

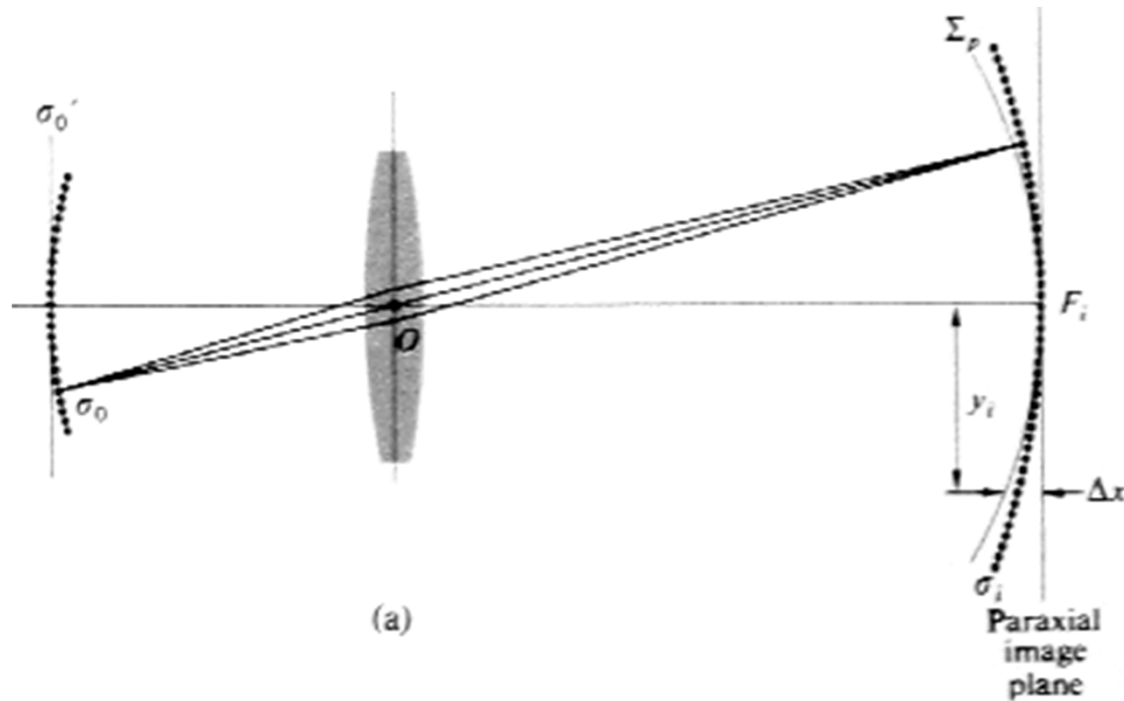
Physics 42200
Waves & Oscillations

Lecture 30 – Geometric Optics

Spring 2015 Semester

Matthew Jones

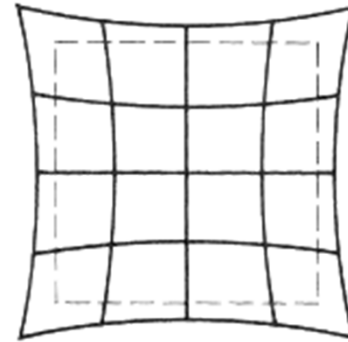
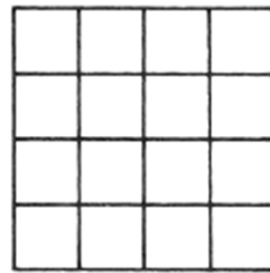
Field Curvature



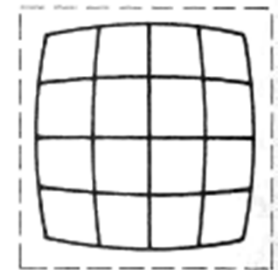
- The focal plane is actually a curved surface
- A negative lens has a field plane that curves away from the image plane
- A combination of positive and negative lenses can cancel the effect

Field Curvature

- Transverse magnification, m_T , can be a function of the off-axis distance:



Positive
(pincushion)
distortion

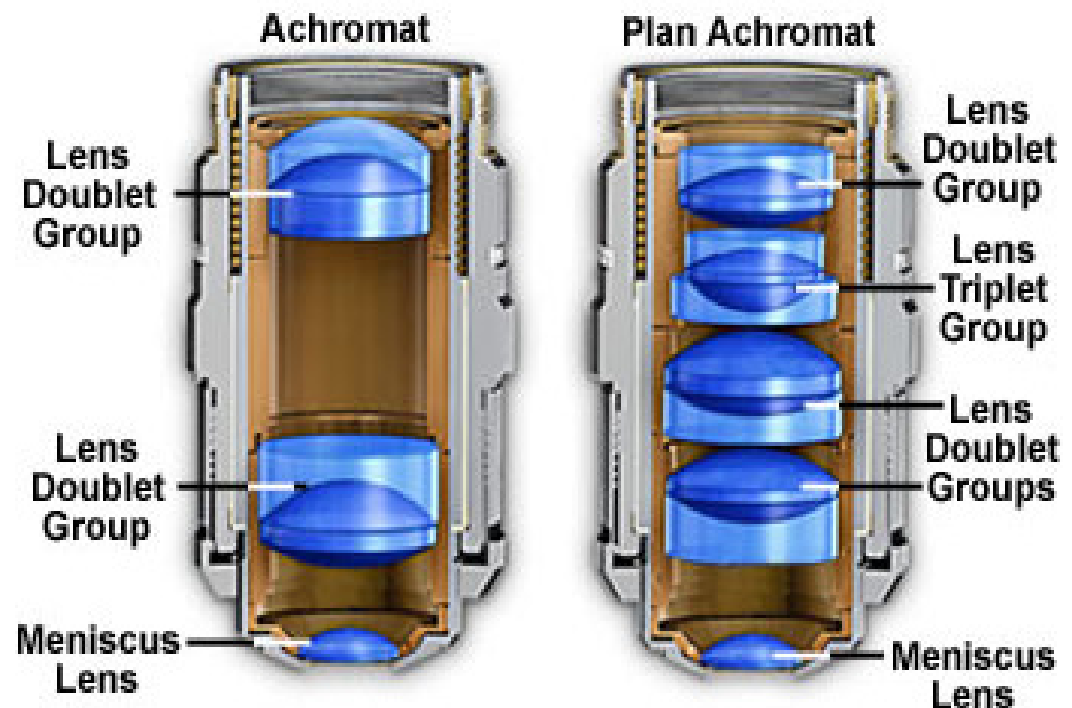


Negative
(barrel)
distortion



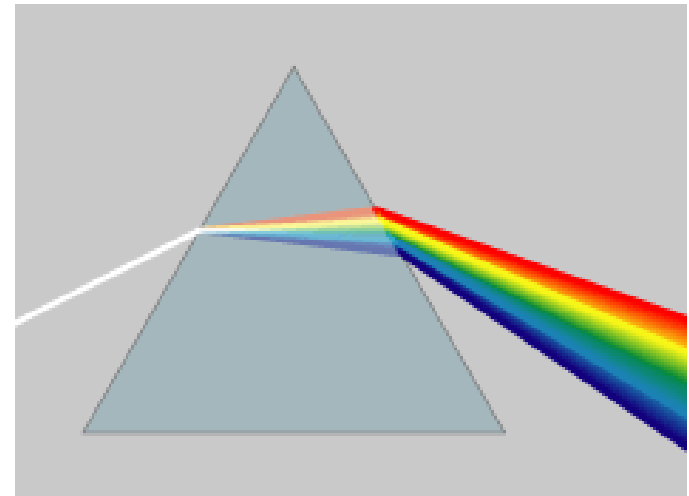
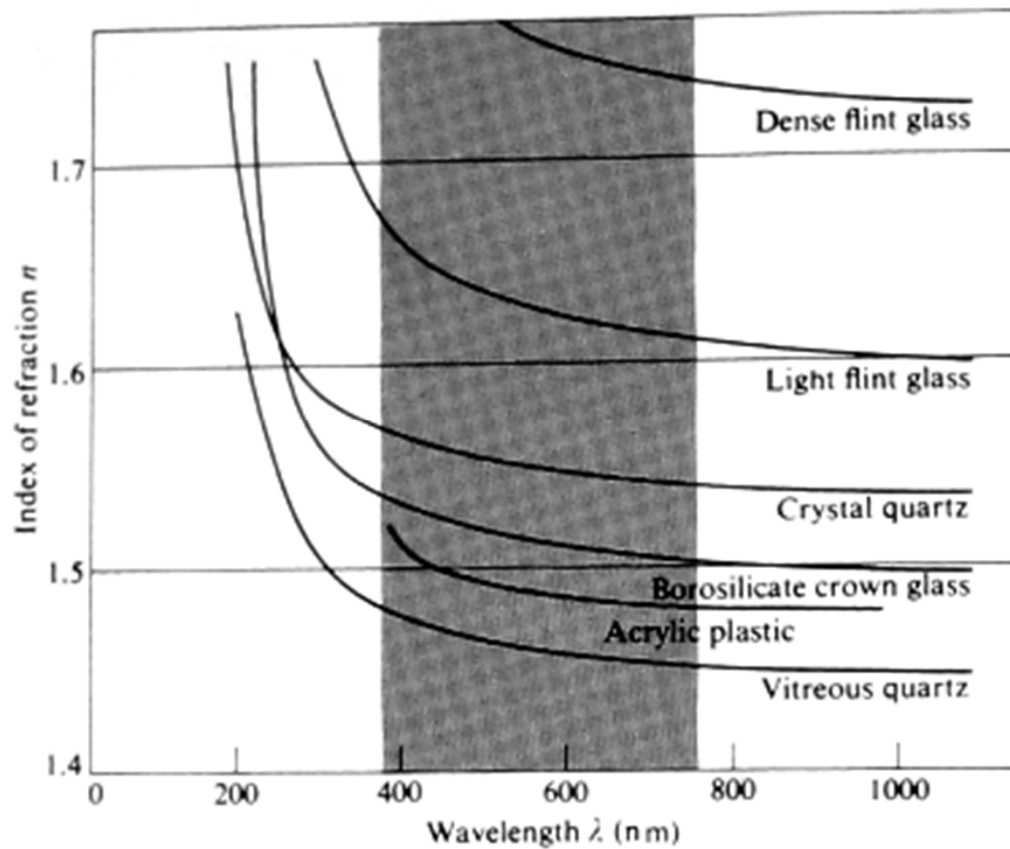
Correcting Monochromatic Aberrations

- Combinations of lenses with mutually cancelling aberration effects
- Apertures
- Aspherical correction elements.



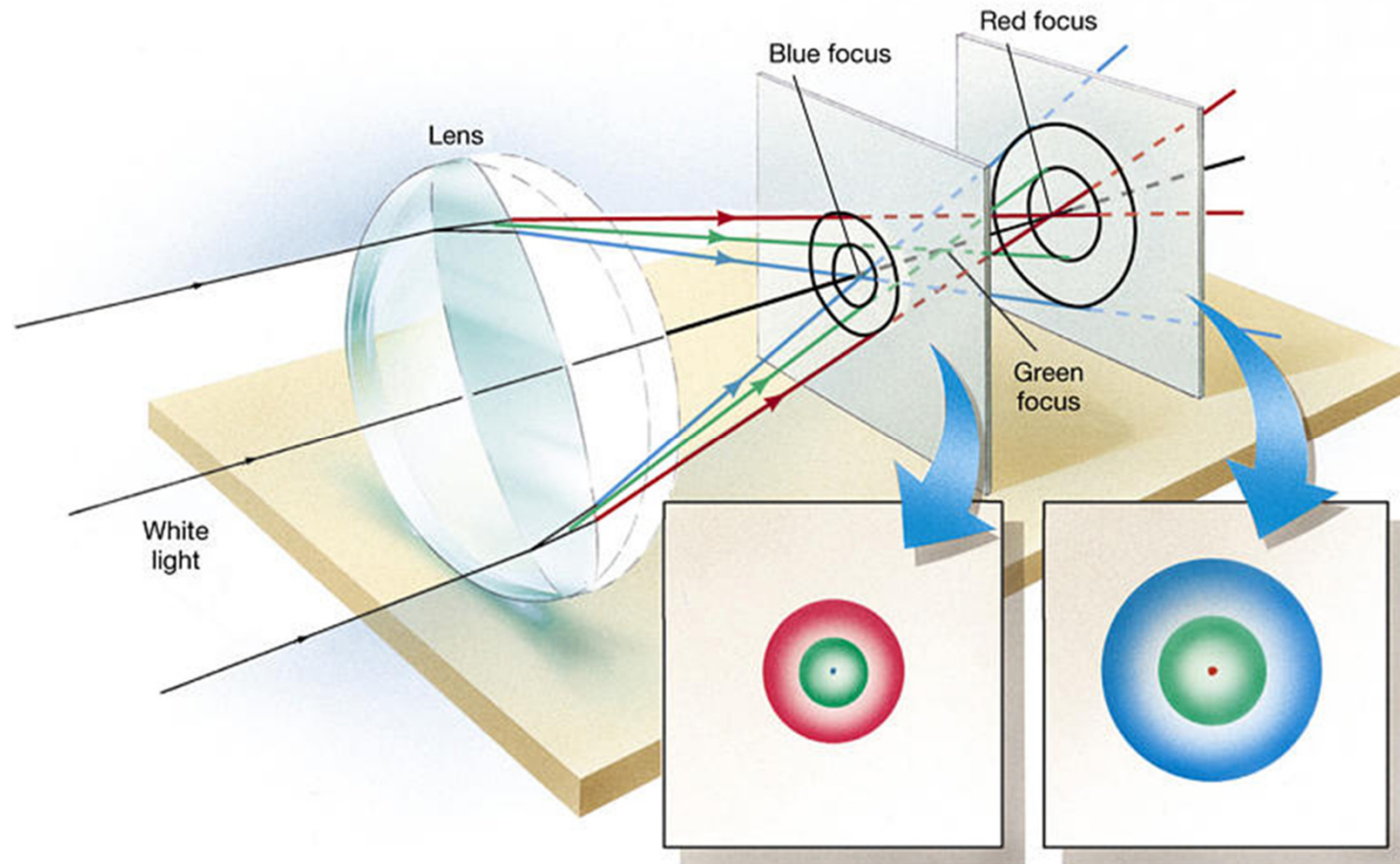
Chromatic Aberrations

- Index of refraction depends on wavelength



$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

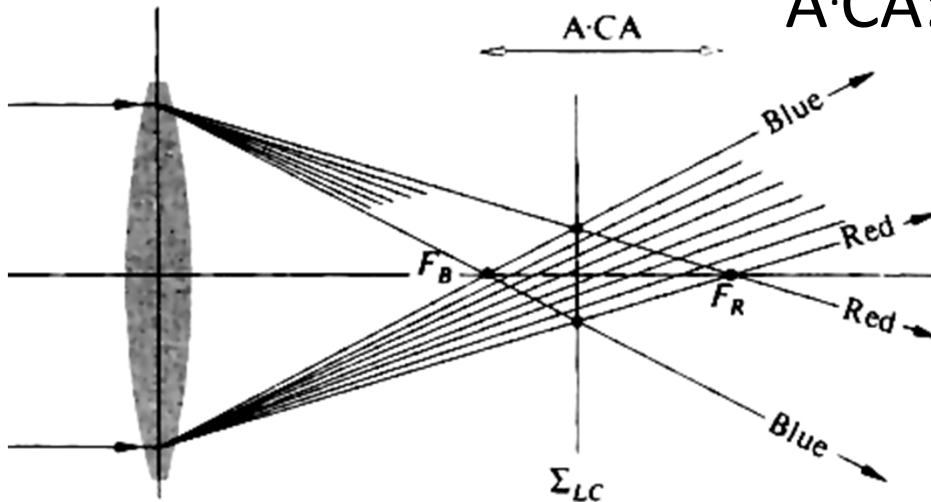
Chromatic Aberrations



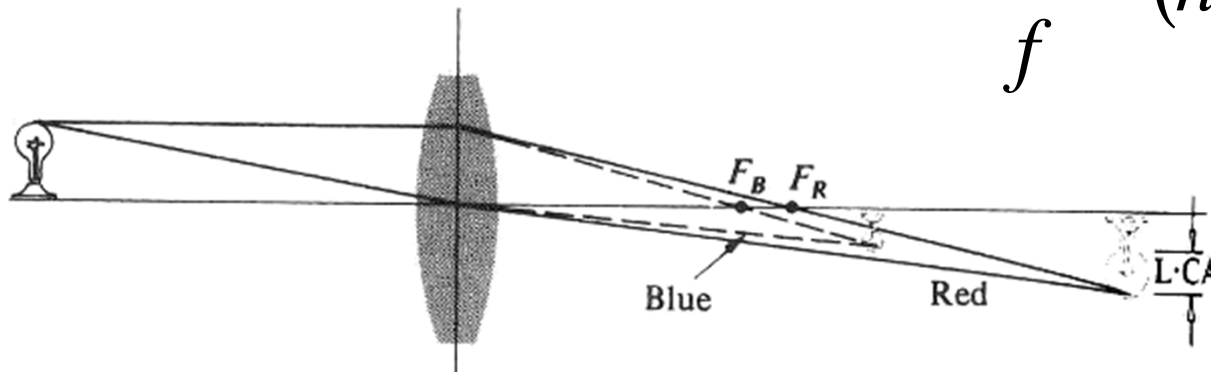
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Chromatic Aberrations

A·CA: axial chromatic aberration



$$\frac{1}{f} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$



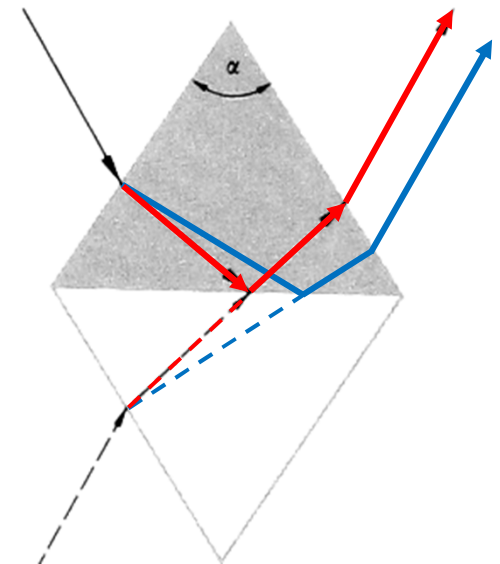
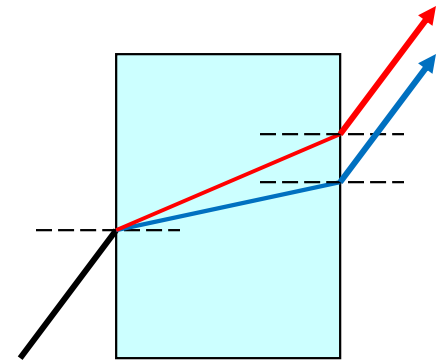
L·CA: lateral chromatic aberration

Chromatic Aberration



Correcting for Chromatic Aberration

- It is possible to have refraction without chromatic aberration even when n is a function of λ :
 - Rays emerge displaced but *parallel*
 - If the thickness is small, then there is no distortion of an image
 - Possible even for non-parallel surfaces:
 - Aberration at one interface is compensated by an opposite aberration at the other surface.



Chromatic Aberration

- Focal length:

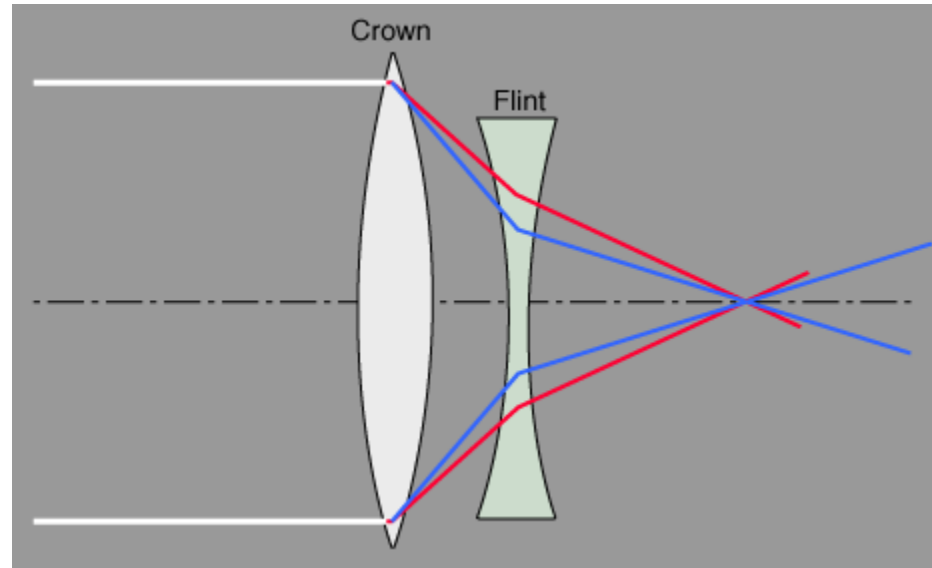
$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

- Thin lens equation:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

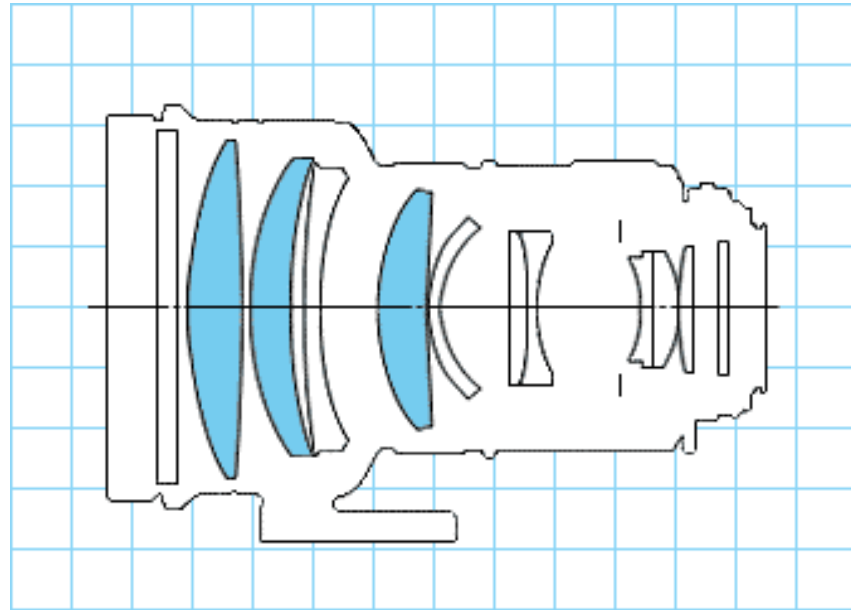
- Cancel chromatic aberration using a combination of concave and convex lenses with different index of refraction

Chromatic Aberration



- This design does not eliminate chromatic aberration completely – only two wavelengths are compensated.

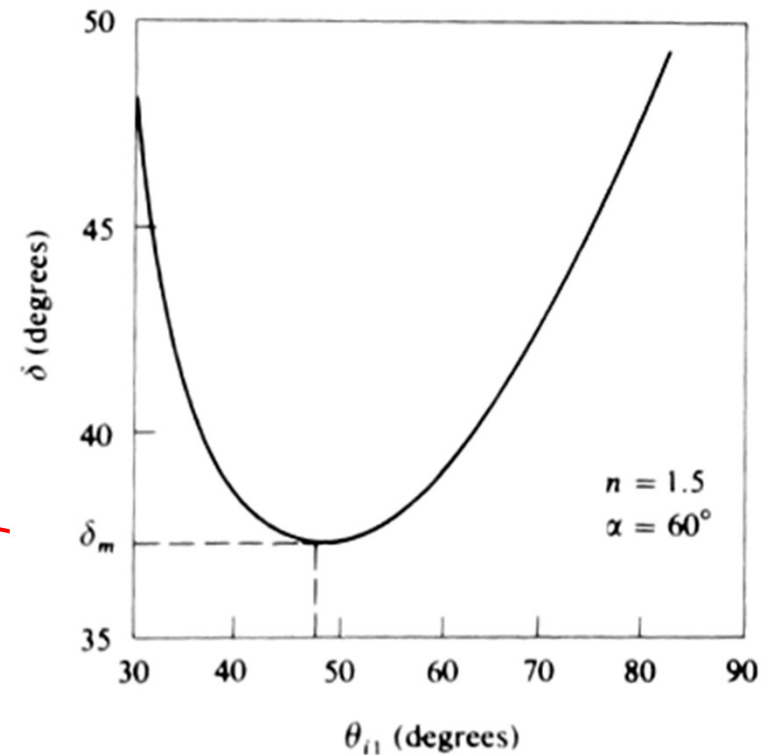
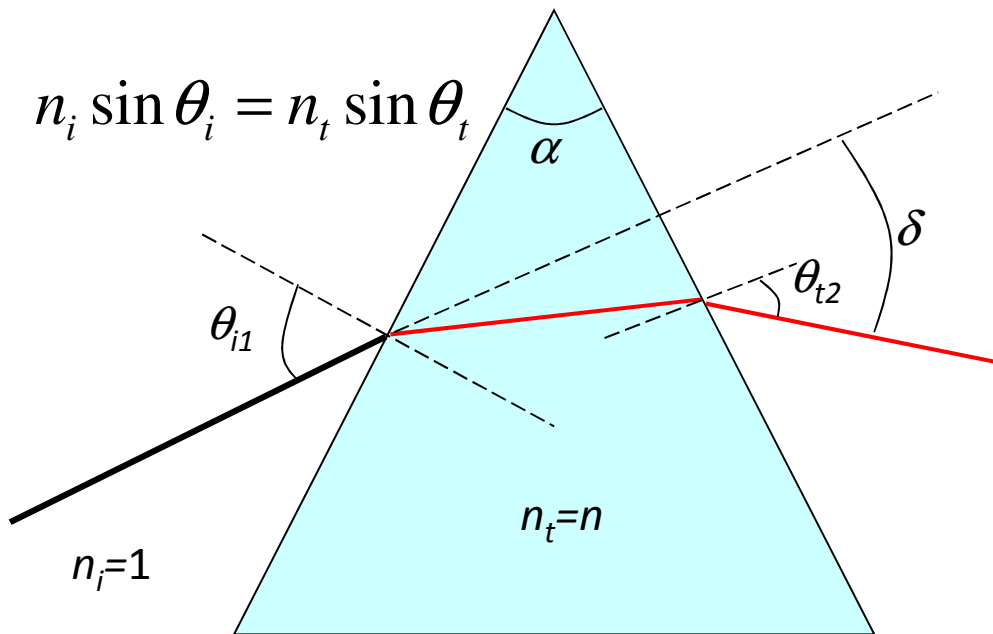
Commercial Lens Assemblies



- Some lens components are made with ultra-low dispersion glass, eg. calcium fluoride

Prisms

- Dispersing prism:

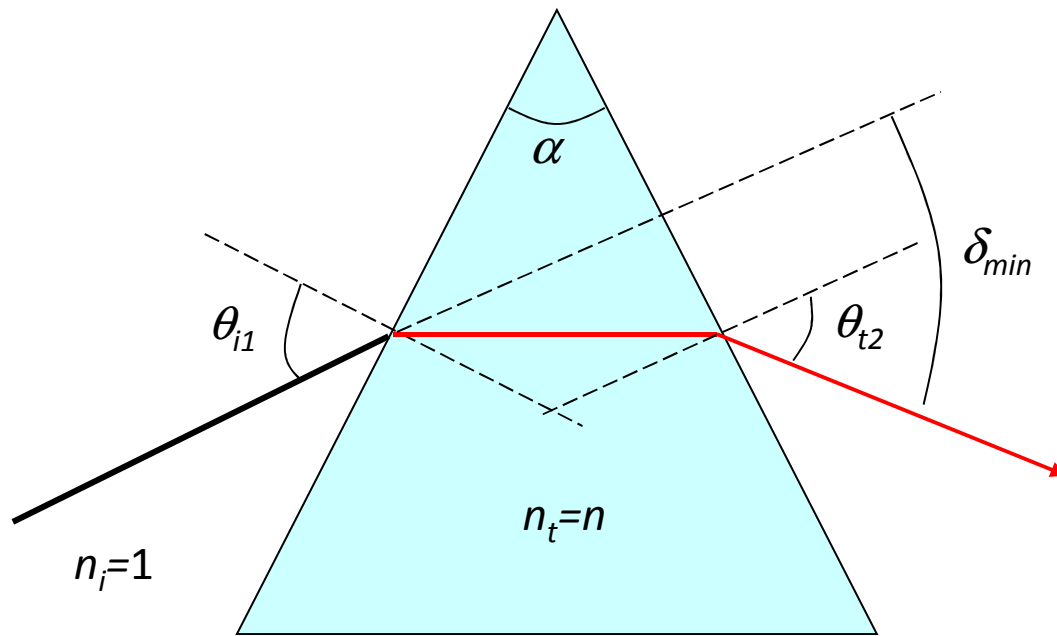


- Total deviation:

$$\delta = \theta_{i1} + \sin^{-1} \left[(\sin \alpha) \sqrt{n^2 - \sin^2 \theta_{i1}} - \sin \theta_{i1} \cos \alpha \right] - \alpha$$

Prisms

- The minimum deflection, δ_{min} , occurs when $\theta_{i1} = \theta_{t2}$:

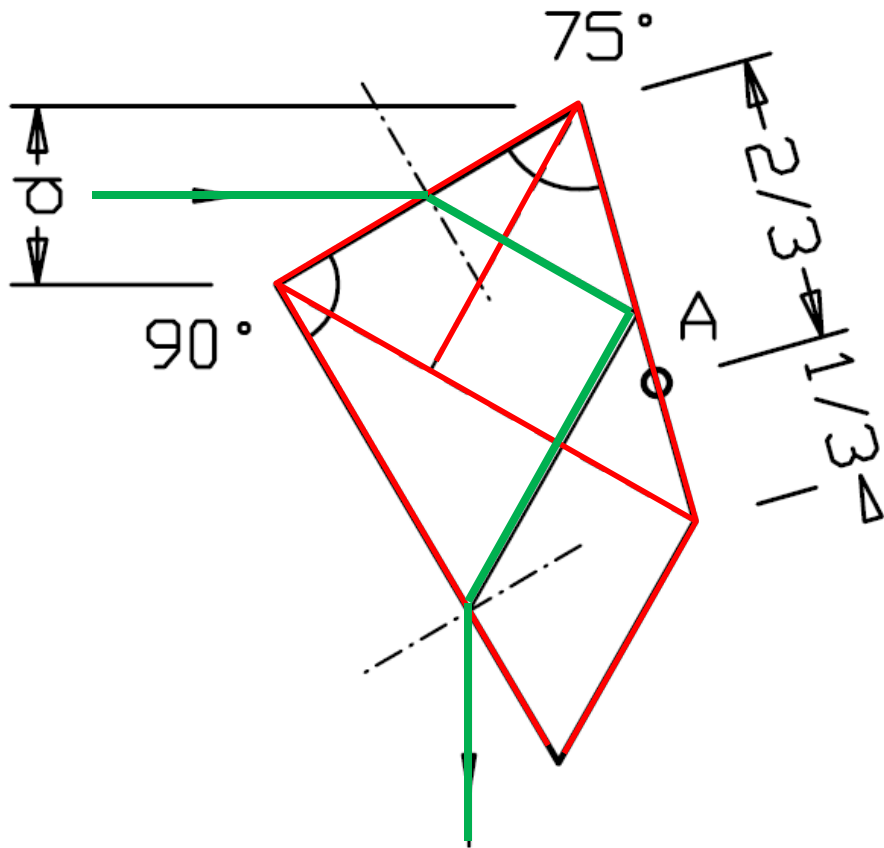


$$n = \frac{\sin[(\delta_{min} + \alpha)/2]}{\sin(\alpha/2)}$$

This can be used as an effective method to measure n

A disadvantage for analyzing colors is the variation of δ_{min} with θ_{i1} - the angle of incidence must be known precisely in order to determine the wavelength as a function of the angle θ_{t2} .

Pellin-Broca Prism



One color is refracted through exactly 90°.

Rotating the prism about point A selects different colors.

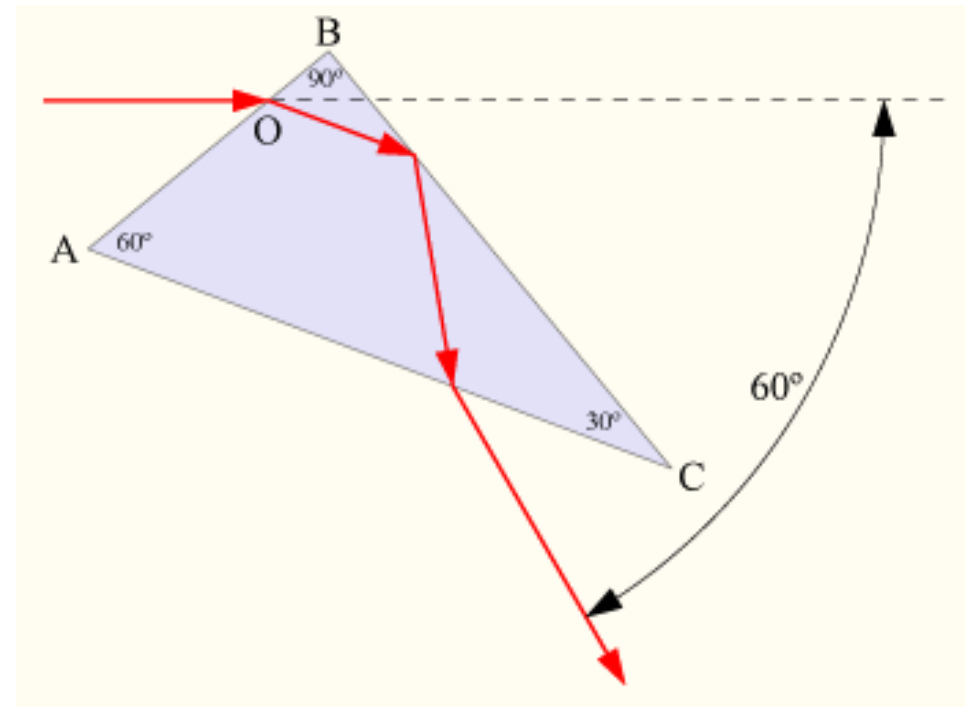
Ideal for selecting a particular wavelength with minimal change to an optical system.

Abbe Prism

- A particular wavelength is refracted through 60°
- Rotating the prism about point O selects different colors.

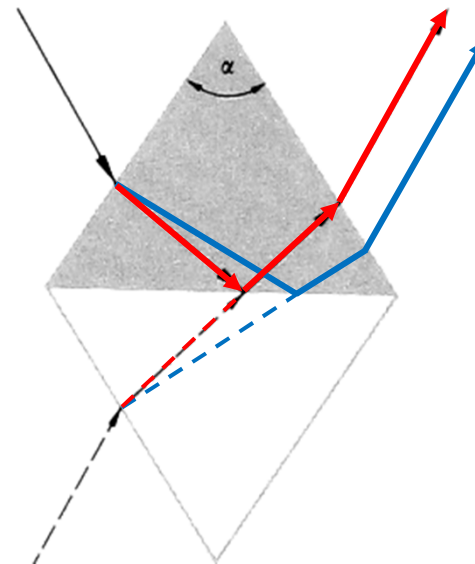
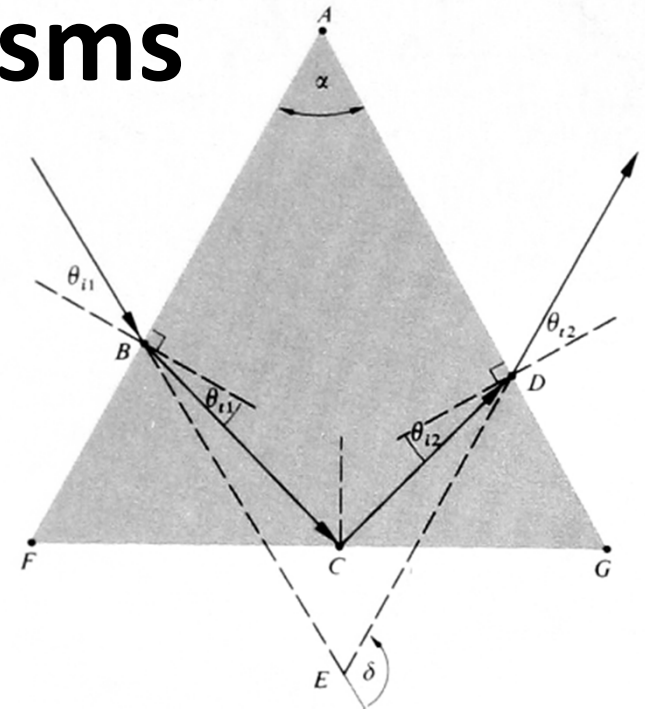


Ernst Abbe
1840-1905



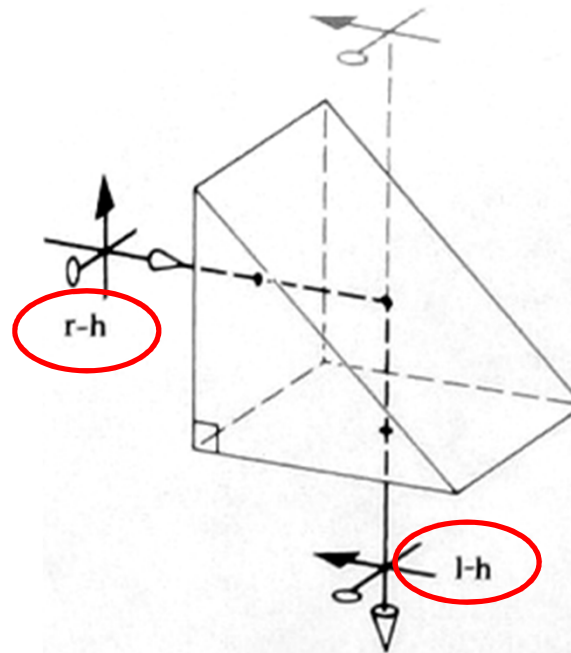
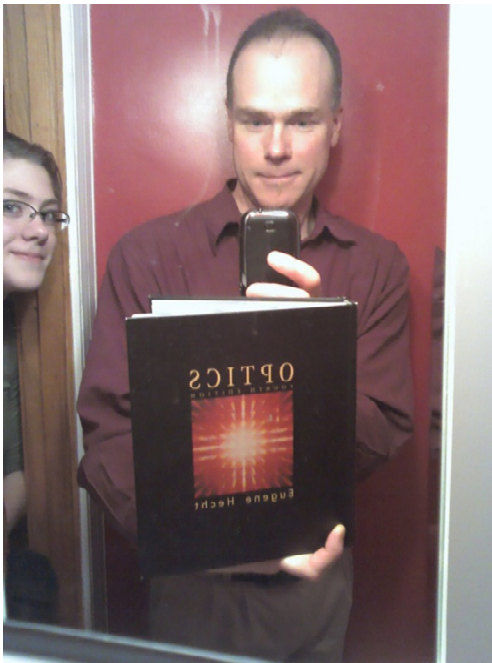
Reflective Prisms

- Total internal reflection on one surface
- Equal and opposite refraction at the other surfaces
- Deflection angle:
$$\delta = 2\theta_{i1} + \alpha$$
- Independent of wavelength (non-dispersive or achromatic prism)



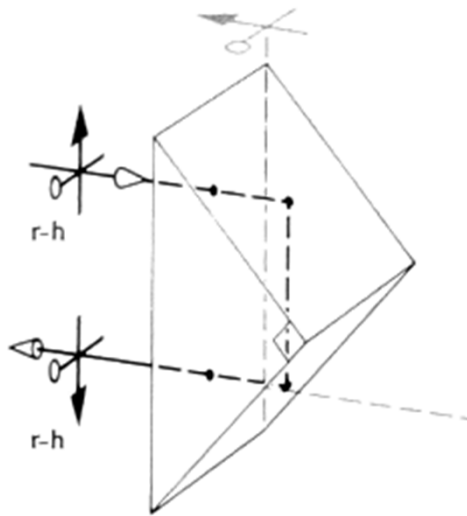
Reflecting Prisms

- Why not just use a mirror?
 - Mirrors produce a reflected image
- Prisms can provide ways to change the direction of light while simultaneously transforming the orientation of an image.

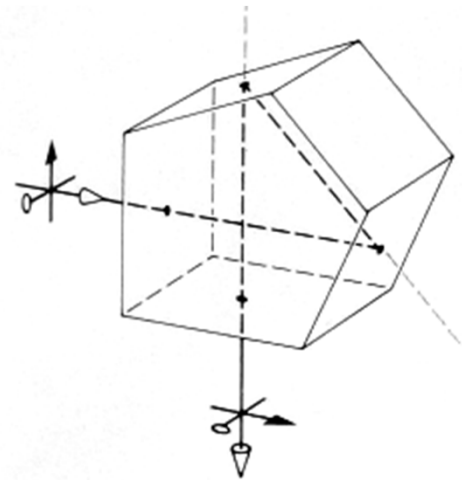


Reflecting Prisms

- Two internal reflections restores the orientation of the original image.

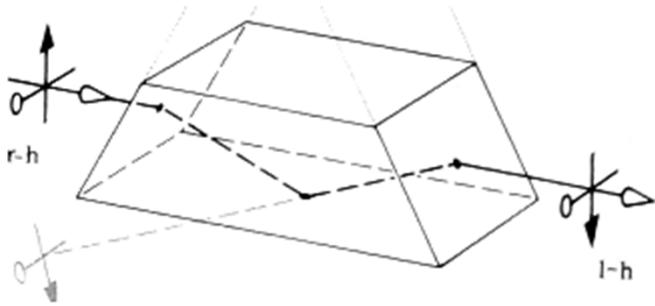


The Porro prism



The penta prism

Dove Prism/Image Rotator

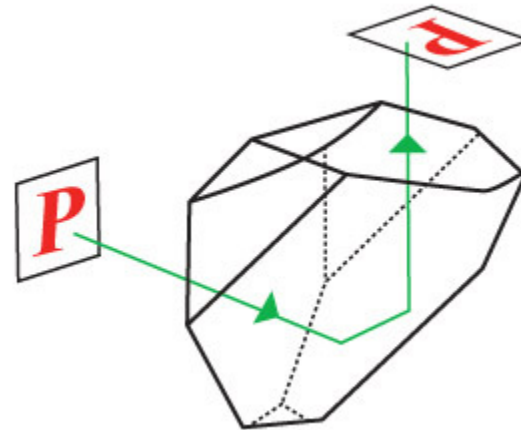
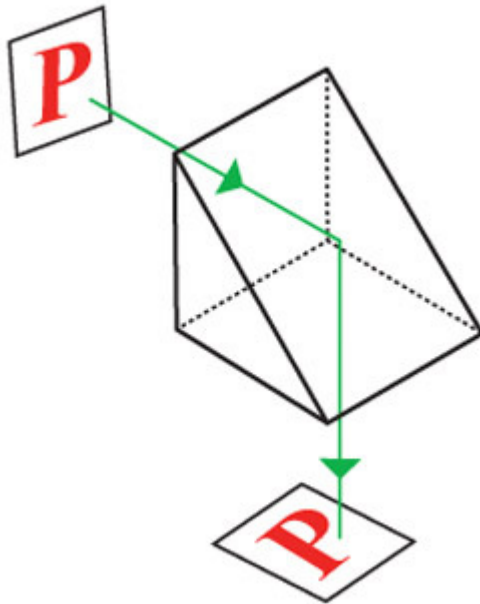


The Dove prism

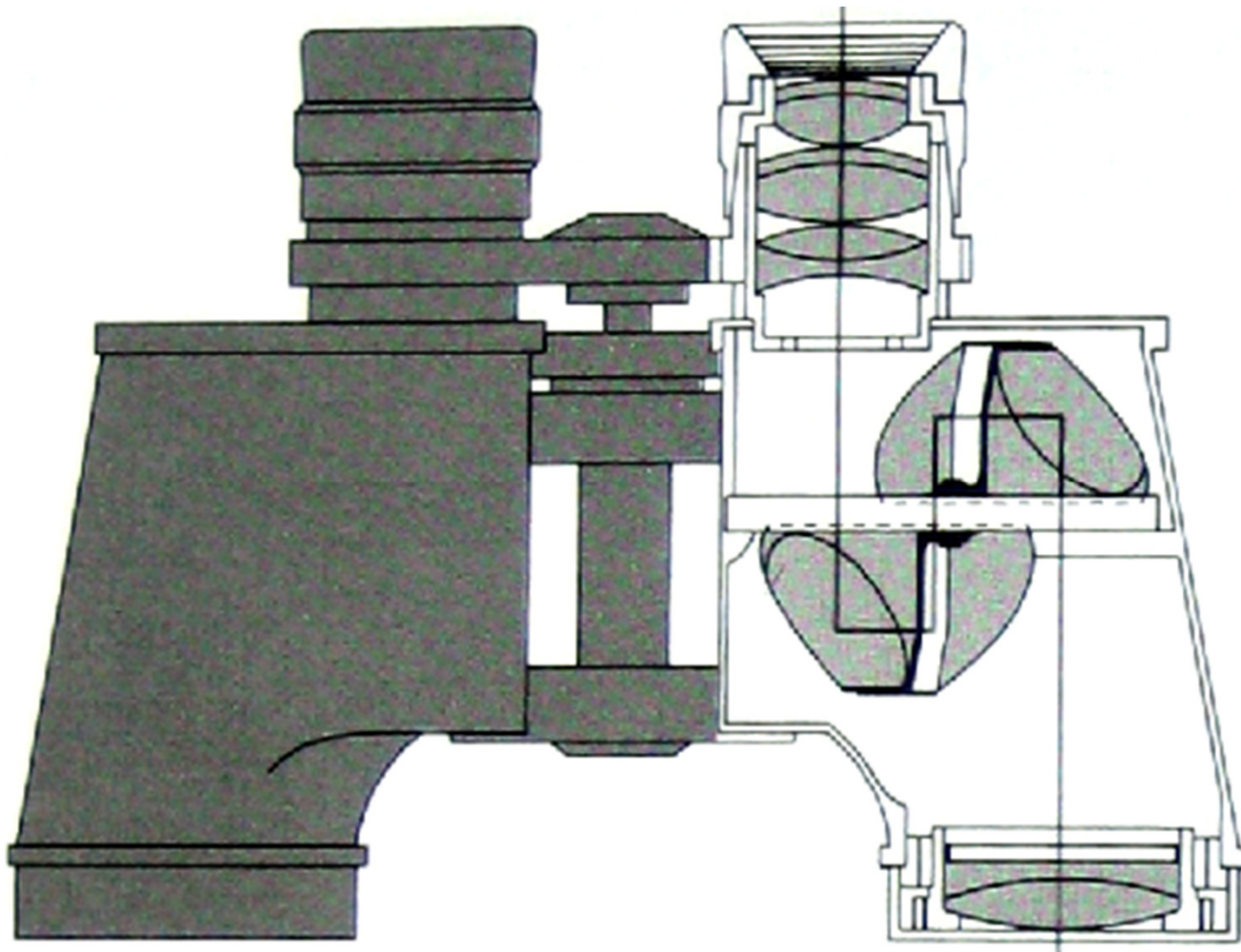


Roof Prism

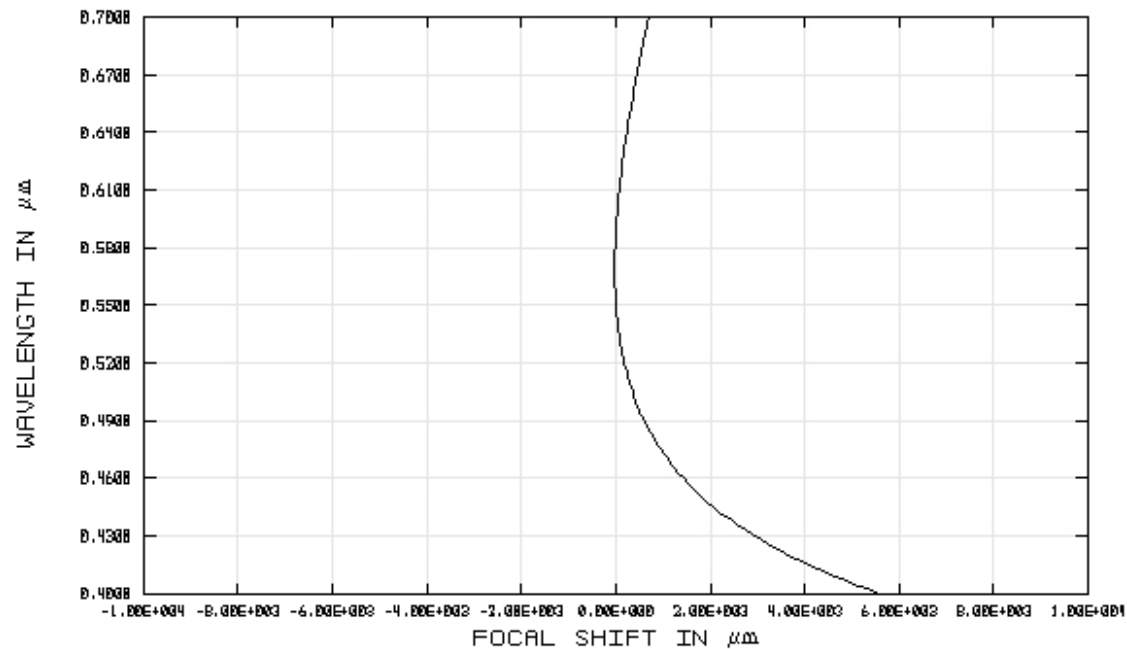
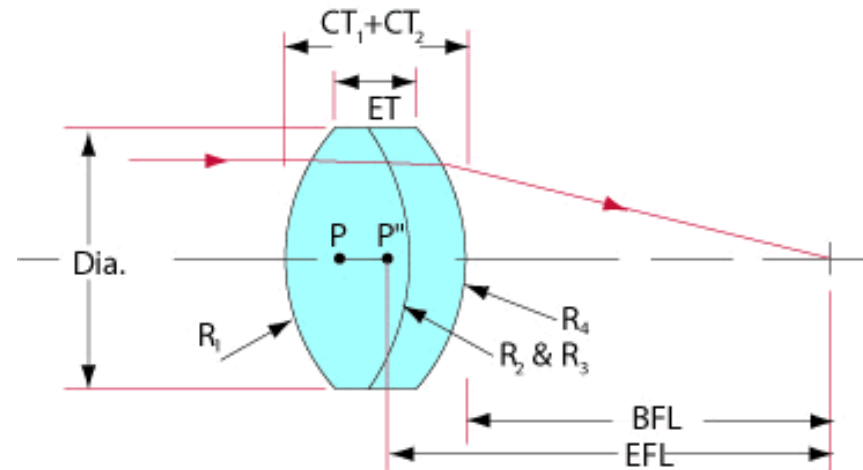
- Right-angle reflection without image reversal (image rotation)



Binoculars

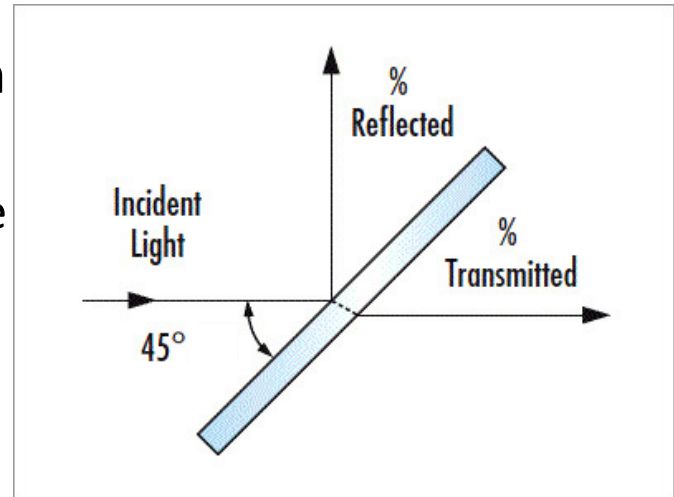


Achromatic Doublets



Beam Splitters

- Reflect half the light in a different direction
- Important application: interferometry
 - Transmitted and reflected beams are phase coherent.
- Beam splitter plate
 - Partially reflective surfaces



- Beam splitter cube:
 - Right angle prisms cemented together
 - Match transmission of both polarization components

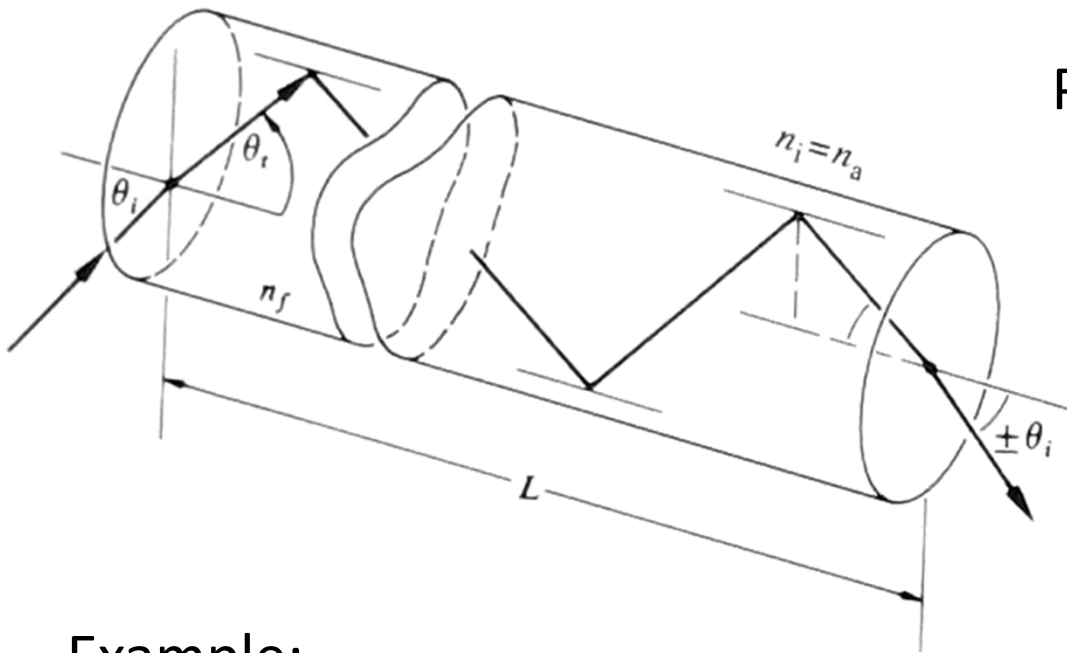


Fiber Optics

- Development of fiber optics:
 - 1854: John Tyndall demonstrated that light could be bent by a curved stream of water
 - 1888: Roth and Reuss used bent glass rods to illuminate body cavities for surgical procedures
 - 1920's: Baird and Hansell patented an array of transparent rods to transmit images
- Significant obstacles:
 - Light loss through the sides of the fibers
 - Cross-talk (transfer of light between fibers)
 - 1954: Van Heel studied fibers clad with a material that had a lower index of refraction than the core

Fiber Optics: Losses

Consider large fiber: diameter $D \gg \lambda \rightarrow$ can use geometric optics



Path length traveled by ray:

$$l = L / \cos \theta_t$$

Number of reflections:

$$N = \frac{l}{D / \sin \theta_t} \pm 1$$

Using Snell's Law for θ_t :

$$N = \frac{L \sin \theta_i}{D \sqrt{n_f^2 - \sin^2 \theta_i}} \pm 1$$

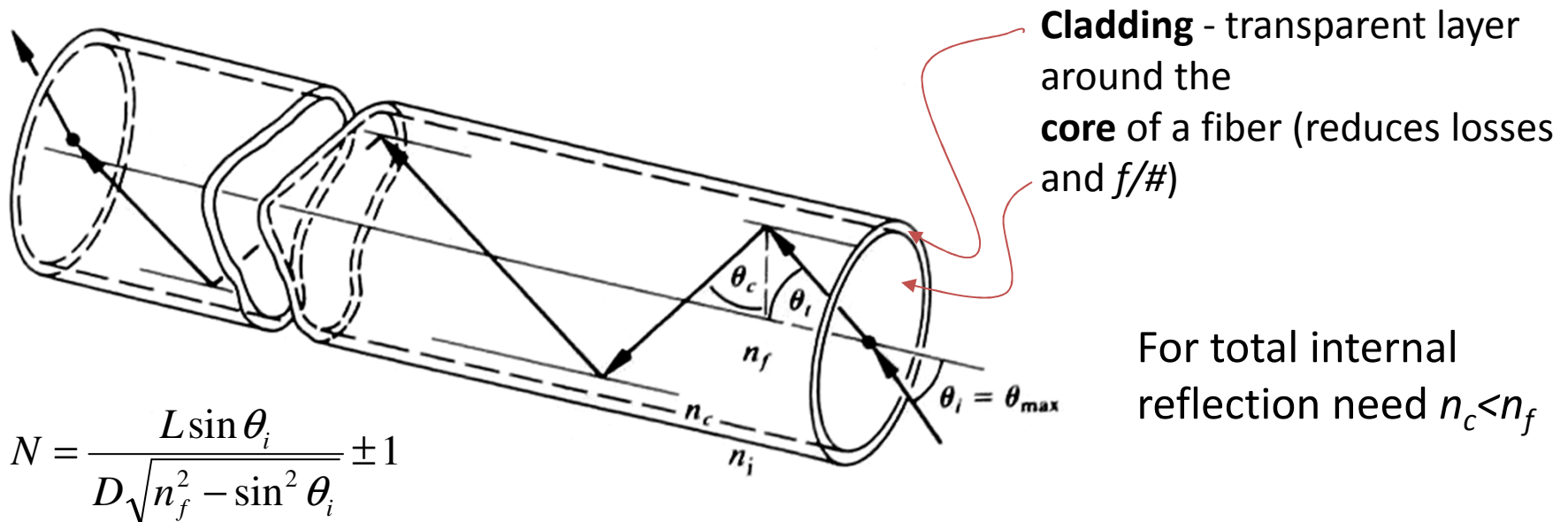
Example:

$$L = 1 \text{ km}, D = 50 \mu\text{m}, n_f = 1.6, \theta_i = 30^\circ$$

$$N = 6,580,000$$

Note: frustrated internal reflection,
irregularities \rightarrow losses!

'Step-index' Fiber

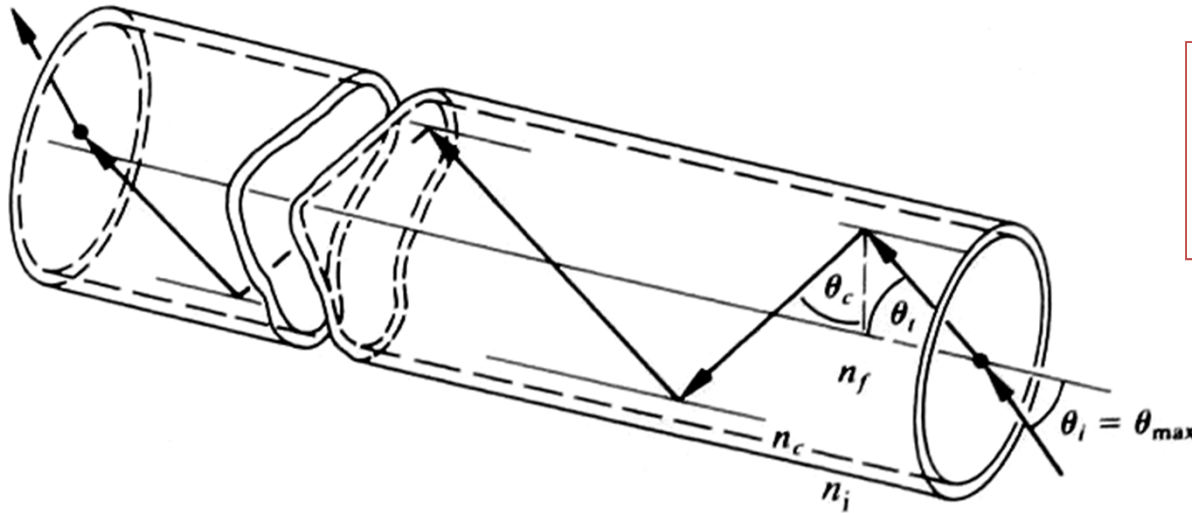


For lower losses need to reduce N , or maximal θ_i , the latter is defined by critical angle for total internal reflection:

$$\sin \theta_c = \frac{n_c}{n_f} = \sin(90^\circ - \theta_t) = \cos(\theta_t) \Rightarrow \sin \theta_{\max} = \frac{\sqrt{n_f^2 - n_c^2}}{n_i}$$

$n_i = 1$ for air

Fiber and $f/\#$



$$\sin \theta_{\max} = \frac{\sqrt{n_f^2 - n_c^2}}{n_i}$$

Angle θ_{\max} defines the light gathering efficiency of the fiber, or numerical aperture NA:

$$NA \equiv n_i \sin \theta_{\max} = \sqrt{n_f^2 - n_c^2}$$

And $f/\#$ is:

$$f/\# \equiv \frac{1}{2(NA)}$$

Largest NA=1
Typical NA = 0.2 ... 1

Data Transfer Limitations

1. Distance is limited by losses in a fiber. Losses α are measured in decibels (dB) per km of fiber (dB/km), i.e. in logarithmic scale:

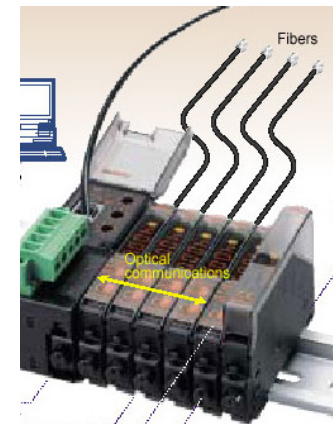
$$\alpha \equiv -\frac{10}{L} \log\left(\frac{P_o}{P_i}\right) \quad \longrightarrow \quad \frac{P_o}{P_i} = 10^{-\alpha L/10}$$

P_o - output power
 P_i - input power
 L - fiber length

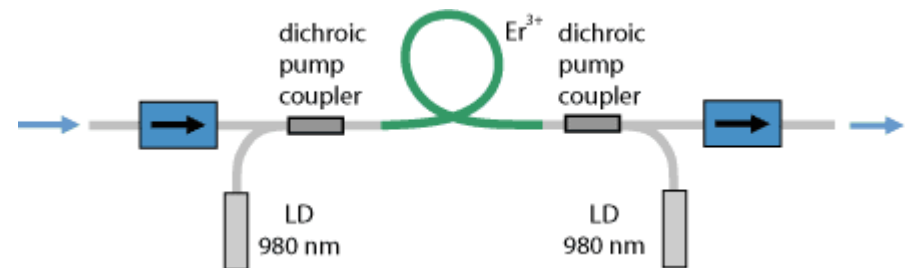
Example:

α	P_o/P_i over 1 km
10 dB	1:10
20 dB	1:100
30 dB	1:1000

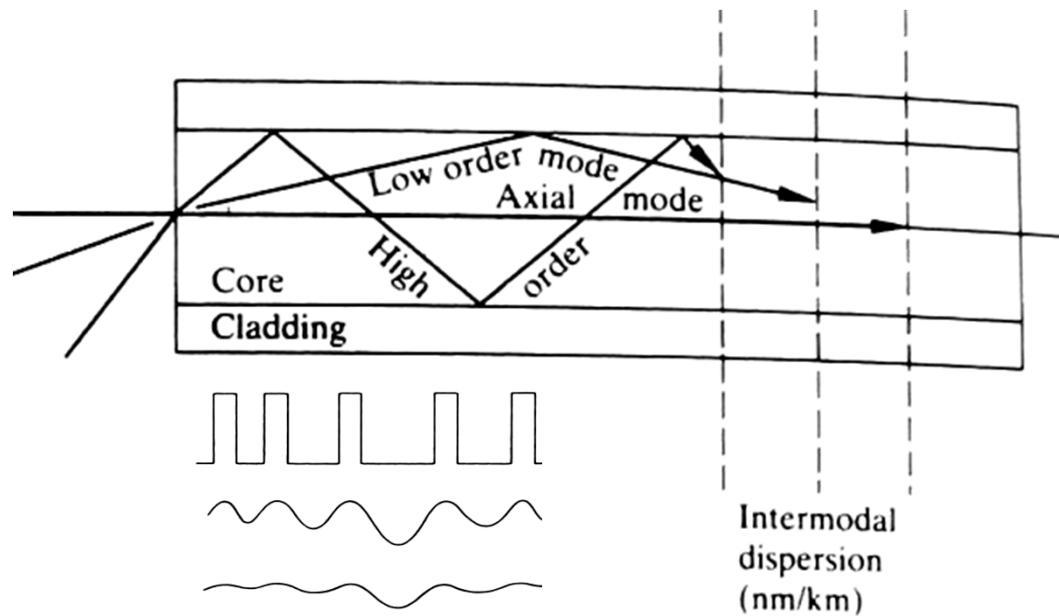
Workaround: use light amplifiers to boost and relay the signal



2. Bandwidth is limited by pulse *broadening* in fiber and processing electronics



Pulse Broadening



Multimode fiber: there are many rays (modes) with different OPLs and initially short pulses will be broadened (*intermodal dispersion*)

For ray along axis:

$$t_{\min} = L/v_f = Ln_f/c$$

For ray entering at θ_{\max} :

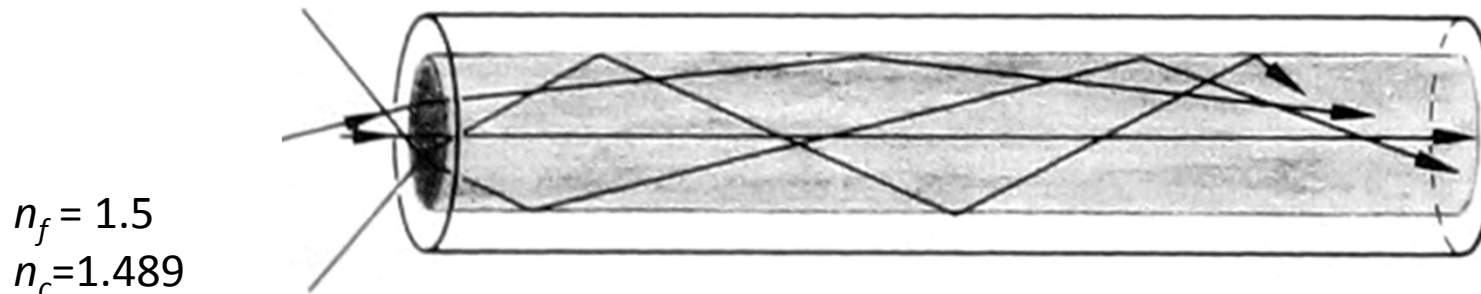
$$t_{\max} = l/v_f = Ln_f^2/(cn_c)$$

The initially short pulse will be broadened by:

Making n_c close to n_f reduces the effect! \longrightarrow

$$\Delta t = t_{\max} - t_{\min} = \frac{Ln_f}{c} \left(\frac{n_f}{n_c} - 1 \right)$$

Pulse Broadening: Example



Estimate the bandwidth limit for 1000 km transmission.

Solution:

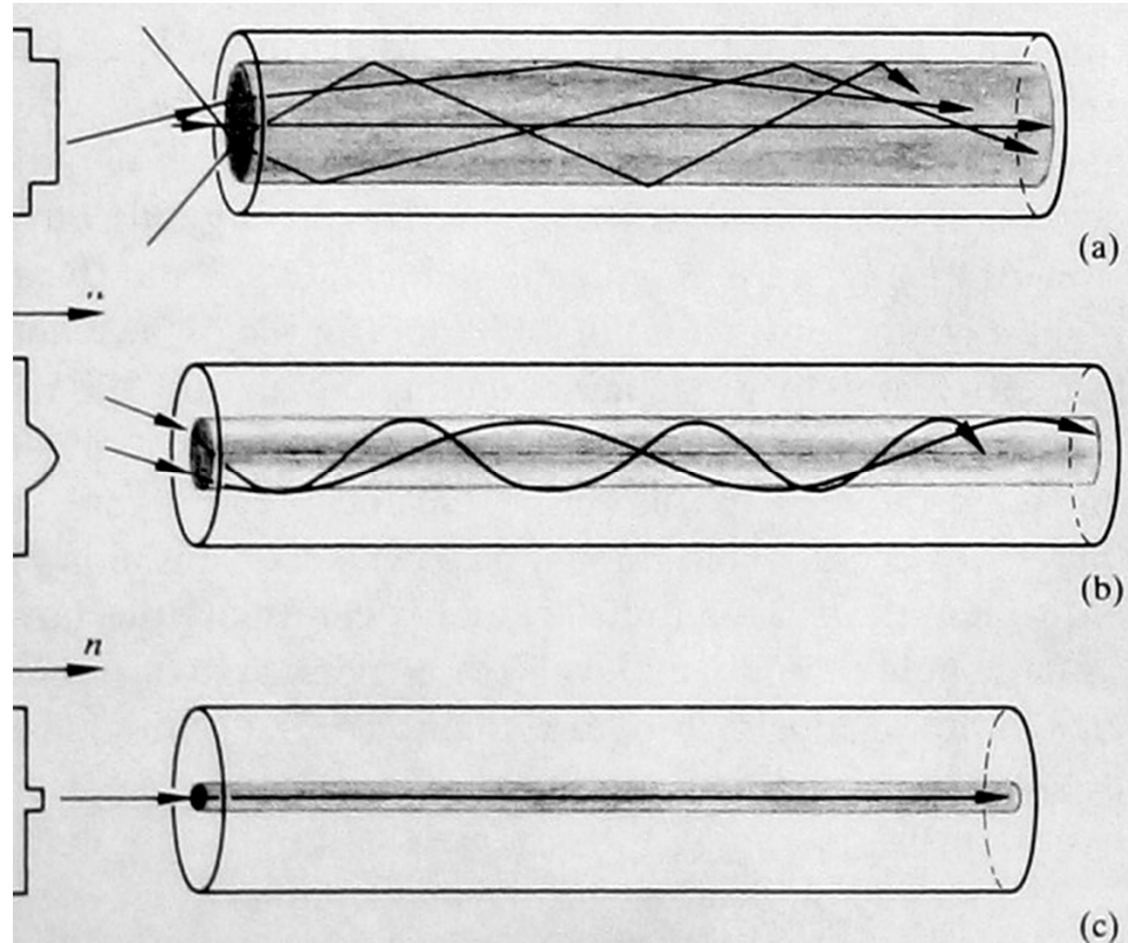
$$\Delta t = \frac{Ln_f}{c} \left(\frac{n_f}{n_c} - 1 \right) = \frac{10^6 \cdot 1.5}{3 \times 10^8} \left(\frac{1.5}{1.489} - 1 \right) s = 3.7 \times 10^{-5} s = 37 \mu s$$

Even the shortest pulse will become $\sim 37 \mu s$ long

$$\text{Bandwidth} \sim \frac{1}{3.7 \times 10^{-5} s} = 27 \text{ kbps} \quad \leftarrow \begin{array}{l} \text{kilobits per second} \\ = \text{ONLY } 3.3 \text{ kbytes/s} \end{array}$$

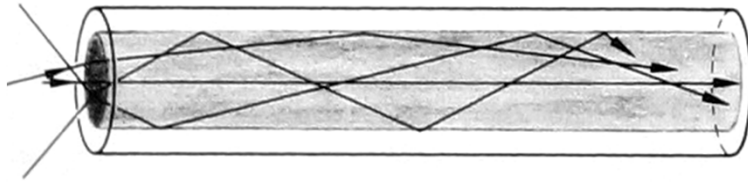
Multimode fibers are not used for communication!

Graded and Step Index Fibers



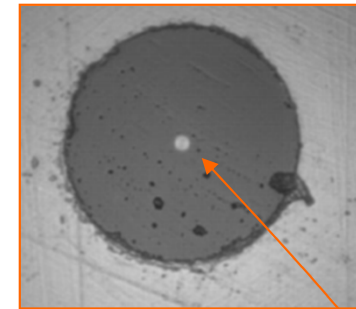
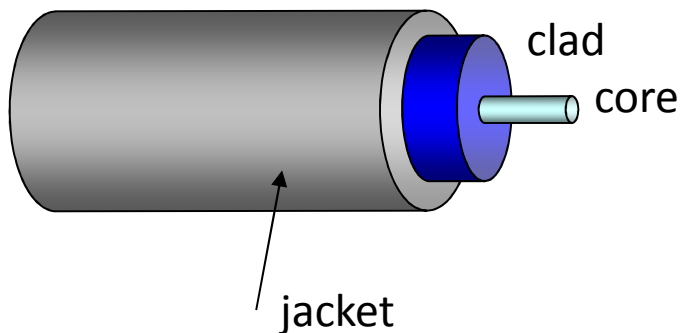
Step index: the change in n is abrupt between cladding and core
Graded index: n changes smoothly from n_c to n_f

Single Mode Fiber



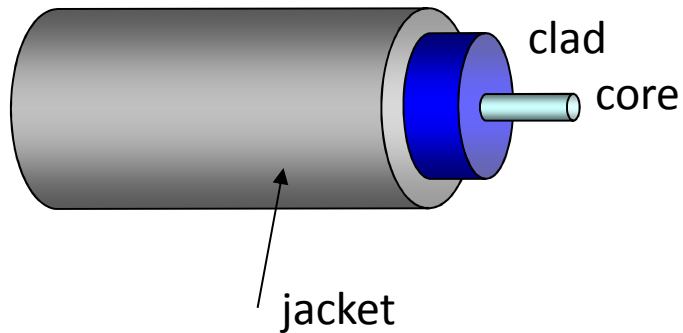
To avoid broadening need to have only one path, or mode

Single mode fiber: there is only one path, all other rays escape from the fiber



Geometric optics does not work anymore: need wave optics.
Single mode fiber core is usually only 2-7 micron in diameter

Single Mode Fiber: Broadening



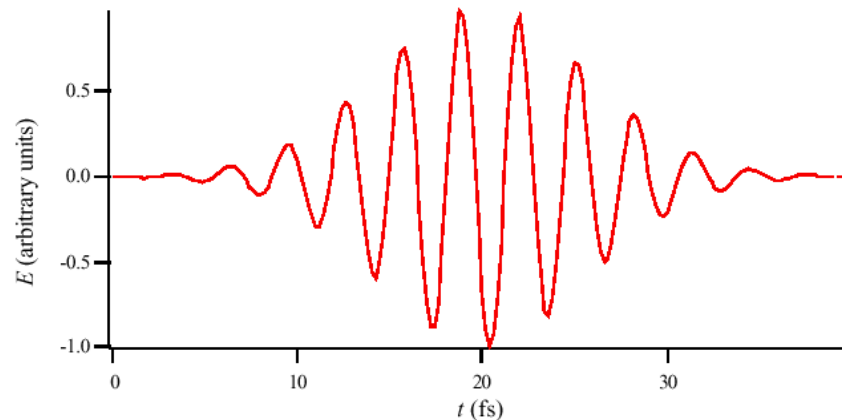
'Transform' limited pulse
product of spectral full width at
half maximum (fwhm) by time
duration fwhm:

$$\Delta f \Delta t \approx 0.2$$

A 10 fs pulse at 800 nm is ~40 nm wide spectrally

If second derivative of n is not zero this pulse will broaden in fiber rapidly

Problem: shorter the pulse, broader
the spectrum.
refraction index depends on
wavelength



Solitons: special pulse shapes that do not change while propagating