

Physics 42200
Waves & Oscillations

Lecture 39 – Review

Spring 2015 Semester

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Final Exam

Date: Tuesday, May 5th

Time: 1:00 to 3:00 pm

Room: Phys 112

You can bring two double-sided pages of notes/formulas.

Bring something to write with.

You shouldn't need a calculator.

Today's Review of Optics

- Polarization
 - Reflection and transmission
 - Linear and circular polarization
 - Stokes parameters/Jones calculus
- Geometric optics
 - Laws of reflection and refraction
 - Spherical mirrors
 - Refraction from one spherical surface
 - Thin lenses
 - Optical systems (multiple thin lenses, apertures, stops)
 - Thick lenses, ray tracing, transfer matrix
 - Aberrations

Wednesday's Review of Optics

- Interference
 - Double-slit experiments
 - Fresnel's double-mirror, double-prism, Lloyd's mirror
 - Thin films, Michelson interferometer
 - Multiple beam interferometry, Fabry-Perot interferometer
- Diffraction
 - Fraunhofer diffraction
 - Single-slit, multiple-slit diffraction
 - Diffraction gratings
 - Fresnel diffraction

Polarization

- General representation:

$$\vec{E}(z, t) = \vec{E}_0 \cos(kz - \omega t + \xi)$$

- Light intensity: $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$, $I = \langle \vec{S} \rangle = \epsilon_0 c \langle E^2 \rangle = \frac{\epsilon_0 c}{2} |E_0|^2$

- Linear polarization:

- $\vec{E}_0$ is constant

- Circular polarization

- Orthogonal components out of phase by $\xi = \pm \frac{\pi}{2}$

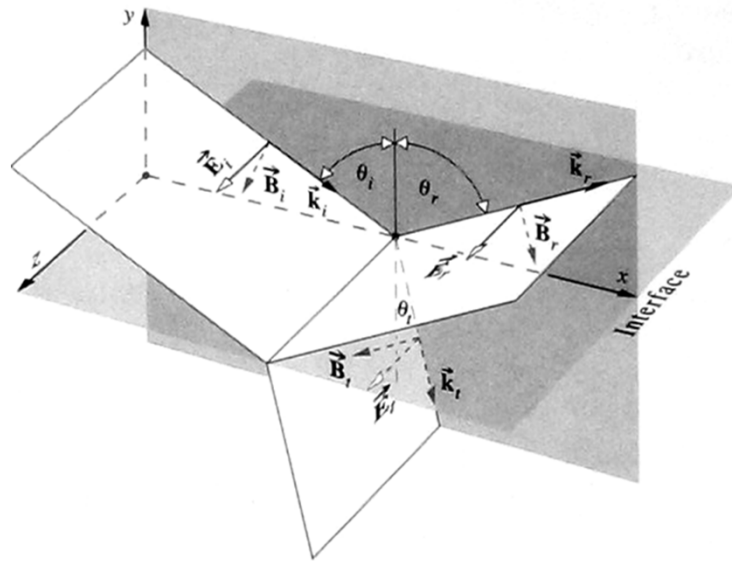
$$\vec{E}(z, t) = E_0(\hat{i} \cos(kz - \omega t) \pm \hat{j} \sin(kz - \omega t))$$

- Right circular polarization: +
 - Left circular polarization: -

- Degree of polarization: $V = I_p / (I_p + I_n)$ or $V = \frac{I_h - I_v}{I_h + I_v}$

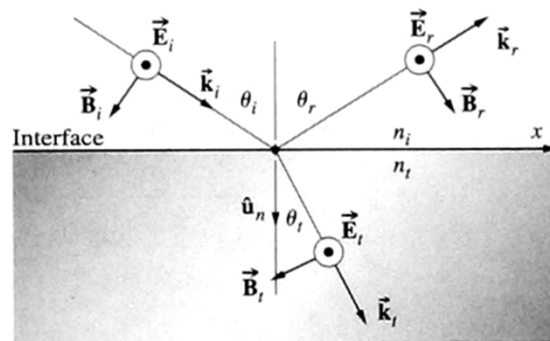
Polarization

- Make sure you understand the following geometry:



$E_{\perp}$ is the component of $\vec{E}$ that is perpendicular to plane of incidence

$E_{\parallel}$ is the component of $\vec{E}$ that is parallel to plane of reflection



In this example, $\vec{E}$ only has an $E_{\perp}$ component ($E_{\parallel} = 0$) while $\vec{B}$ only has a $B_{\parallel}$ component ($B_{\perp} = 0$).

Polarization

- Fresnel equations:

$$\left(\frac{E_r}{E_i}\right)_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$\left(\frac{E_r}{E_i}\right)_{\parallel} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\left(\frac{E_t}{E_i}\right)_{\perp} = \frac{(n_1/n_2)\sin(2\theta_i)}{\sin(\theta_i + \theta_t)}$$

$$\left(\frac{E_t}{E_i}\right)_{\parallel} = \frac{2 \cos(\theta_i) \sin(\theta_t)}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)}$$

- You have to calculate θ_t using Snell's law.

Polarization

- Brewster's angle, θ_B : $\theta_i + \theta_t = 90^\circ$
 - $r_{\parallel} \rightarrow 0$ when $\theta_i \rightarrow \theta_B$ (polarization by reflection)
- When light is at normal incidence,

$$\theta_i = \theta_t = 0 \text{ and } E_{\parallel} = 0...$$

$$r_{\perp} = \left(\frac{E_r}{E_i} \right)_{\perp} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t_{\perp} = \left(\frac{E_t}{E_i} \right)_{\perp} = \frac{2n_1}{n_1 + n_2}$$

- When $n_1 < n_2$, $r_{\perp} = -1$ (phase changes by π).

Polarization

- Remember that the reflected and transmitted intensities depend on the square of r and t .
- Also remember that intensity depends on ν

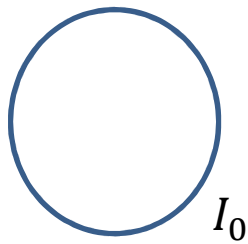
$$I = \langle \vec{S} \rangle = \epsilon_0 \nu \langle E^2 \rangle = \frac{\epsilon_0 c}{n} \langle E^2 \rangle$$

- Energy conservation:

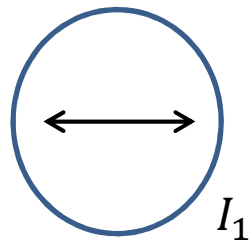
$$R = \frac{I_r}{I_i} = r^2 \quad T = \frac{I_t \cos \theta_t}{I_i \cos \theta_i} = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} t^2$$

Stokes Parameters

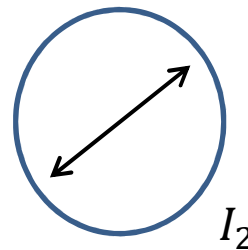
- Stokes selected four filters
- Each filter transmits exactly half the intensity of unpolarized light



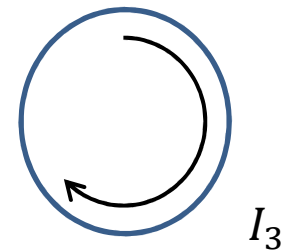
Unpolarized:
filters out $\frac{1}{2}$ the
intensity of any
incident light.



Linear: transmits
only horizontal
component



Linear: transmits
only light
polarized at 45°



Circular:
transmits only R-
polarized light

$$\left. \begin{aligned} S_0 &= 2I_0 \\ S_1 &= 2I_1 - 2I_0 \\ S_2 &= 2I_2 - 2I_0 \\ S_3 &= 2I_3 - 2I_0 \end{aligned} \right\} \text{Stokes parameters}$$

Stokes Parameters

- The Stokes parameters that describe a mixture of polarized light components is the weighted sum of the Stokes parameters of each component.
- Example: Two components
 - 40% has vertical linear polarization
 - 60% has right circular polarization

- Calculate Stokes parameters:

$$S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = 0.4 \times \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + 0.6 \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.4 \\ 0 \\ 0.6 \end{bmatrix}$$

- Degree of polarization:

$$V = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} = \sqrt{(0.4)^2 + (0.6)^2} = 0.72$$

Mueller Matrices

$$S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \rightarrow S' = MS$$

Example: right circular polarizer

- Incident unpolarized light

$$S_0 = 1, S_1 = 0, S_2 = 0, S_3 = 0$$

- Emerging circular polarization

$$S_0 = \frac{1}{2}, S_1 = 0, S_2 = 0, S_3 = \frac{1}{2}$$

- Mueller matrix:

$$M = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Jones Calculus

- Applies to $\vec{E}$, not intensity
 - Light must be coherent

- Electric field vectors:

$$\vec{E}_x(z, t) = E_{0x} \hat{i} \cos(kz - \omega t + \varphi_x)$$

$$\vec{E}_y(z, t) = E_{0y} \hat{j} \cos(kz - \omega t + \varphi_y)$$

- Jones vector:

$$\tilde{E} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0y} e^{i\varphi_y} \end{bmatrix} \Rightarrow \vec{E} = \frac{1}{\sqrt{E_{0x}^2 + E_{0y}^2}} \begin{bmatrix} E_{0x} \\ E_{0y} e^{i\xi} \end{bmatrix}$$

Jones Calculus

- Examples:

- Horizontal linear polarization: $\vec{E}_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

- Vertical linear polarization: $\vec{E}_y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

- Linear polarization at 45° : $\vec{E}_{45^\circ} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

- Right circular polarization: $\vec{E}_R = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ e^{-i\pi/2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$

- Left circular polarization: $\vec{E}_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ e^{+i\pi/2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$

Jones Calculus

- $\vec{E'}$ and $\vec{E}$ are related by a 2x2 matrix (the Jones matrix):

$$\vec{E'} = A \vec{E}$$

- If light passes through several optical elements, then

$$\vec{E'} = A_n \cdots A_2 A_1 \vec{E}$$

- Examples:

- Transmission through an optically inactive material:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

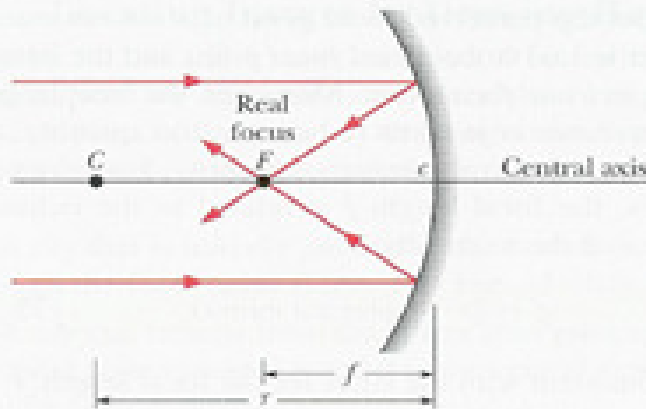
- Rotation of the plane of linear polarization (eg, propagation through a sugar solution)

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Geometric Optics

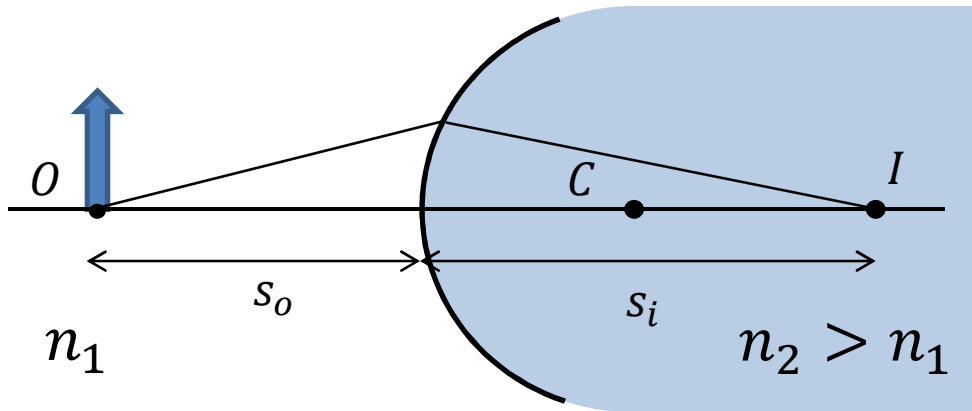
- Law of reflection: $\theta'_1 = \theta_1$
- Law of refraction (Snell's law): $n_1 \sin \theta_1 = n_2 \sin \theta_2$
- Make sure you understand the geometry and sign conventions for lenses and mirrors:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{where } f = -\frac{r}{2} \text{ (for a concave mirror)}$$



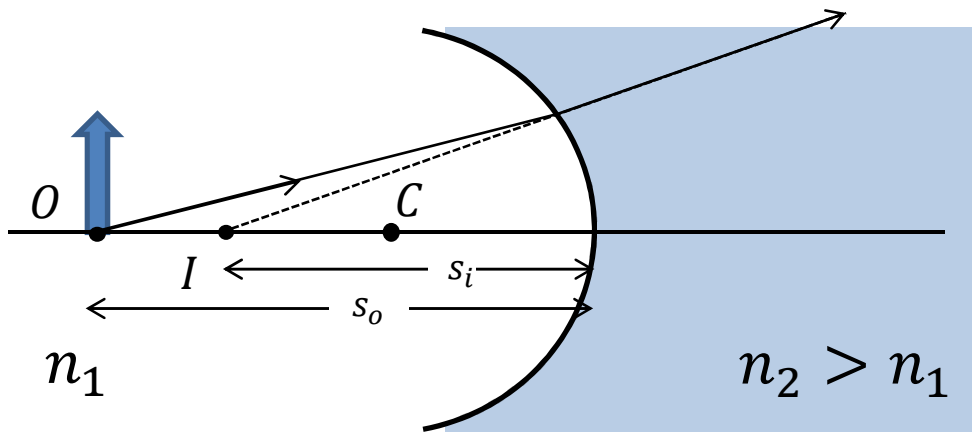
In this diagram, r is negative, but f , s and s' are all positive.

Spherical Refracting Surfaces



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

All quantities are positive here.



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

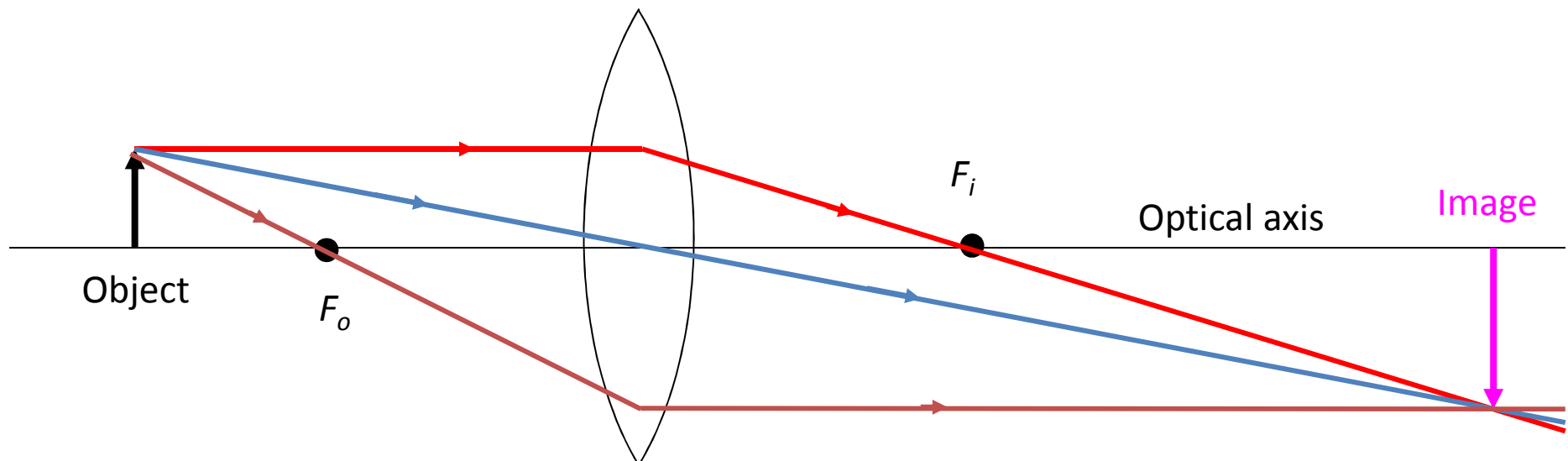
(same formula but now $R < 0$)

Thin Lenses

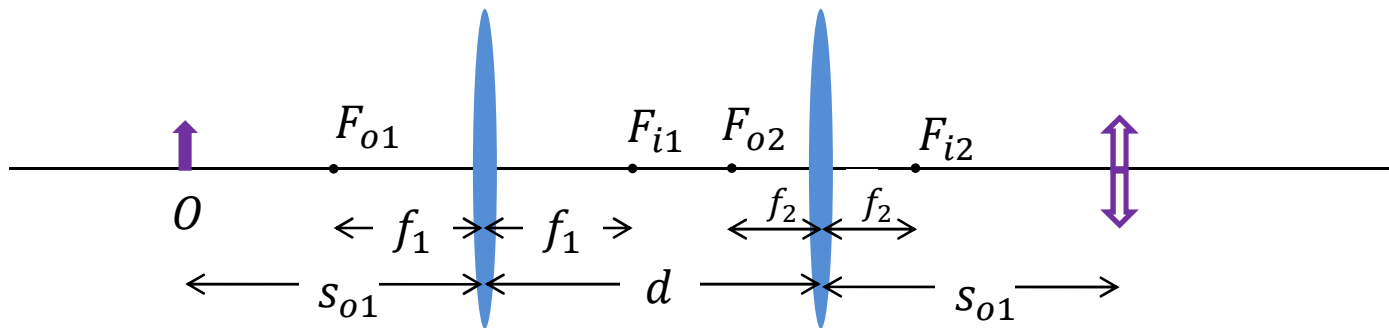
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}$$

$$m_T = -s_i/s_o \quad (\text{transverse magnification})$$

- You should be able to calculate image positions and draw ray diagrams:



Multiple Thin Lenses



- Two techniques:
 - Calculate position of intermediate image formed by first lens, then the final image formed by second lens
 - Use front/back focal lengths:

- Front focal length:

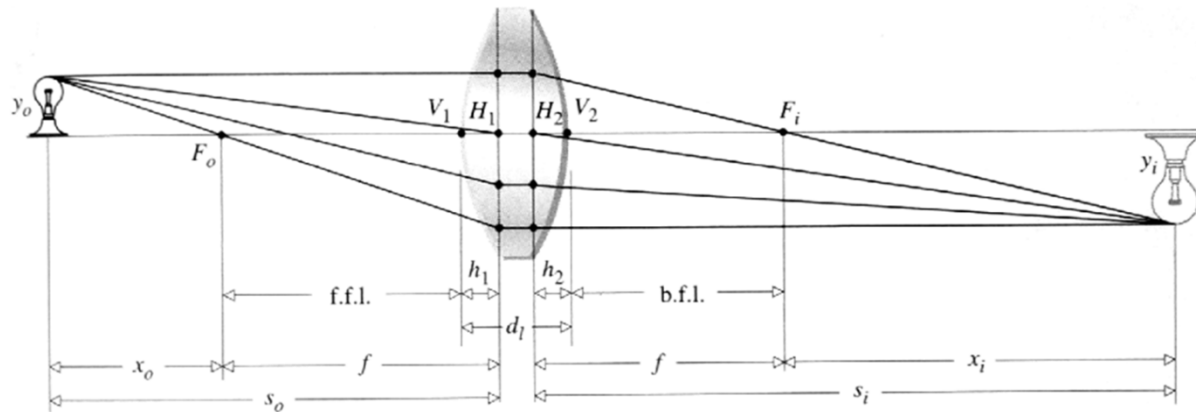
$$\text{f. f. l.} = \frac{f_1(d - f_2)}{d - (f_1 + f_2)}$$

- Back focal length:

$$\text{b. f. l.} = \frac{f_2(d - f_1)}{d - (f_1 + f_2)}$$

Thick Lenses

- Two approaches:
 - Calculate positions of intermediate images formed by each spherical refracting surface
 - Calculate positions of principle planes:

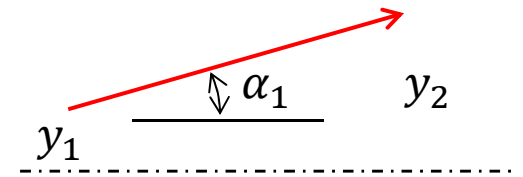


$$\text{Focal length: } \frac{1}{f} = (n - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2} \right]$$

$$\text{Principal planes: } h_1 = -\frac{f(n-1)d}{nR_2}, h_2 = -\frac{f(n-1)d}{nR_1}$$

Ray Tracing

- Ray vector: $\vec{r}_i = \begin{bmatrix} n_i \alpha_i \\ y_i \end{bmatrix}$



- Transfer matrix: $T = \begin{bmatrix} 1 & 0 \\ d/n & 1 \end{bmatrix}$

- Refraction matrix: $R = \begin{bmatrix} 1 & -D \\ 0 & 1 \end{bmatrix}, D = \frac{n_t - n_i}{R}$

- Mirror matrix: $M = \begin{bmatrix} -1 & -2n/R \\ 0 & 1 \end{bmatrix}$

Aberrations

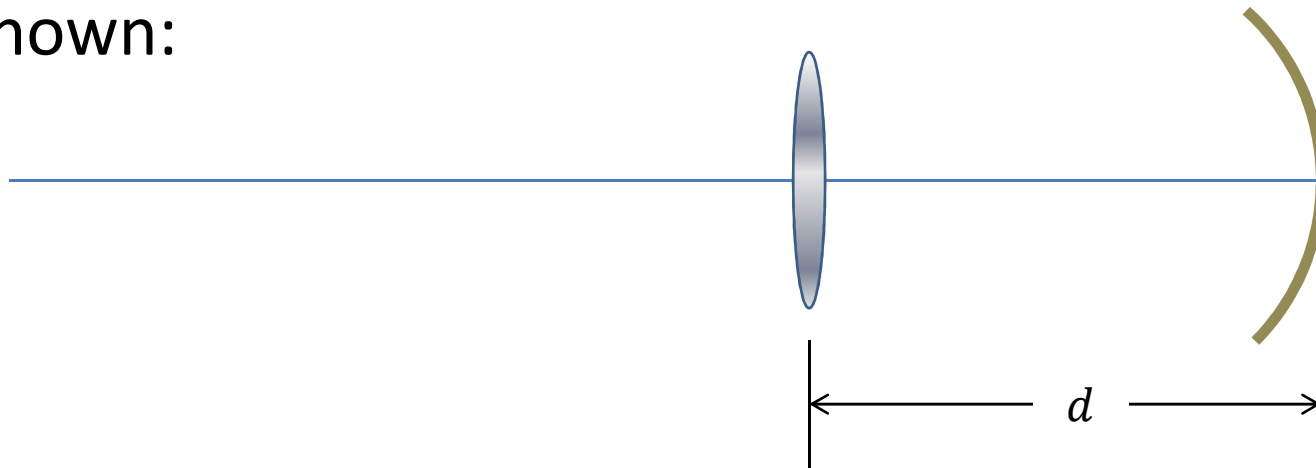
- Images usually suffer from some degree of distortion
 - Seidel's primary aberrations
 - Spherical
 - Coma
 - Astigmatism
 - Field curvature
 - Distortion
 - Chromatic aberrations

Typical Exam Questions

- An optical system consists of a horizontal linear polarizer, a solution of dextrorotary sugar, which rotates the plane of polarized light by $+45^\circ$, followed by a left circular polarizing filter.
 - (a) Write the Mueller matrices for each component
 - (b) Calculate the intensity of transmitted light if the incident light is unpolarized
 - (c) Calculate the intensity of transmitted light if the incident light is left circular polarized
 - (d) Is the system symmetric? That is, is the intensity of transmitted light the same if the paths of all light rays are reversed?

Typical Exam Questions

- An optical system consists of a thin lens with focal length f and a concave spherical mirror with radius R as shown:



- Where is the effective focal point of this combination of optical elements?
- What is the transverse magnification?

Typical Exam Questions

- Answer one, or the other, but not both...
 - (a) Describe two types of optical aberration and techniques that can be used to minimize them.
 - (b) Compare and contrast the assumptions and main features of Fraunhofer and Fresnel diffraction. Under what circumstances is Fraunhofer diffraction a special case of Fresnel diffraction?