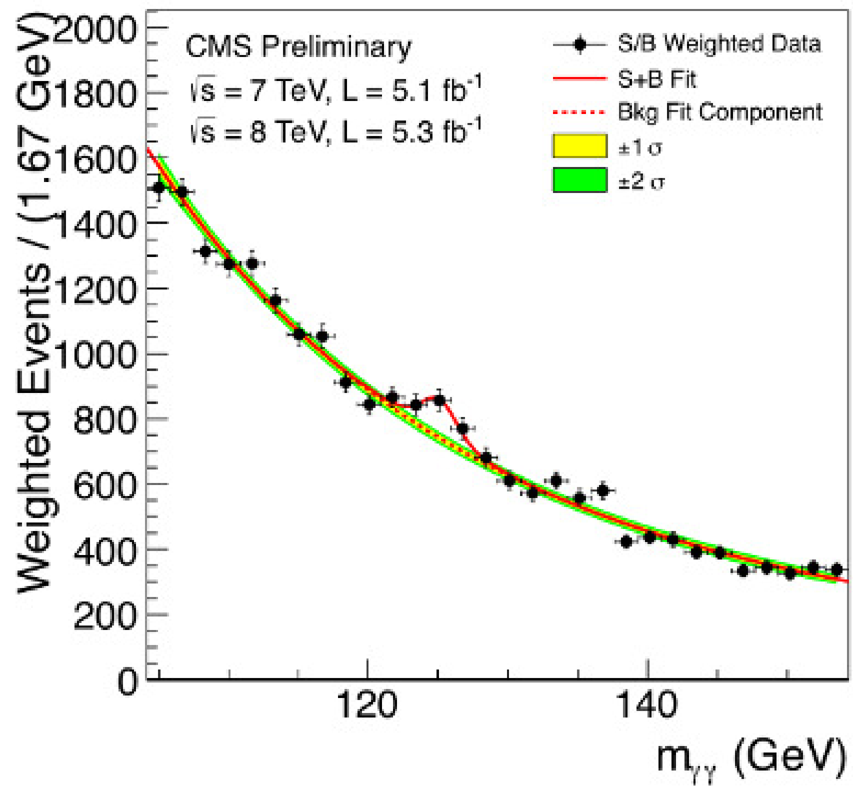


FIG. 7. γ-ray yield curves for both series of measurements of F(pγ) resonance reaction. Yield values are all on same relative scale.

"Lifetime" is defined in terms of the width of the resonance.

Resonance

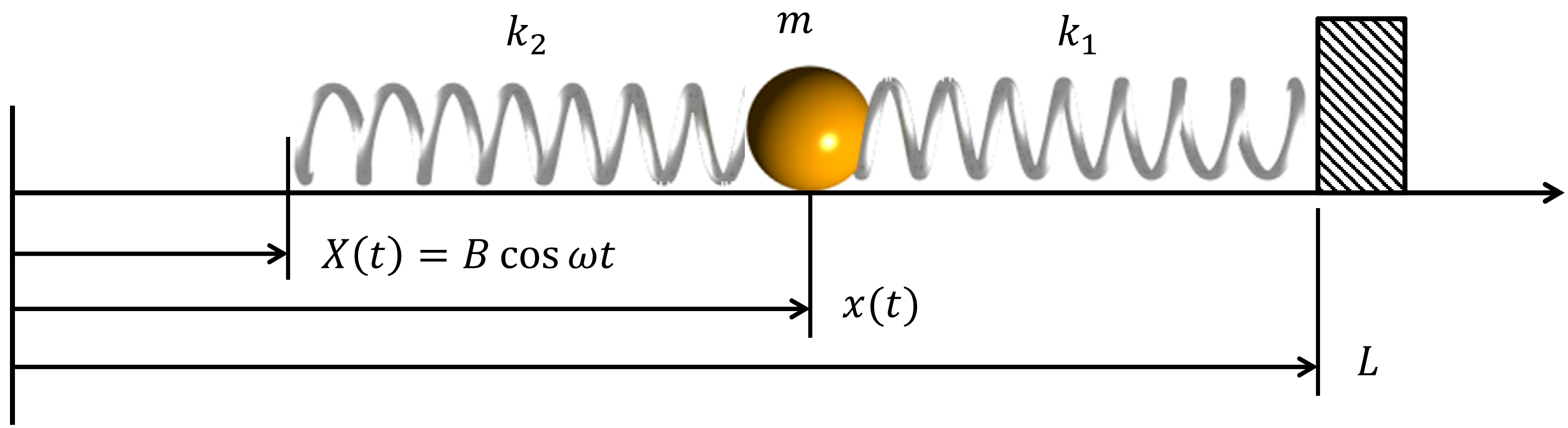




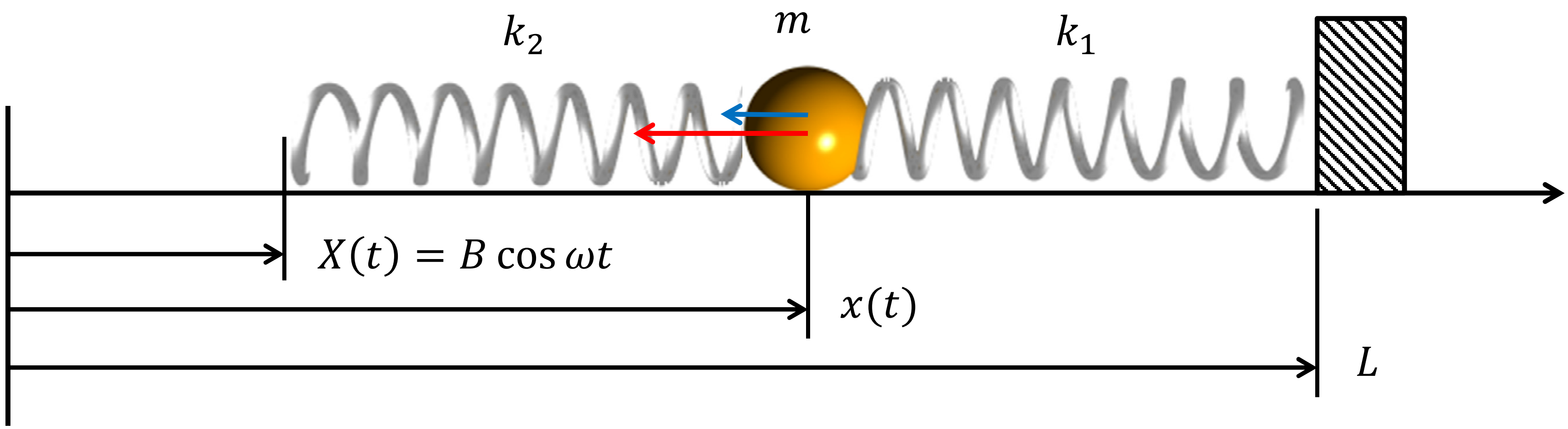
- Resonances are the main way we "observe" fundamental particles.

Forced Harmonic Motion

- In practice, how can we apply such a force to a mechanical system?
- Example:



Forced Harmonic Motion



- Force acting on the mass:

$$F = -k_2\big(x(t) - X(t)\big) - k_1\, x(t)$$
$$= -(k_1 + k_2)x(t) + k_2 X(t)$$
$$= -(k_1 + k_2)x(t) + k_2 B\ \cos\omega t$$
$$= m\frac{d^2x}{dt^2}$$

Forced Harmonic Motion

- Let's write this in the standard form:

$$-(k_1 + k_2)x(t) + k_2 B\ \ cos\, \omega t = m\frac{d^2x}{dt^2}$$

$$m\,\ddot{x} + (k_1 + k_2)x = k_2 B\ \ \cos \omega t$$

$$\boldsymbol{\ddot{x} + \omega_0^2 x = \frac{F_0}{m}\cos \omega t}$$

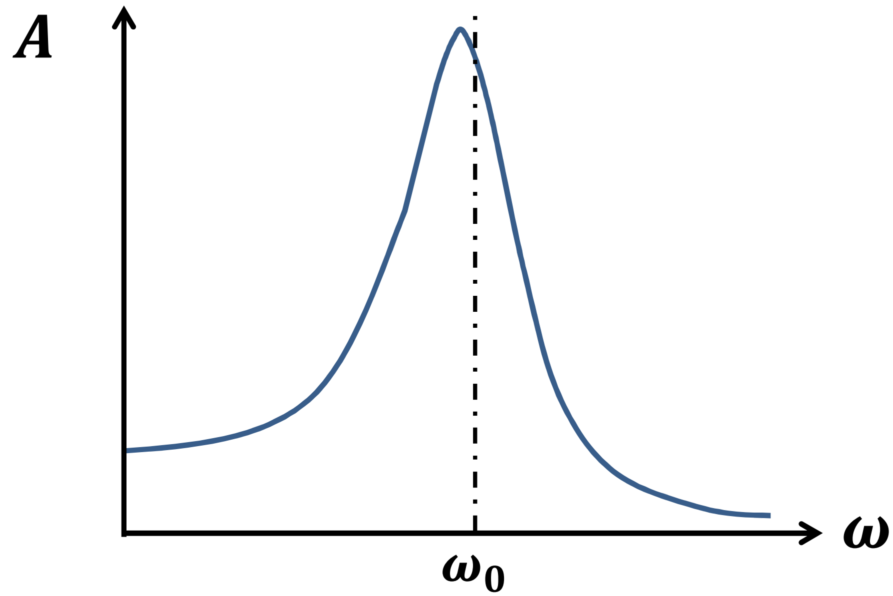
where

$$\omega_0^2 = \frac{k_1 + k_2}{m}$$

$$F_0 = k_2 B$$

Forced Harmonic Motion

$$A = \frac{F_0/m}{\sqrt{\left((\omega_0)^2 - \omega^2\right)^2 + (\omega\gamma)^2}}$$



Non-Inertial Reference Frames

- An accelerated reference frame is one way to provide a periodic driving force.
- Analyzing mechanical systems in accelerated reference frames can be very convenient.
- An accelerated reference frame is an example of a non-inertial reference frame.
- Let’s analyze the motion of a particle in an accelerated reference frame…

Non-Inertial Reference Frame

- Newton's second law applies to forces and motion in an inertial reference frame:

$$F = m\frac{d^2x}{dt^2}$$

- The force is a real force (eg, Coulomb's law)
- An inertial reference frame is one in which Newton's second law applies.
  - For example, a "stationary" reference frame or one that moves with constant velocity.
  - This is sort of a circular argument but it is still useful.

Non-Inertial Reference Frames

- It is frequently very convenient to measure the position of an object with respect to some moving reference frame.
- Example:
  - The position of your coffee cup relative to some fixed point on the dashboard of your car.
  - The only real forces are gravity and the normal force which acts in the vertical direction.
  - But your car is *not* an inertial reference frame and Newton's second law does not apply!

Non-Inertial Reference Frames

- How do we analyze this problem?
- Express the position of the object in an inertial reference frame:

$$x(t) = X(t) + u(t)$$

  - $x(t)$ is the position of the object in the inertial reference frame
  - $X(t)$ is the position of the non-inertial reference frame, measured in the inertial reference frame
  - $u(t)$ is the position of the object in the non-inertial reference frame.

Non-Inertial Reference Frames

- Example:
  - The non-inertial reference frame is your car.
  - $u(t)$ is the position of the coffee cup in your car.
    - Let $u(t) > 0$ be towards the windshield
    - Then $u(t) < 0$ is in your lap… (foreshadowing)
  - $X(t)$ is the position of your car measured along Northwestern Ave, with $X(t) = 0$ located just outside the main entrance to the Physics building.
  - $x(t)$ is the position of the coffee cup in the inertial reference frame, but it is difficult to understand the resulting motion in terms of $x(t)$.

Non-Inertial Reference Frames

- Newton's second law only applies to $x(t)$:

$$F = m\frac{d^2x}{dt^2}$$

- Write this in terms of $X(t)$ and $u(t)$:

$$x(t) = X(t) + u(t)$$

$$F = m\frac{d^2X}{dt^2} + m\frac{d^2u}{dt^2}$$

- Write this so that it "looks" like Newton's second law, but in the non-inertial reference frame:

$$F - m\frac{d^2X}{dt^2} = m\frac{d^2u}{dt^2}$$

Non-Inertial Reference Frames

$$F - m\frac{d^2X}{dt^2} = m\frac{d^2u}{dt^2}$$

The term $\left(-m\frac{d^2X}{dt^2}\right)$ looks like a force but it is not a real force. It is sometimes called a "fictitious" force.

- Example:
  - Suppose you accelerate with constant acceleration a in the $+X$ direction.
  - The only "real" force in the $+u$ direction is the force of friction acting on the coffee cup.
  - The equation of motion for $u(t)$:

$$-m\,a + \mu m g = m\frac{d^2u}{dt^2}$$

Non-Inertial Reference Frames

$$-m\,a + \mu m g = m\frac{d^2u}{dt^2}$$

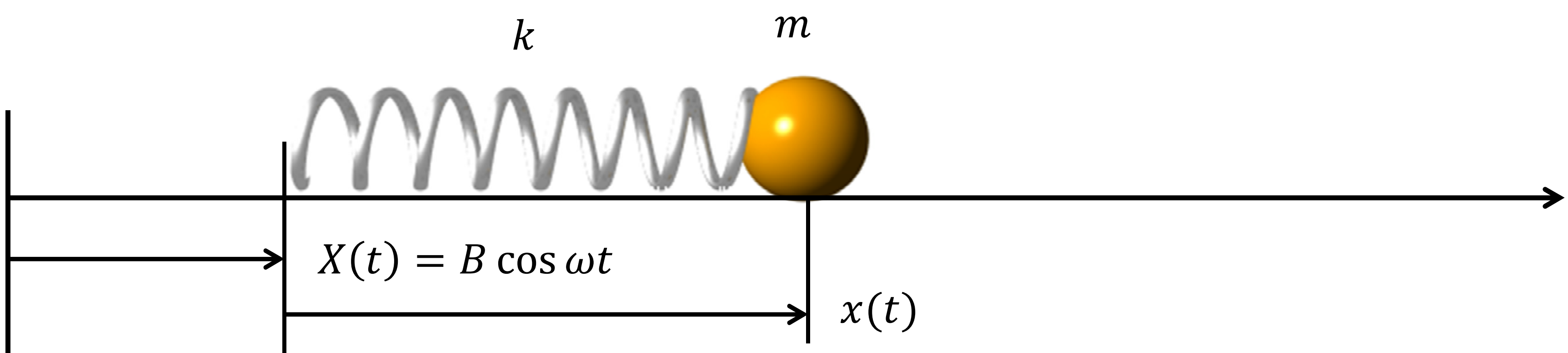
When $ma > \ \mu m g$ the net acceleration will be negative and the coffee cup will land in your lap.

- The coffee cup was not pushed!
  - It tried to stay motionless but the car accelerated.
  - If the coefficient if friction was large enough, then the force of static friction would have caused the coffee cup to accelerate with the car.

$$(\mu m g = ma)$$

Non-Inertial Reference Frames

- Applied to forced harmonic motion:



- $x(t)$ is the position in the non-inertial reference frame
- The only real force is $F = -k\,x(t)$
- Equation of motion in the inertial reference frame:

$$F = m\frac{d}{dt^2}\left(X(t) + x(t)\right)$$

Non-inertial Reference Frames

$$F = m\frac{d}{dt^2}(X(t) + x(t))$$

$$F = -k\,x(t)$$

$$X(t) = B\ \cos\omega t$$

$$\frac{d^2X}{dt^2} = -B\ \omega^2\ \cos\omega t$$

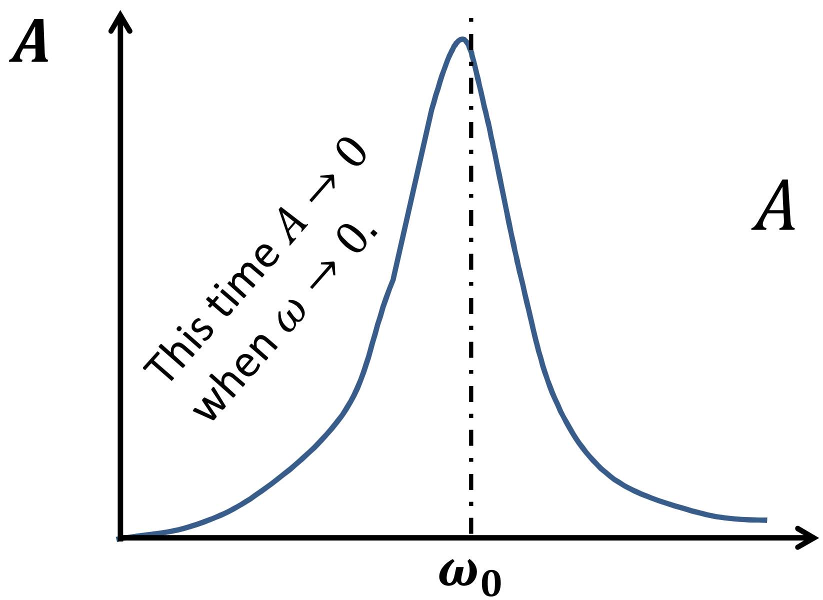
In the standard form:

$$\ddot{x} + \omega_0^2 x = -\frac{d^2X}{dt^2} = B\omega^2\cos\omega t = \frac{F_0}{m}\cos\omega t$$

where

$$F_0 = m\ B\ \omega^2$$

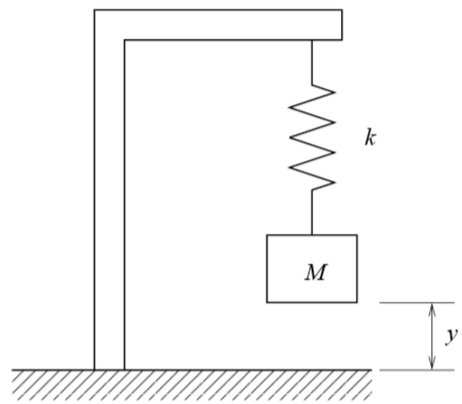
Non-inertial Reference Frames



$$A = \frac{B\ \omega^2}{\sqrt{\left((\omega_0)^2 - \omega^2\right)^2 + (\omega\gamma)^2}}$$

Application

2. (*French, 4-6*) Imagine a simple seismograph consisting of a mass M hung from a spring on a rigid framework attached to the earth, as shown:



Spring 2013 Assignment #3.

The spring force and the damping force depend on the displacement and velocity relative to the earth's surface, but this is not an inertial reference frame if its surface is moving which would be the case in the event of an earthquake.

- Let $\eta(t)$ be the height of the surface of the earth in an inertial reference frame.
- Then, $x(t) = y(t) + \eta(t)$ is the height of the mass, in the inertial reference frame.

$$m\,\ddot{x} + b\,\dot{x} + k\,x = -m\,d^2\eta/dt^2.$$