

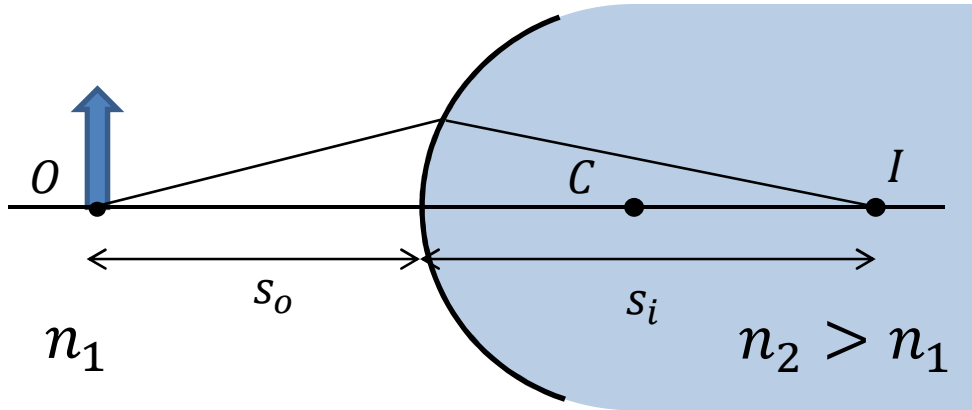
Physics 42200
Waves & Oscillations

Lecture 27 – Geometric Optics

Spring 2015 Semester

Matthew Jones

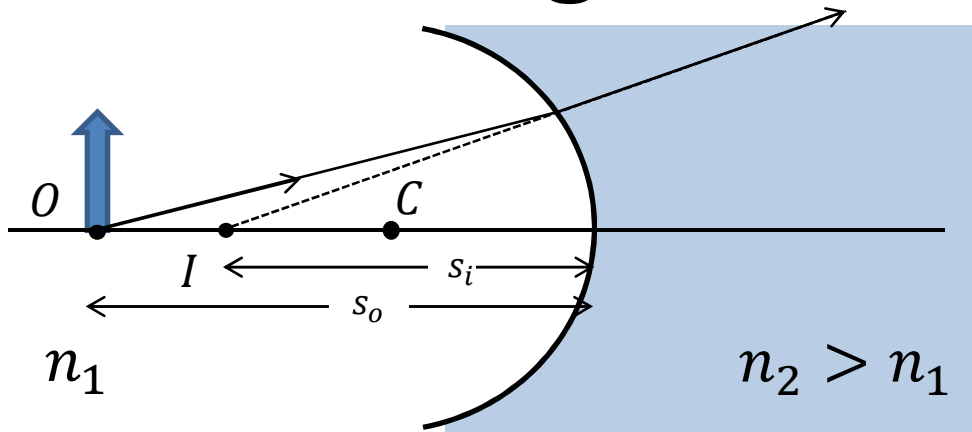
Sign Conventions



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

- Convex surface:
 - s_o is positive for objects on the incident-light side
 - s_i is positive for images on the refracted-light side
 - R is positive if C is on the refracted-light side

Sign Conventions



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

(same formula)

- Concave surface:
 - s_o is positive for objects on the incident-light side
 - s_i is negative for images on the incident-light side
 - R is negative if C is on the incident-light side

Magnification

- Using these sign conventions, the magnification is

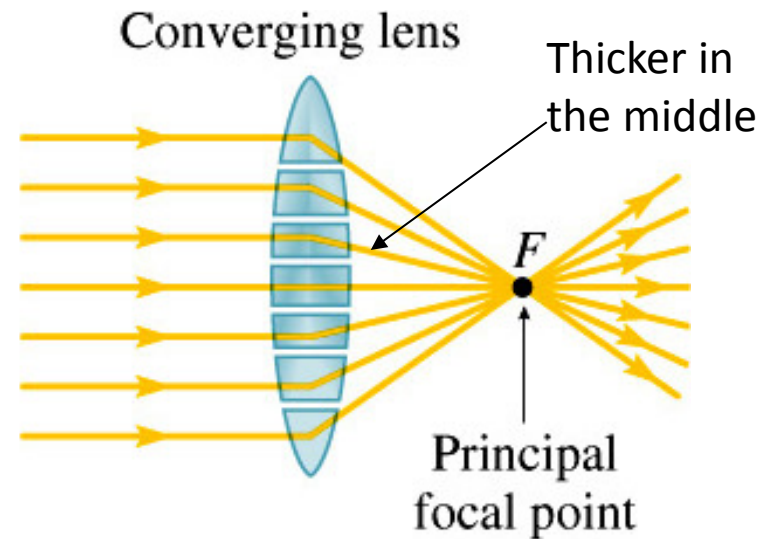
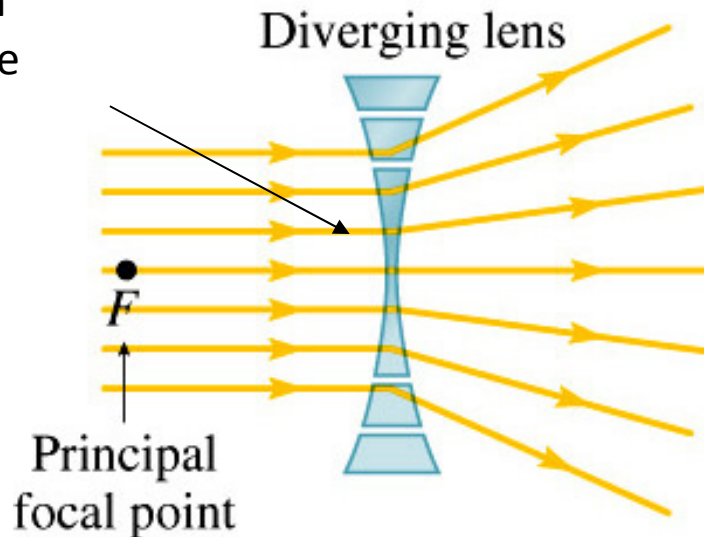
$$m = -\frac{n_1 S_i}{n_2 S_o}$$

- Ratio of image height to object height
- Sign indicates whether the image is inverted

Thin Lenses

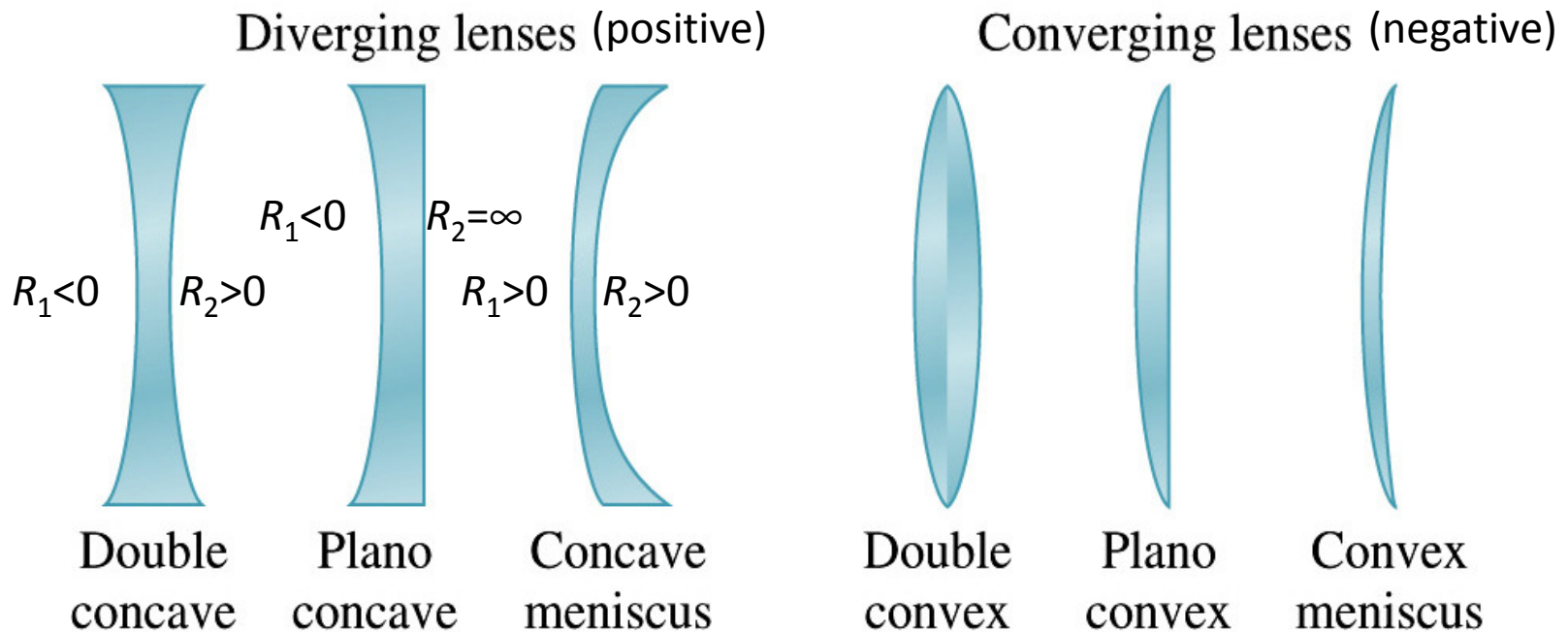
- The previous examples were for one spherical surface.
- Two spherical surfaces make a thin lens

Thinner in the middle

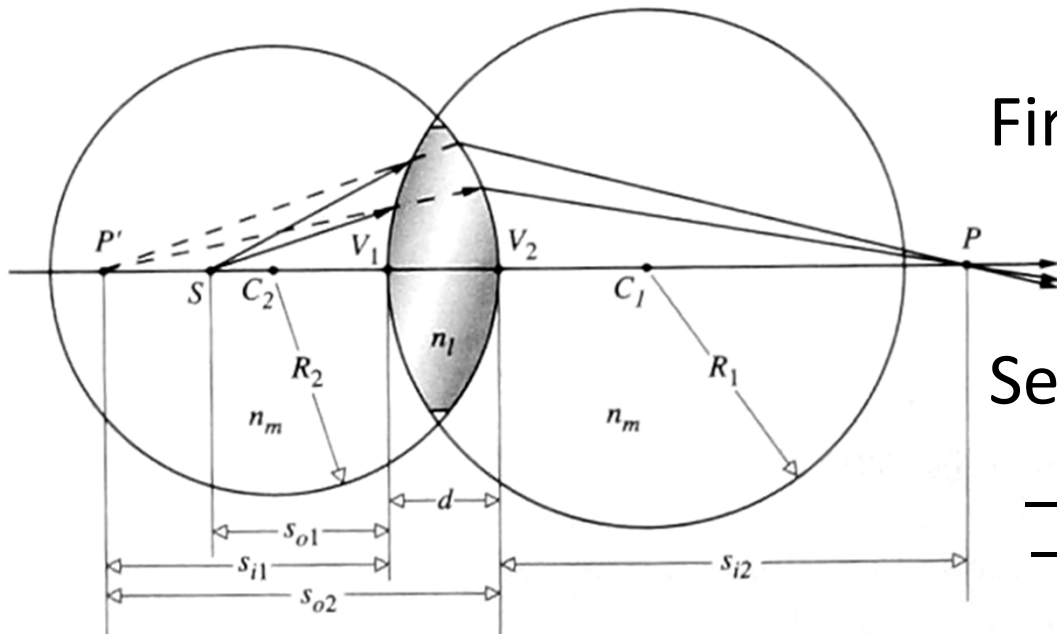


Thin Lens Classification

- A flat surface corresponds to $R \rightarrow \infty$
- All possible combinations of two surfaces:



Thin Lens Equation



First surface:

$$\frac{n_m}{s_{o1}} + \frac{n_l}{s_{i1}} = \frac{n_l - n_m}{R_1}$$

Second surface:

$$\frac{n_l}{-s_{i1} + d} + \frac{n_m}{s_{i2}} = \frac{n_m - n_l}{R_2}$$

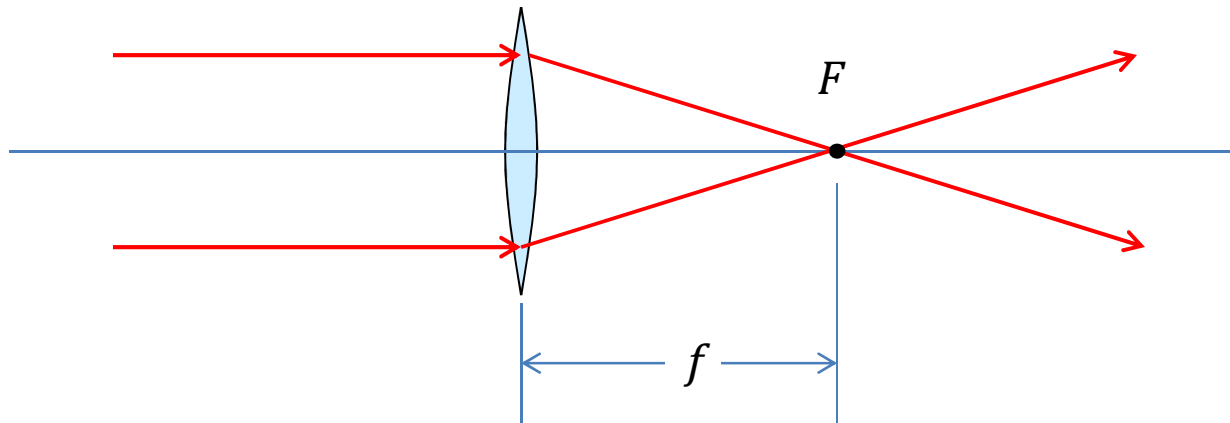
Add these equations and simplify using $n_m = 1$ and $d \rightarrow 0$:

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

(Thin lens equation)

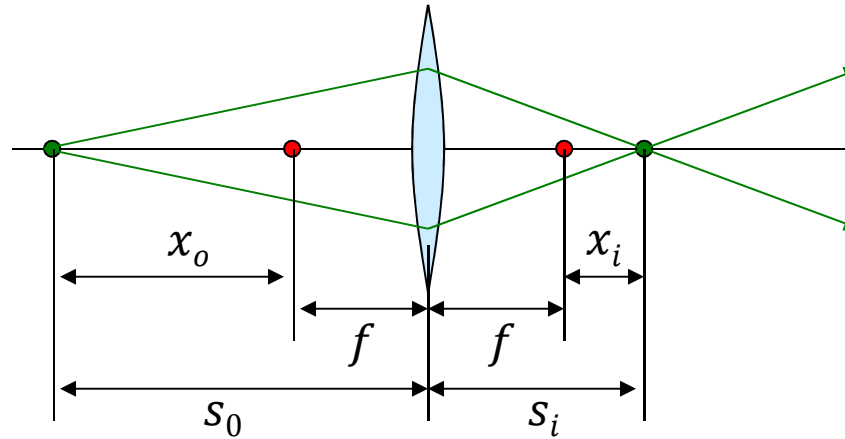
Gaussian Lens Formula

- Recall that the focal point was the place to which parallel rays were made to converge



- Parallel rays from the object correspond to $s_o \rightarrow \infty$ and $s_i \rightarrow f$: $\frac{1}{f} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$
- This lens equation: $\frac{1}{s_i} + \frac{1}{s_o} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}$

Gaussian Lens Formula



- Gaussian lens formula:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

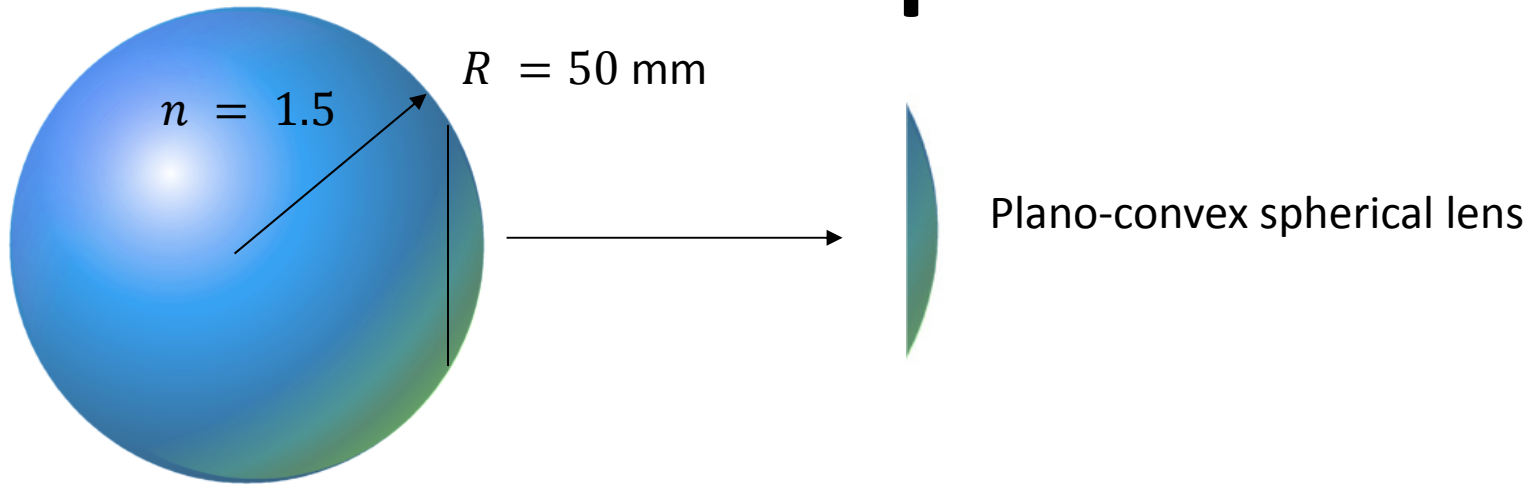
- Newtonian form:

$$x_o x_i = f^2$$

(follows from the Gaussian formula after about 5 lines of algebra)

- All you need to know about a lens is its focal length

Example



- What is the focal length of this lens?

– Let $s_o \rightarrow \infty$, then $s_i \rightarrow f$

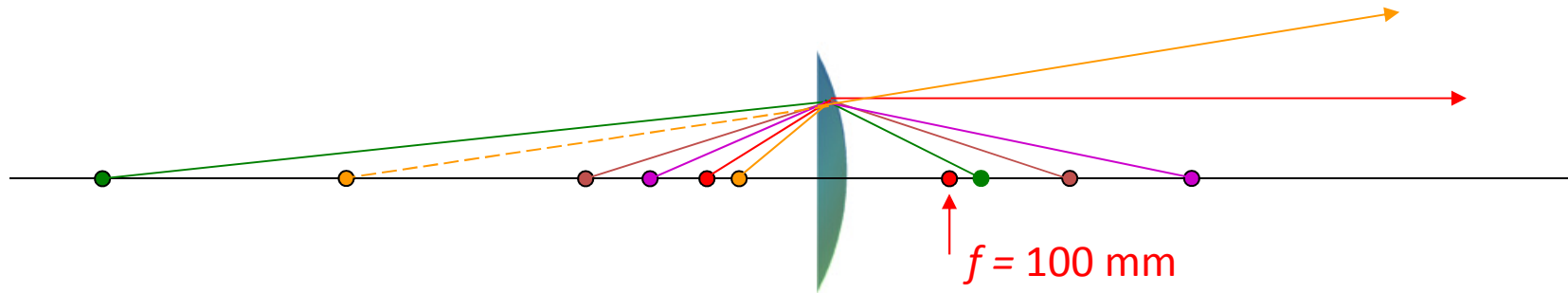
$$\frac{1}{f} = (n_l - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

– The flat surface has $R_1 \rightarrow \infty$ and we know that $R_2 = -50 \text{ mm}$

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{\infty} - \frac{1}{-50 \text{ mm}} \right) = \frac{1}{100 \text{ mm}}$$

$$\mathbf{f = 100 \text{ mm}}$$

Example

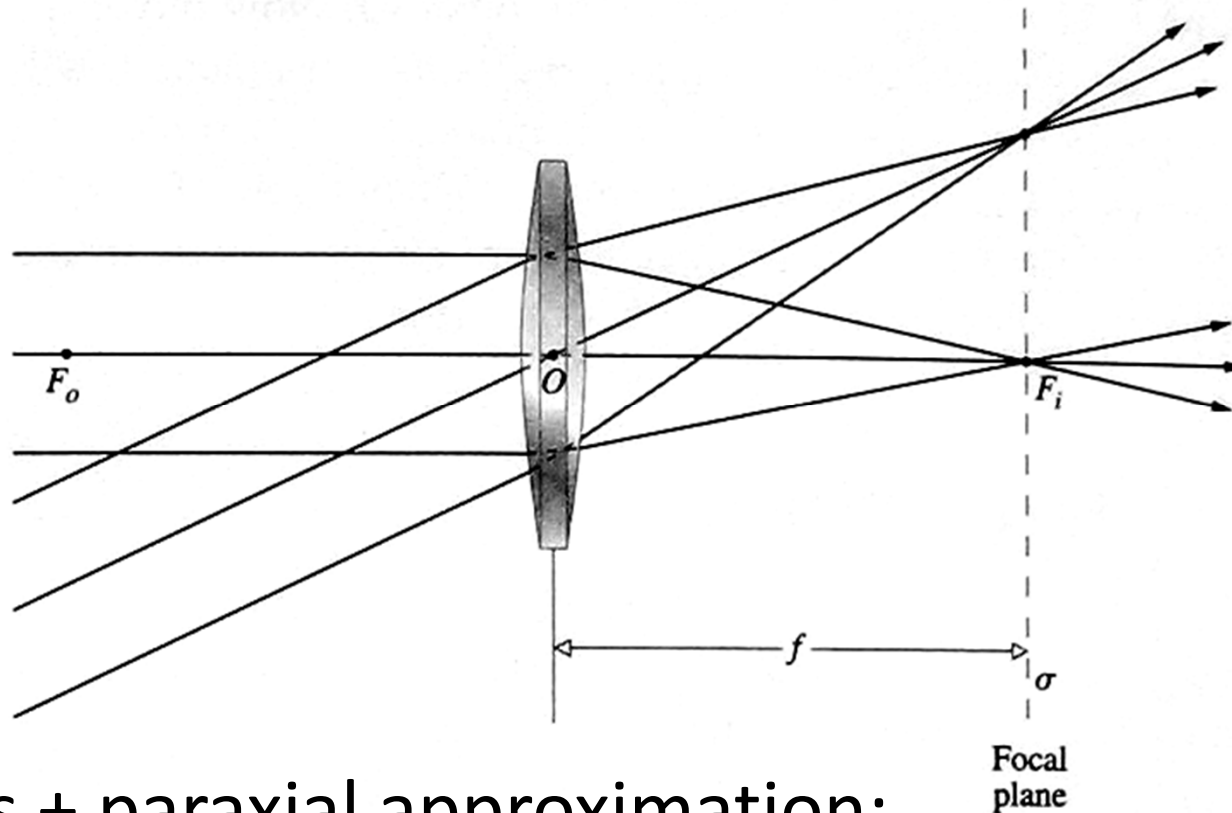


- Objects are placed at $s_o = 600$ mm, 200 mm, 150 mm, 100 mm, 80 mm
- Where are their images?

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad \Rightarrow \quad s_i = \frac{s_o f}{s_o - f}$$

$$s_i = 120 \text{ mm}, 200 \text{ mm}, 300 \text{ mm}, \infty, -400 \text{ mm}$$

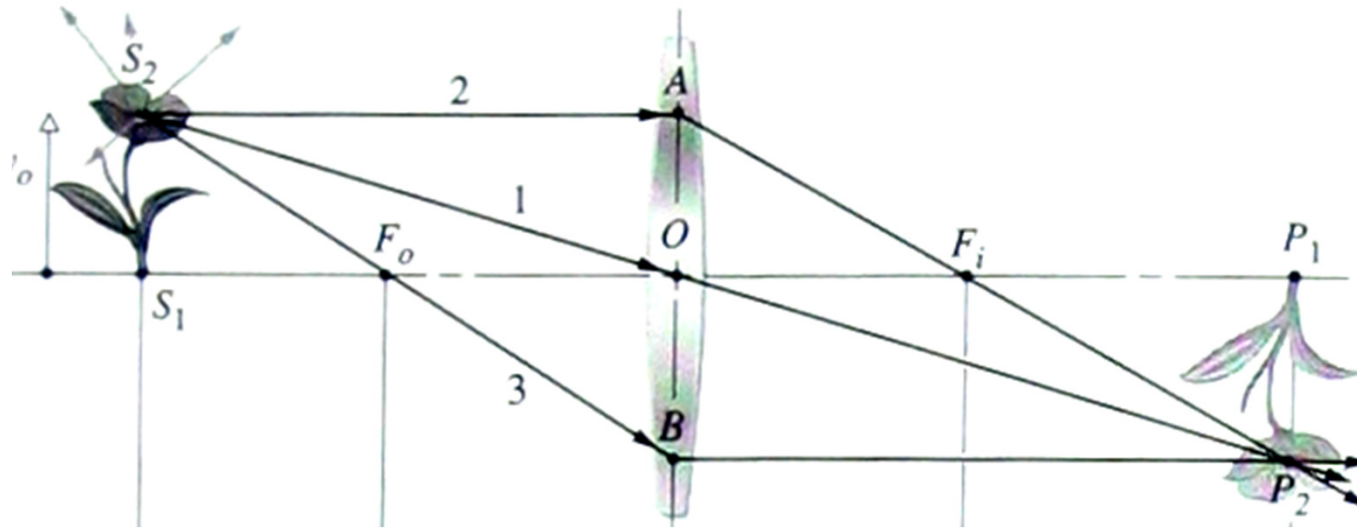
Focal Plane



Thin lens + paraxial approximation:

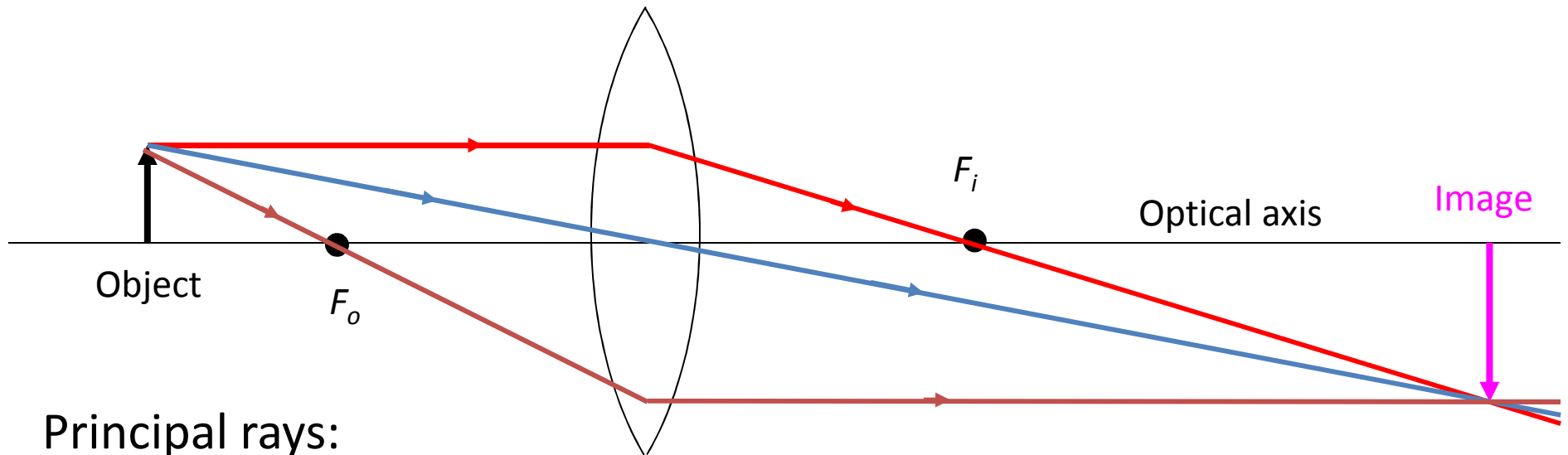
- All rays that pass through the center, O , do not bend
- All rays converge to points in the focal plane (back focal plane)
- F_o lies in the front focal plane

Imaging with a Thin Lens



- For each point on the object we can draw three rays:
 1. A ray straight through the center of the lens
 2. A ray parallel to the central axis, then through the image focal point
 3. A ray through the object focal point, then parallel to the central axis.

Converging Lens: Principal Rays



Principal rays:

- 1) Rays **parallel** to principal axis pass through focal point F_i .
- 2) Rays through **center** of lens are not refracted.
- 3) Rays **through** F_o emerge parallel to principal axis.

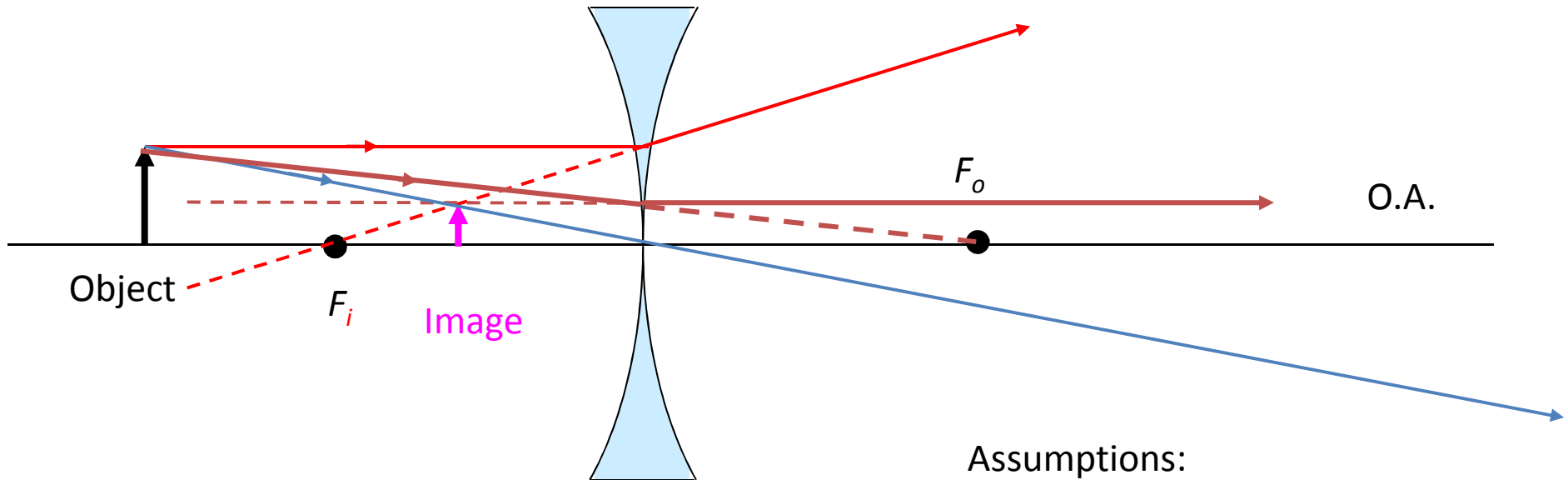
In this case image is real, inverted and enlarged

Assumptions:

- Monochromatic light
- Thin lens
- Paraxial rays (near the optical axis)

Since n is function of λ , in reality each color has different focal point: *chromatic aberration*. Contrast to mirrors: angle of incidence/reflection not a function of λ

Diverging Lens: Forming Image



Assumptions:

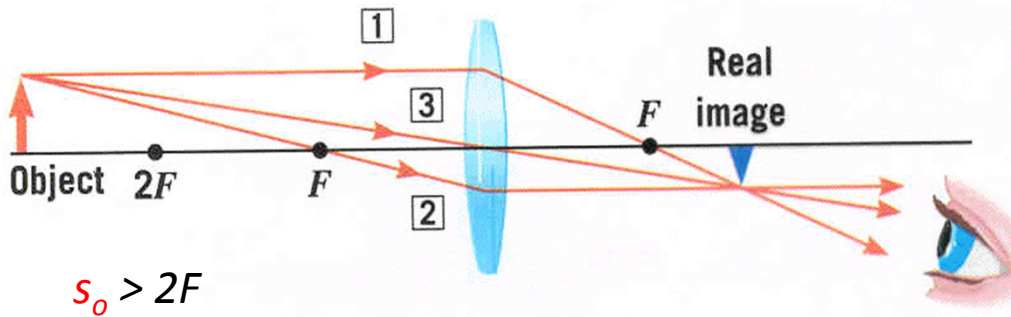
- paraxial monochromatic rays
- thin lens

Principal rays:

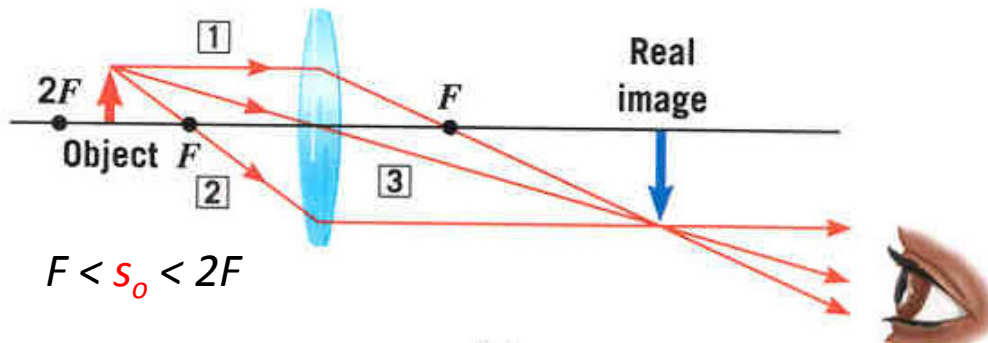
- 1) Rays **parallel** to principal axis appear to come from focal point F_i .
- 2) Rays through **center** of lens are not refracted.
- 3) Rays **toward** F_o emerge parallel to principal axis.

Image is virtual, upright and reduced.

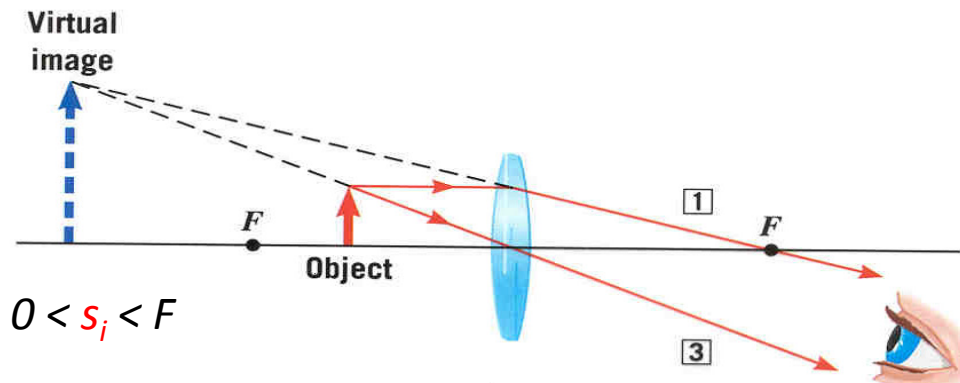
Converging Lens: Examples



This could be used in a camera. Big object on small film

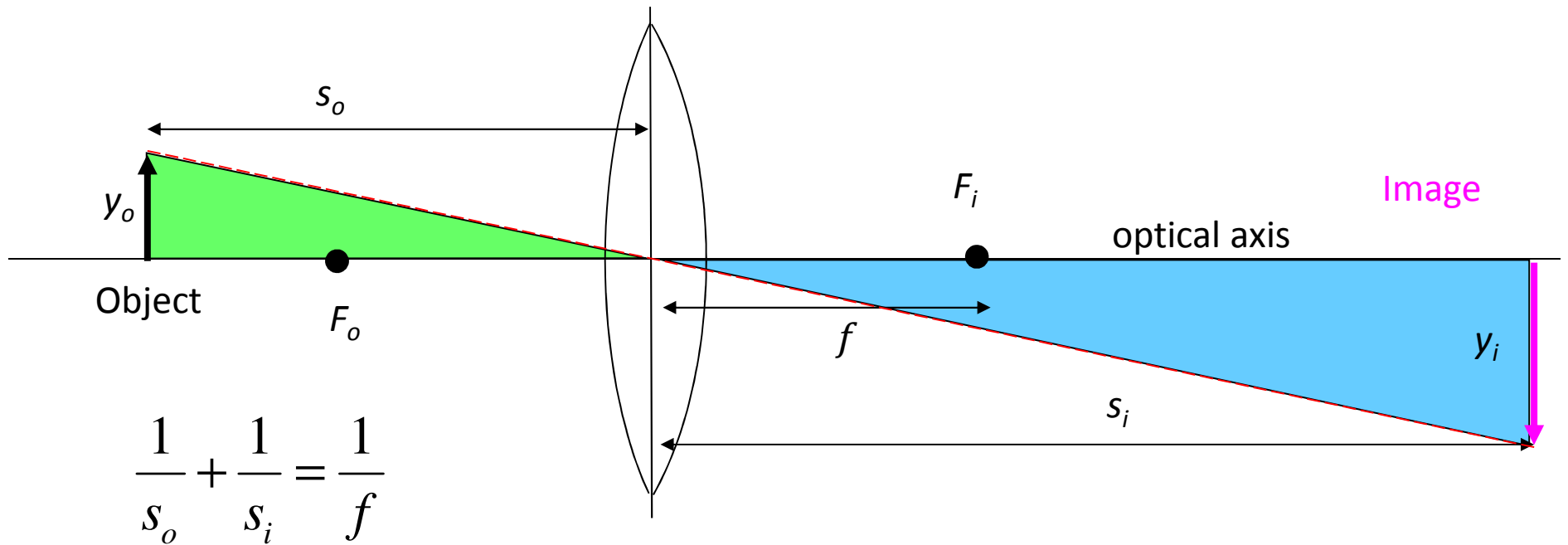


This could be used as a projector. Small slide(object) on big screen (image)



This is a magnifying glass

Lens Magnification



Green and blue triangles are similar:

Magnification equation:

$$M_T \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o}$$

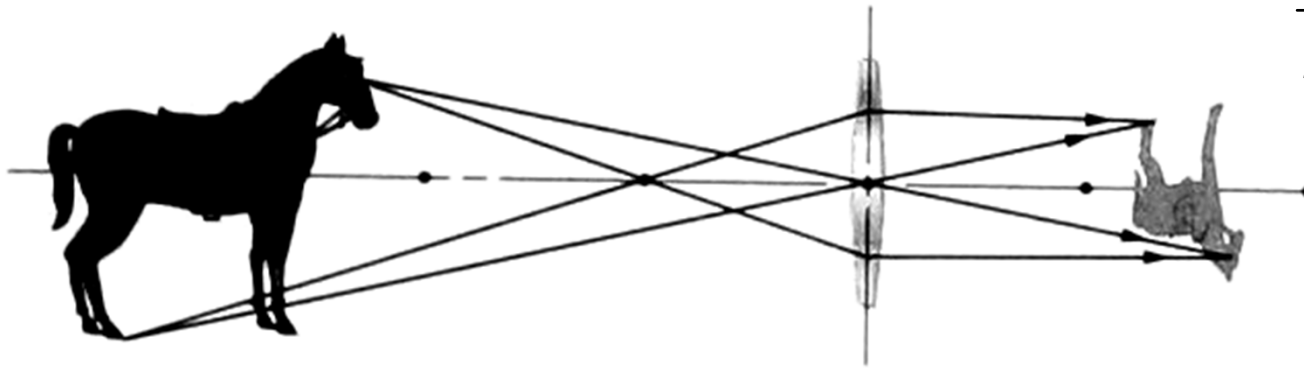
T = transverse

Example: $f=10\text{ cm}$, $s_o=15\text{ cm}$

$$\frac{1}{15\text{cm}} + \frac{1}{s_i} = \frac{1}{10\text{cm}} \quad \longrightarrow \quad s_i = 30\text{ cm}$$

$$M_T = -\frac{30\text{cm}}{15\text{cm}} = -2$$

Longitudinal Magnification



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

The 3D image of the horse is distorted:

- transverse magnification changes along optical axis
- longitudinal magnification is not linear

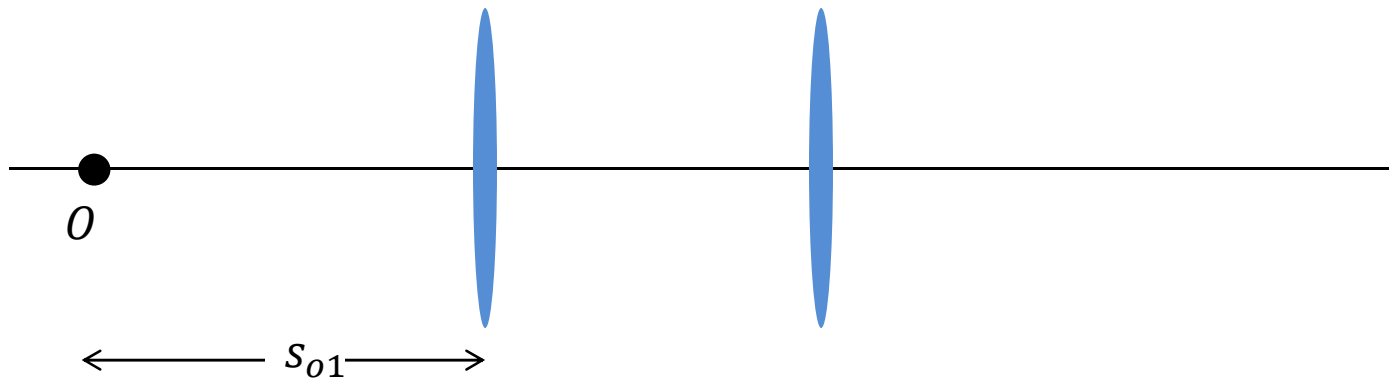
Longitudinal magnification:

$$M_L \equiv \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$$

Negative: a horse looking towards the lens forms an image that looks away from the lens

$$x_o x_i = f^2 \rightarrow x_i = f^2 / x_o \rightarrow \frac{dx_i}{dx_o} = \frac{d}{dx_o} (f^2 / x_o) = -(f^2 / x_o^2)$$

Two Lens Systems

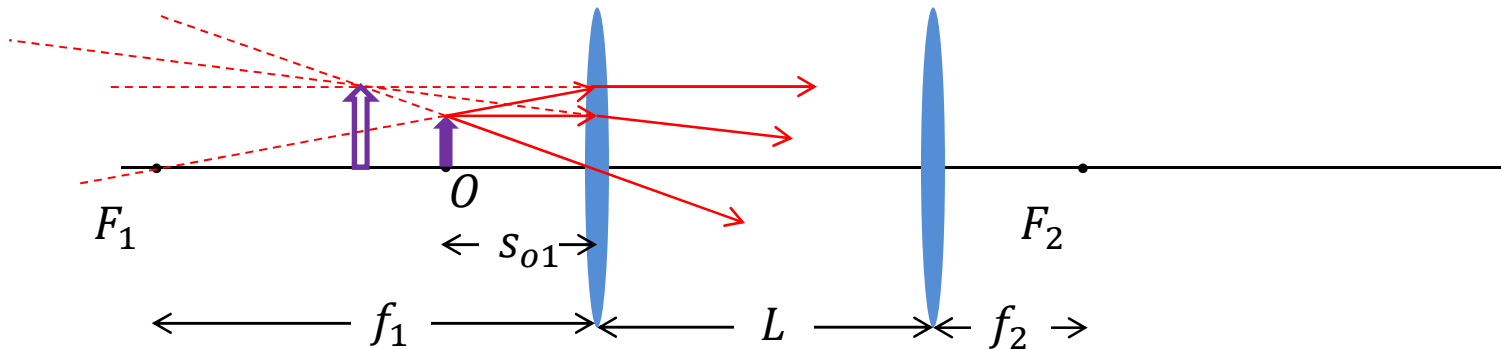


- Calculate s_{i1} using $\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}}$
- Ignore the first lens, treat s_{i1} as the object distance for the second lens. Calculate s_{i2} using $\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$
- Overall magnification: $M = m_1 m_2 = \left(-\frac{s_{i1}}{s_{o1}}\right) \left(-\frac{s_{i2}}{s_{o2}}\right)$

Example: Two Lens System

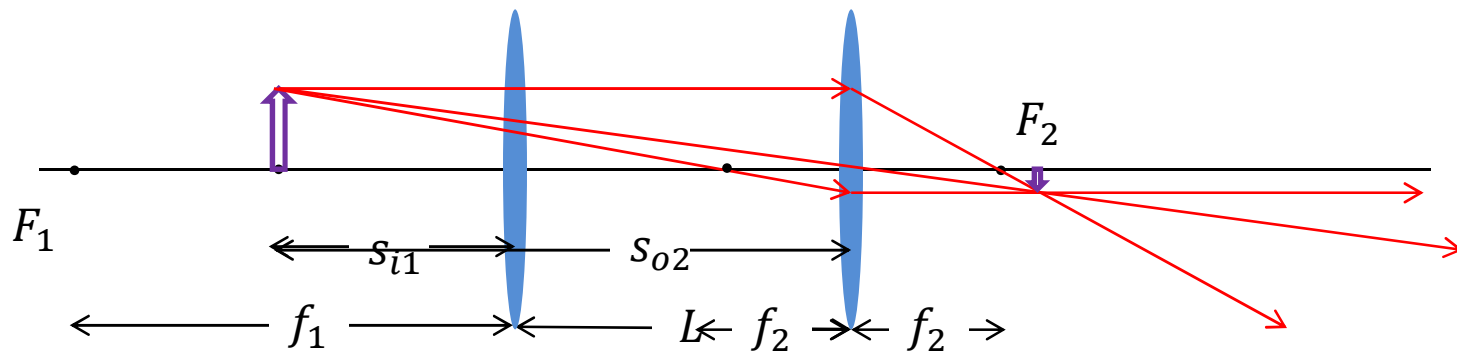
An object is placed in front of two thin symmetrical coaxial lenses (lens 1 & lens 2) with focal lengths $f_1=+24$ cm & $f_2=+9.0$ cm, with a lens separation of $L=10.0$ cm. The object is 6.0 cm from lens 1.

Where is the image of the object?



Example: Two Lens System

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(not really to scale...)

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Where is the image of the object?

Lens 1: $\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \quad \longrightarrow \quad s_{i1} = -8 \text{ cm}$

Image 1 is virtual.

Lens 2: Treat image 1 as O_2 for lens 2. O_2 is outside the focal point of lens 2. So, image 2 will be real & inverted on the other side of lens 2.

$$\begin{aligned} s_{o2} &= L - s_{i1} & \longrightarrow & \quad s_{i2} = 18.0 \text{ cm} \\ \frac{1}{f_2} &= \frac{1}{s_{o2}} + \frac{1}{s_{i2}} & \text{Image 2 is real.} \end{aligned}$$

Magnification: $M_T = \left(-\frac{-8 \text{ cm}}{6 \text{ cm}}\right) \left(-\frac{18 \text{ cm}}{18 \text{ cm}}\right) = -1.33$