Assignment #7

1. (a) At normal incidence the reflection coefficient is

$$r_\perp = \frac{n_1 - n_2}{n_1 + n_2}$$

where $n_1 = 1$ and $n_2 = 1.33$.
The intensity of reflected light is

$$I_R = r_\perp^2 I_i$$

Thus, $\frac{I_R}{I_i} = r_\perp^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{1 - 1.33}{1 + 1.33}\right)^2$

$$= 0.020$$

(b) Brewster's angle θ_P is defined such that

$$\theta_P + \theta_t = \pi/2 .$$

or $\theta_P = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}(1.33) = 53.1^\circ$

(c) When light is incident at an angle of θ_P,

$$\theta_t = \sin^{-1}\left(\frac{n_1 \sin \theta_P}{n_2}\right) = 36.9^\circ$$

Fresnel's equations give

$$r_\perp = -\frac{\sin(\theta_P - \theta_t)}{\sin(\theta_P + \theta_t)} \qquad r_\parallel = 0 \quad \text{when } \theta_i = \theta_P.$$

and $\frac{I_R}{I_i} = \frac{r_\perp^2}{2} = \frac{1}{2}\frac{\sin^2(\theta_P - \theta_t)}{\sin^2(\theta_P + \theta_t)} = \frac{\sin^2(16.2^\circ)}{2\sin^2(90^\circ)} = \frac{\sin^2 16.2^\circ}{2}$

$$= \frac{0.078}{2} = 0.039.$$

2. The optical rotation is expected to vary linearly with respect to the path length and/concentration:

$$\alpha = \alpha_0 \left(\frac{L}{L_0} \right) \left(\frac{c}{c_0} \right)$$

where $L_0 = 10$ cm is the reference path length and $c_0 = 1$ g/cm^3 is the reference concentration.

When $L = 100$ cm and $c = 10$g/1000 cm^3 the optical rotation will be

$$\alpha = +66.45° \left(\frac{100 \text{ cm}}{10 \text{ cm}} \right) \left(\frac{0.01 \text{g/cm}^3}{1 \text{ g/cm}^3} \right) = (66.45°)(0.1)$$

$$= +6.645° .$$

3. The Jones matrix is

$$J = \begin{pmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{pmatrix}$$

(a) Light polarized at an angle θ to the horizontal has a Jones vector given by

$$\tilde{E} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}.$$

The emerging light is then

$$\tilde{E}' = J\tilde{E} = \begin{pmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\alpha\cos\theta + \cos\alpha\sin\alpha\sin\theta \\ \cos\alpha\sin\alpha\cos\theta + \sin^2\alpha\sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha(\cos\alpha\cos\theta + \sin\alpha\sin\theta) \\ \sin\alpha(\cos\alpha\cos\theta + \sin\alpha\sin\theta) \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha\cos(\alpha-\theta) \\ \sin\alpha\cos(\alpha-\theta) \end{pmatrix}$$

$$= \cos(\alpha-\theta)\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

(b) The resulting light is polarized at an angle α with respect to the horizontal axis. The filter is therefore a linear polarizing filter with its transmission axis at an angle α.

(c) Suppose $\theta = \alpha$. Then $\cos(\alpha-\theta) = 1$ and the light emerges with no change in polarization or intensity, as expected.