

Physics 42200

Waves & Oscillations

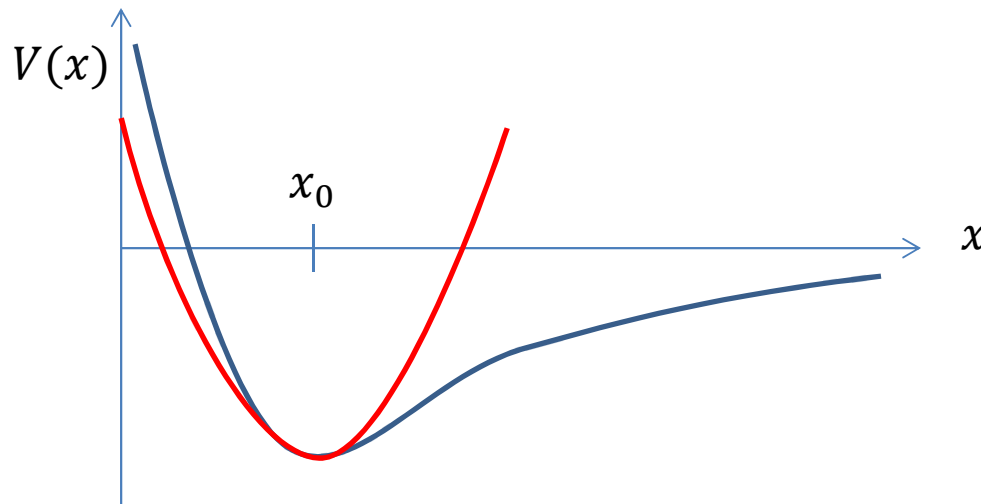
Lecture 9 – French, Chapter 4

Spring 2014 Semester

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Comments on the Assignment

- Question 1 – Primarily concerned with the case when $\gamma \ll \omega$ (weak damping).
 - In this case $\omega \approx \omega_0$ and the total energy changes much more slowly than the position, $x(t)$.
- Question 2 – A graph will help a lot...



Forced Oscillators and Resonance

$$m\ddot{x} + b\dot{x} + kx = F_0 \cos \omega t$$

- Natural oscillation frequency:

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad \omega_{free} = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}$$

- Amplitude of steady-state oscillations:

$$A = \frac{F_0/m}{\sqrt{((\omega_0)^2 - \omega^2)^2 + (\omega\gamma)^2}}$$

- Phase difference:

$$\delta = \tan^{-1} \left(\frac{\omega\gamma}{(\omega_0)^2 - \omega^2} \right)$$

Resonance Phenomena

- Change of variables:

$$\gamma = \frac{b}{m} \quad Q = \frac{\omega_0}{\gamma} \quad \begin{array}{l} \text{Strong damping force} \rightarrow \text{large } \gamma \\ \text{Strong damping force} \rightarrow \text{small } Q \end{array}$$

- Natural oscillation frequency:

$$\omega_0 = \sqrt{\frac{k}{m}} \quad \omega_{free} = \sqrt{\frac{k}{m} - \frac{\gamma^2}{2}} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

- Amplitude of steady-state oscillations:

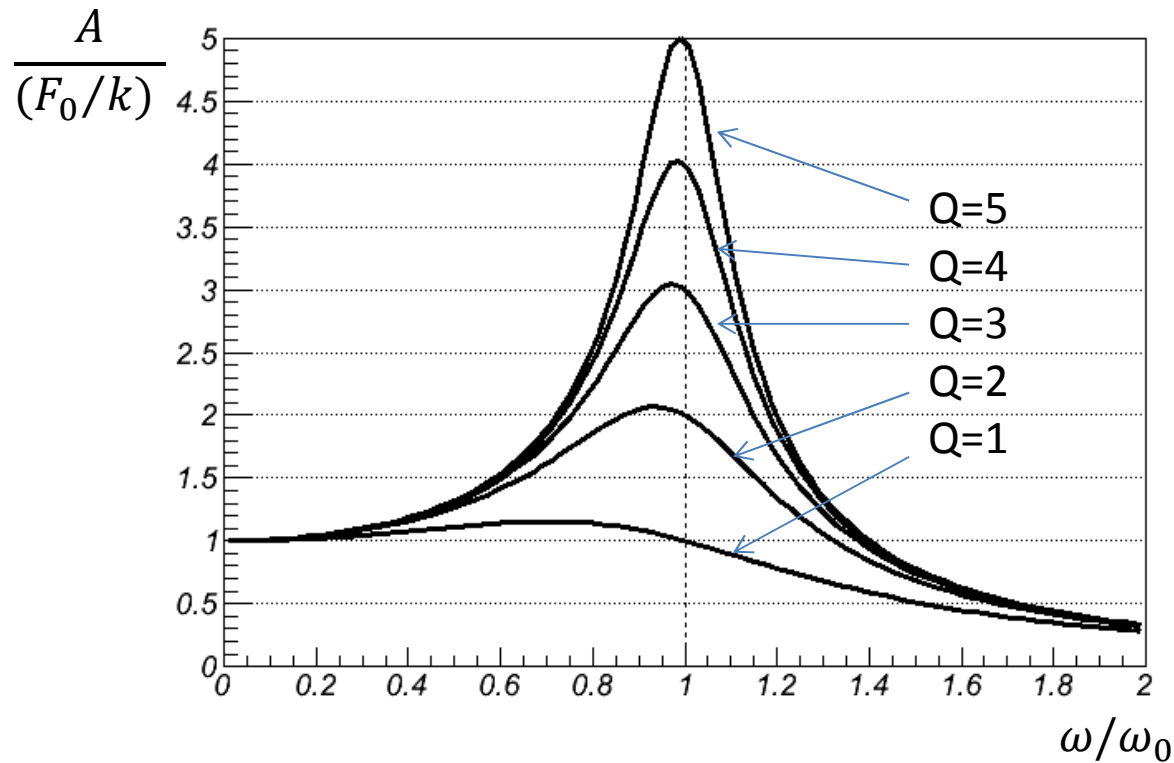
$$A(\omega) = \frac{F_0}{k} \frac{\omega_0/\omega}{\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)^2 + \frac{1}{Q^2} \right]^{1/2}}$$

- Phase difference:

$$\delta = \tan^{-1} \left(\frac{1/Q}{\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}} \right)$$

Resonance Phenomena

- Steady state amplitude:

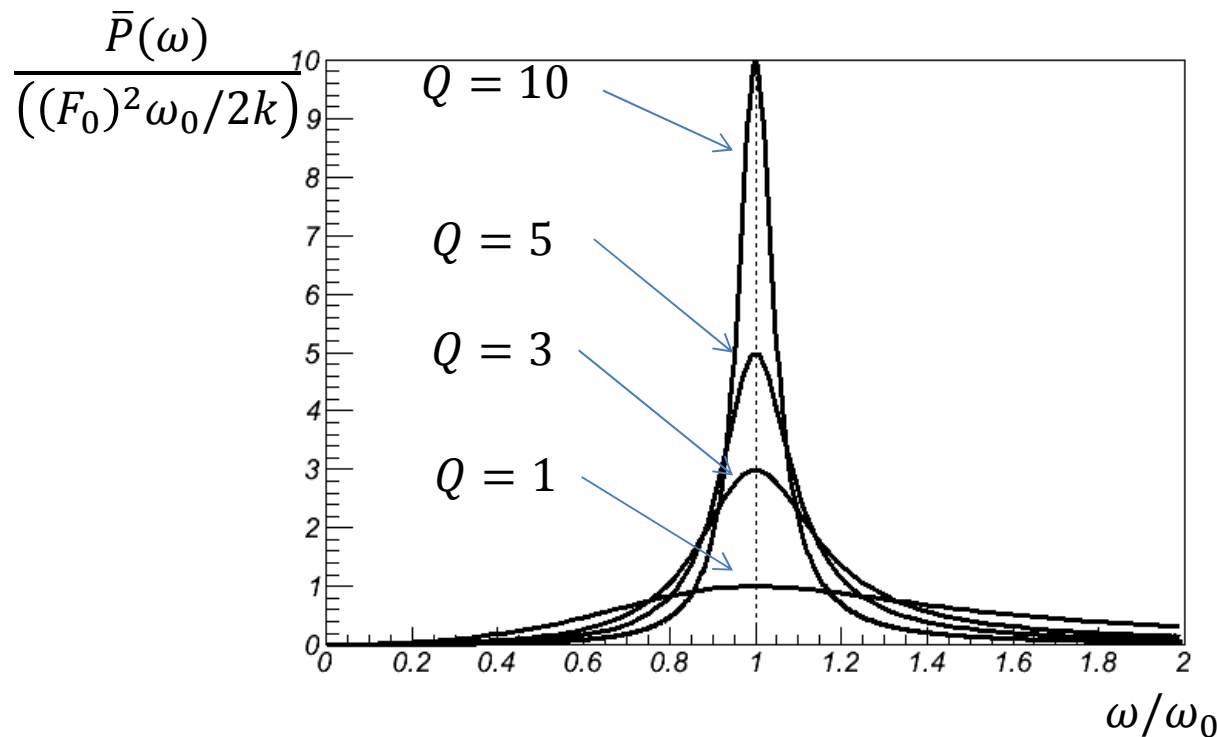


Peak is near $\omega/\omega_0 \approx 1$. The peak occurs at *exactly* $\omega/\omega_{free} = 1$.

Average Power

- The rate at which the oscillator absorbs energy is:

$$\bar{P}(\omega) = \frac{(F_0)^2 \omega_0}{2kQ} \frac{1}{\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2 + \frac{1}{Q^2}}$$

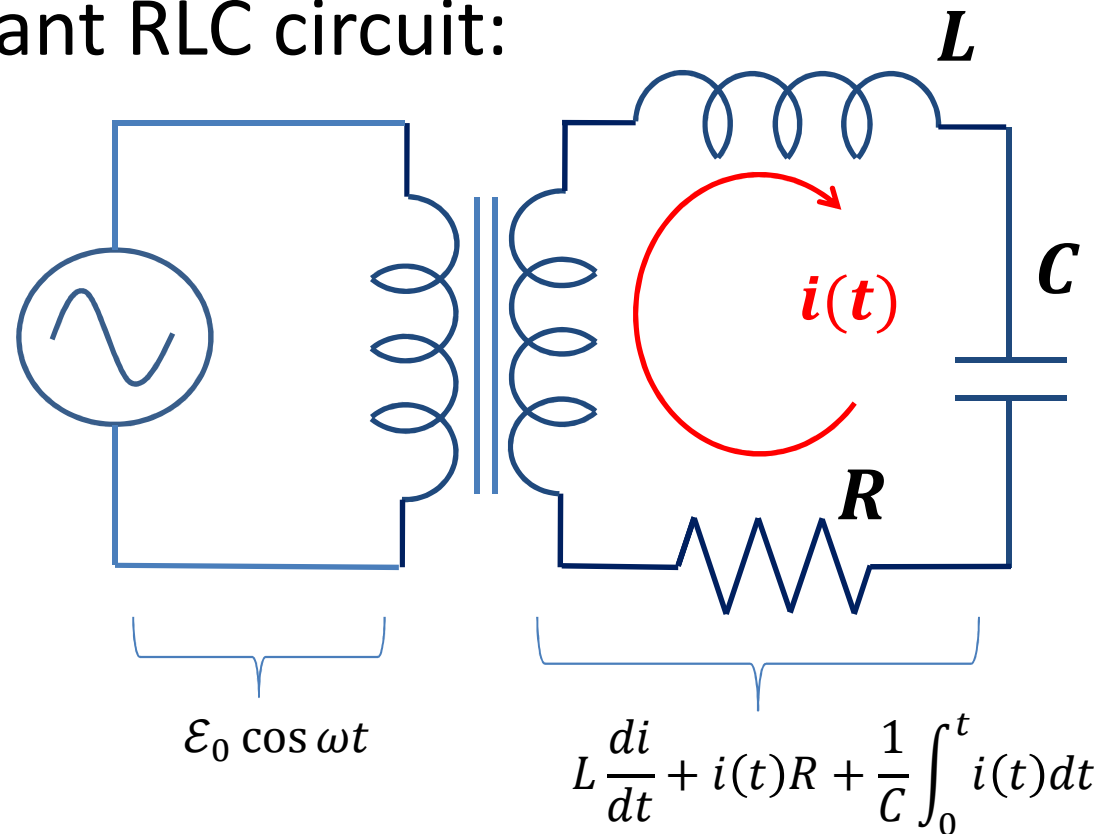


Full-Width-at-Half-Max:

$$FWHM = \frac{\omega_0}{Q} = \gamma$$

Examples

- Resonant RLC circuit:



The transformer does not play a role in the analysis of the circuit. It is just a convenient way to isolate the driving voltage source from the part of the circuit that oscillates.

Resonant Circuit

$$L \frac{di}{dt} + i(t)R + \frac{1}{C} \int_0^t i(t)dt = \mathcal{E}_0 \cos \omega t$$

- Differentiate once:

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i(t) = -\mathcal{E}_0 \omega \sin \omega t$$

- Redefine what we mean by “ $t = 0$ ”:

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i(t) = \mathcal{E}_0 \omega \cos \omega t$$

- Change of variables:

$$\frac{d^2 i}{dt^2} + \gamma \frac{di}{dt} + (\omega_0)^2 i(t) = \frac{\mathcal{E}_0 \omega}{L} \cos \omega t$$

Resonant Circuit

$$\frac{d^2 i}{dt^2} + \gamma \frac{di}{dt} + (\omega_0)^2 i(t) = \frac{\mathcal{E}_0 \omega}{L} \cos \omega t$$

- Amplitude of steady state current oscillations:

$$A(\omega) = \mathcal{E}_0 \omega C \frac{\omega_0 / \omega}{\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)^2 + \frac{1}{Q^2} \right]^{1/2}}$$

- Voltage across the capacitor:

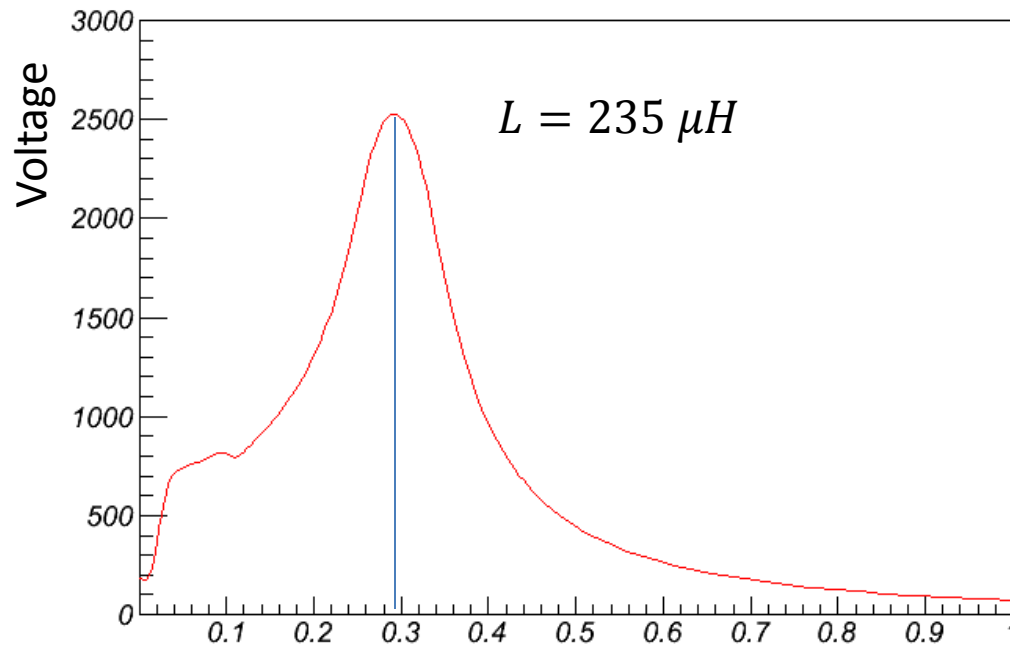
$$V(t) = i(t) X_C = \frac{i(t)}{\omega C}$$

- Amplitude of voltage oscillations measured across C:

$$V(\omega) = \mathcal{E}_0 \frac{\omega_0 / \omega}{\left[\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)^2 + \frac{1}{Q^2} \right]^{1/2}}$$

Actual Data

- If we know that $L = 235 \mu H$, can we estimate R and C ?



Peak voltage is at

$$f_{peak} = 290 \text{ kHz}$$

$$\omega_{peak} = 2\pi(f_{peak})$$

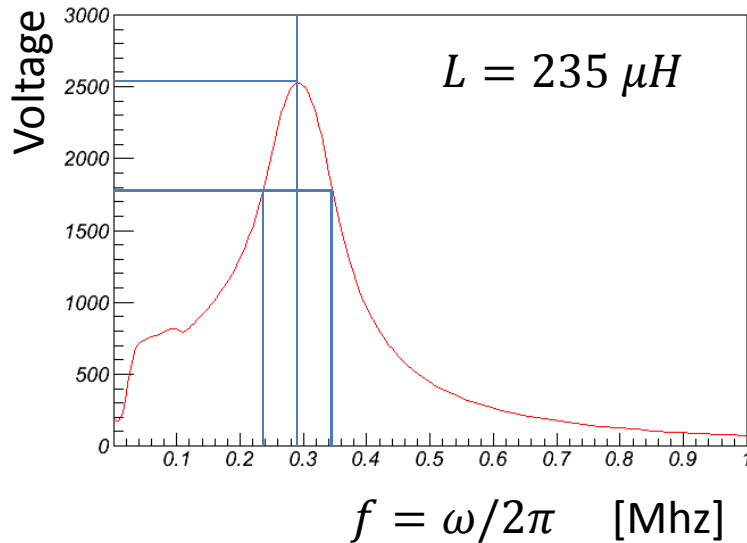
$$f = \omega/2\pi$$

[Mhz]

Energy

- Energy stored on a capacitor is $E = \frac{1}{2} CV^2$
- The graph of the stored energy is proportional to the square of the voltage graph.
- If we defined $2\Delta\omega$ as the FWHM on the graph of power vs frequency, then it will correspond to $1/\sqrt{2}$ of the peak voltage.

Resonant Circuit



$$2\Delta f = 110 \text{ kHz}$$

$$\gamma = 2\pi(\Delta f) = 6.91 \times 10^5 \text{ s}^{-1}$$

But $\gamma = R/L$ so we can find R :

$$R = \gamma L = (6.91 \times 10^5 \text{ s}^{-1})(235 \mu H) = 162 \Omega$$

Peak position is $\omega_{peak} = \sqrt{\frac{1}{LC} - \frac{\gamma^2}{2}} \rightarrow C = \frac{1}{L\left((\omega_{peak})^2 + \frac{\gamma^2}{2}\right)}$

$$C = 1.20 \text{ nF}$$

Lifetime of Oscillations

- Amplitude of a damped harmonic oscillator:

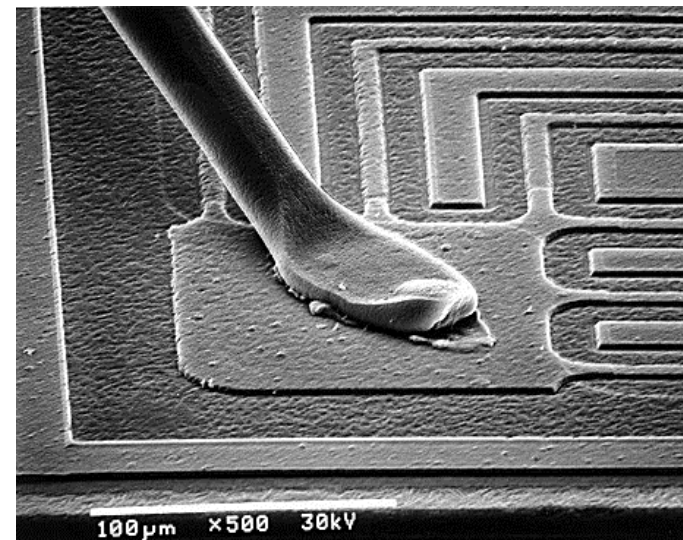
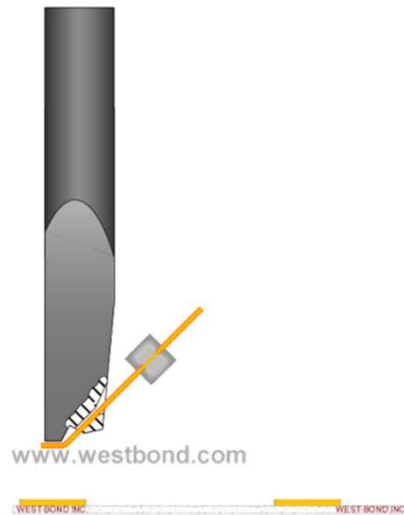
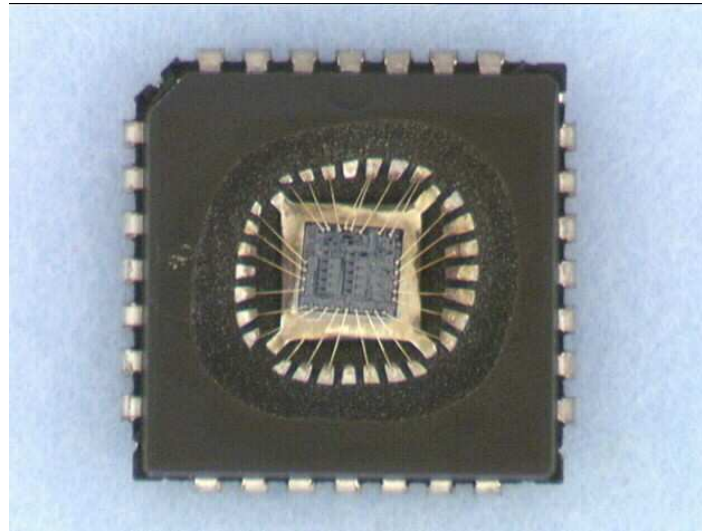
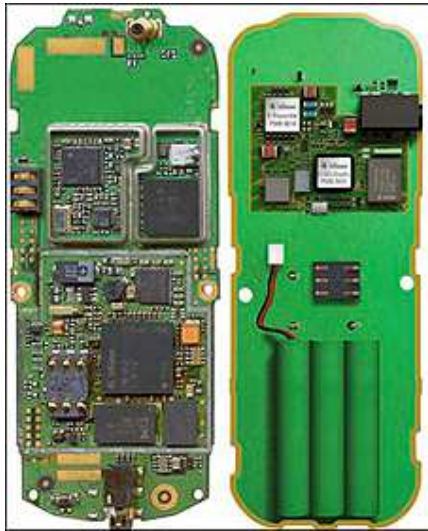
$$x(t) = A e^{-\gamma t/2} \cos \omega t$$

- Maximum potential energy:

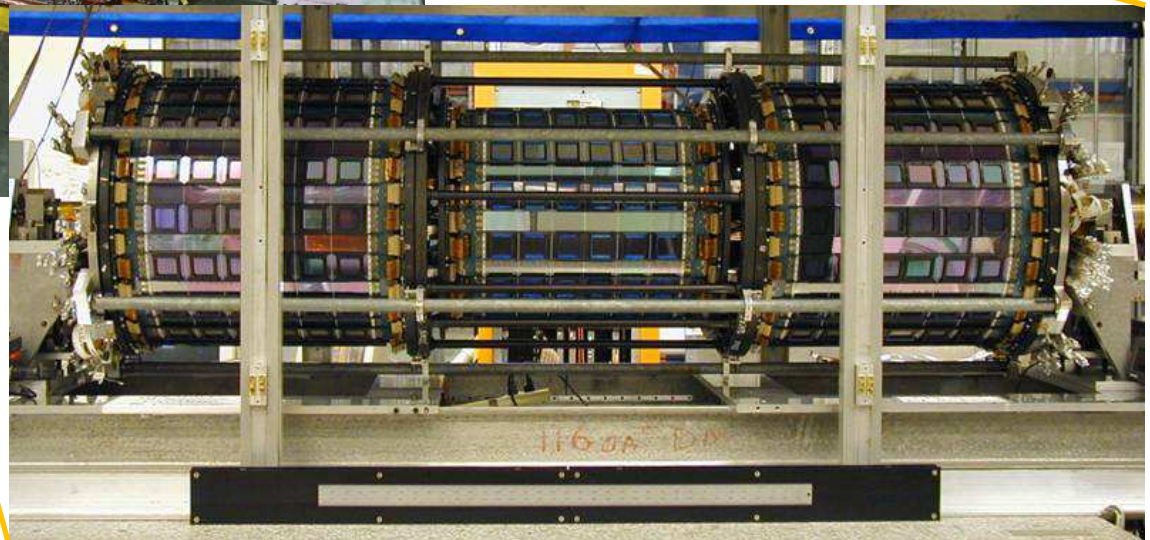
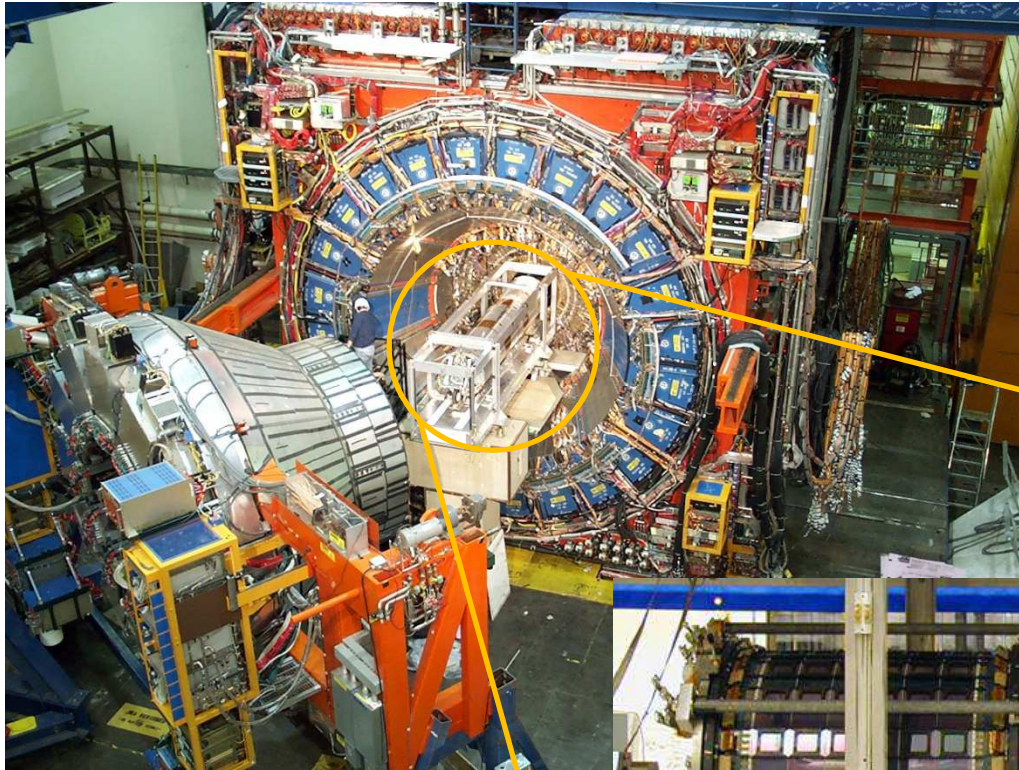
$$U = \frac{1}{2} k x^2 \propto e^{-\gamma t}$$

- After time $t = 1/\gamma$, the energy is reduced by the factor $1/e$.
- We call $\tau = 1/\gamma$ the “lifetime” of the oscillator.

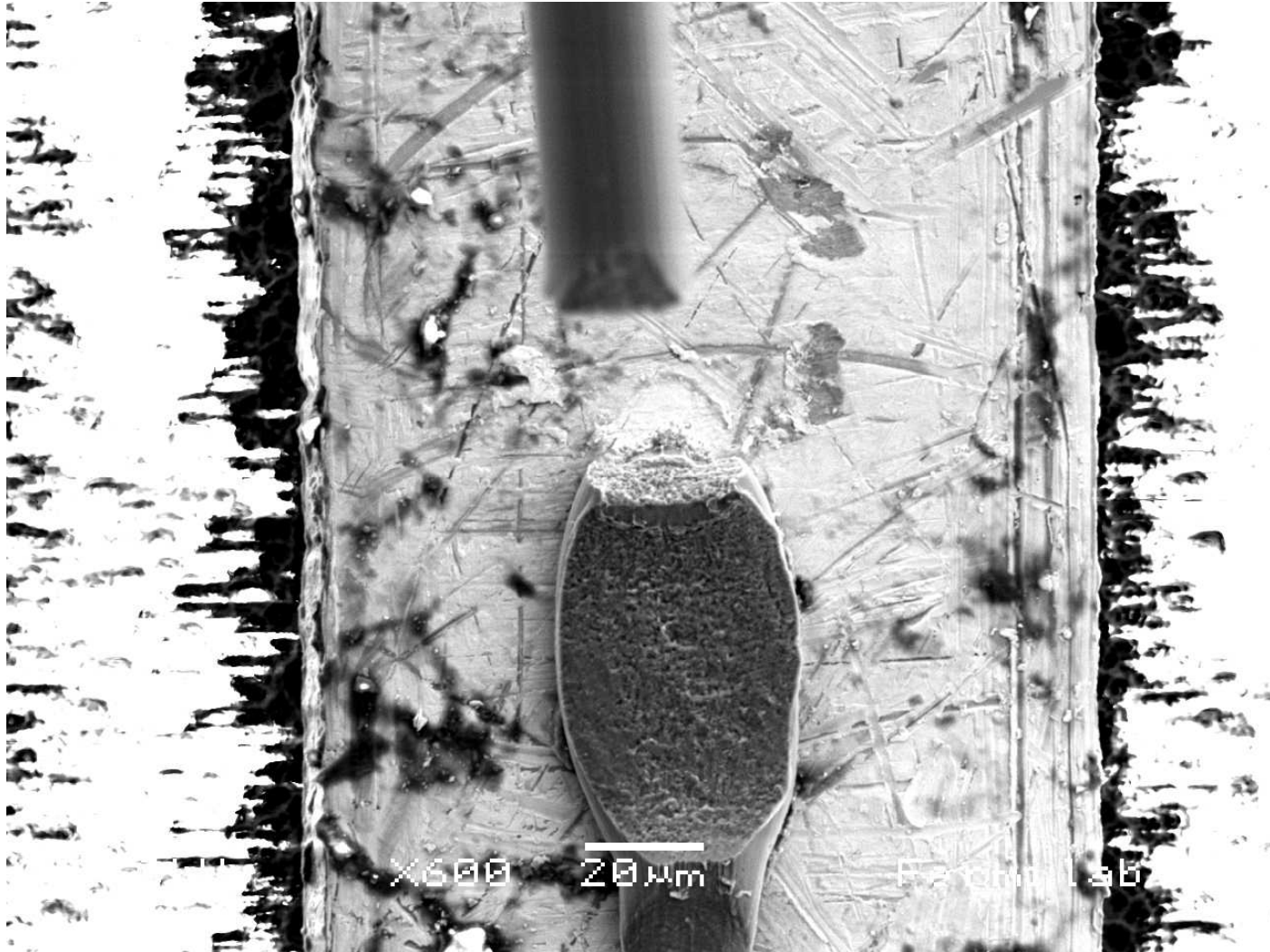
Other Resonant Systems



Other Resonant Systems

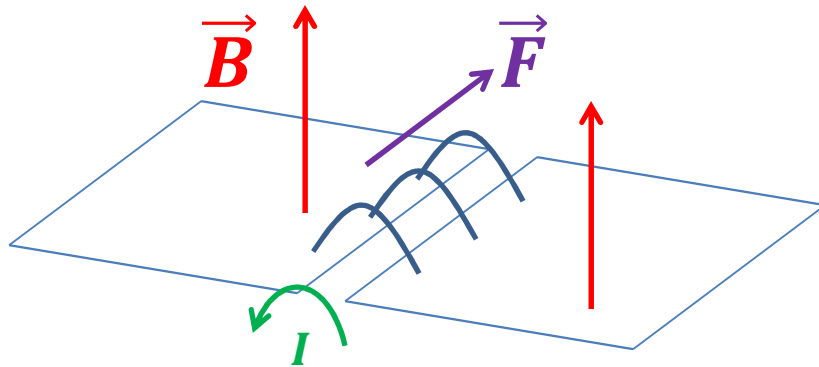


Other Resonant Systems



Wire Bond Resonance

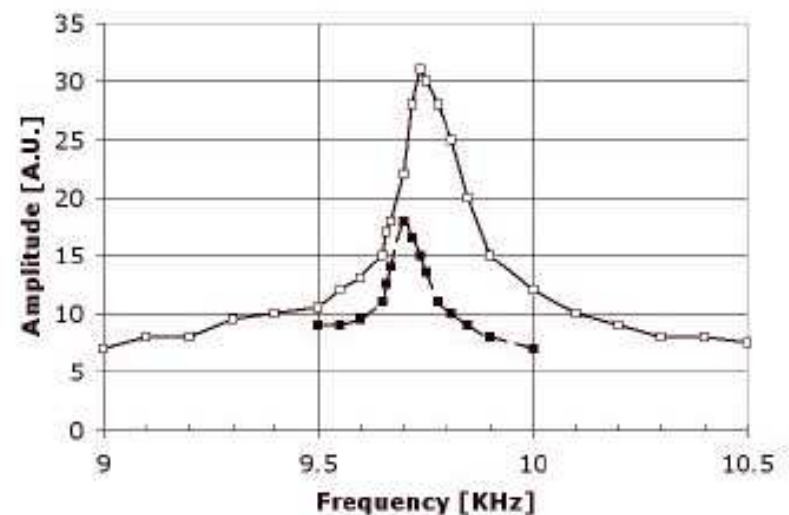
- Wire bonds in a magnetic field:



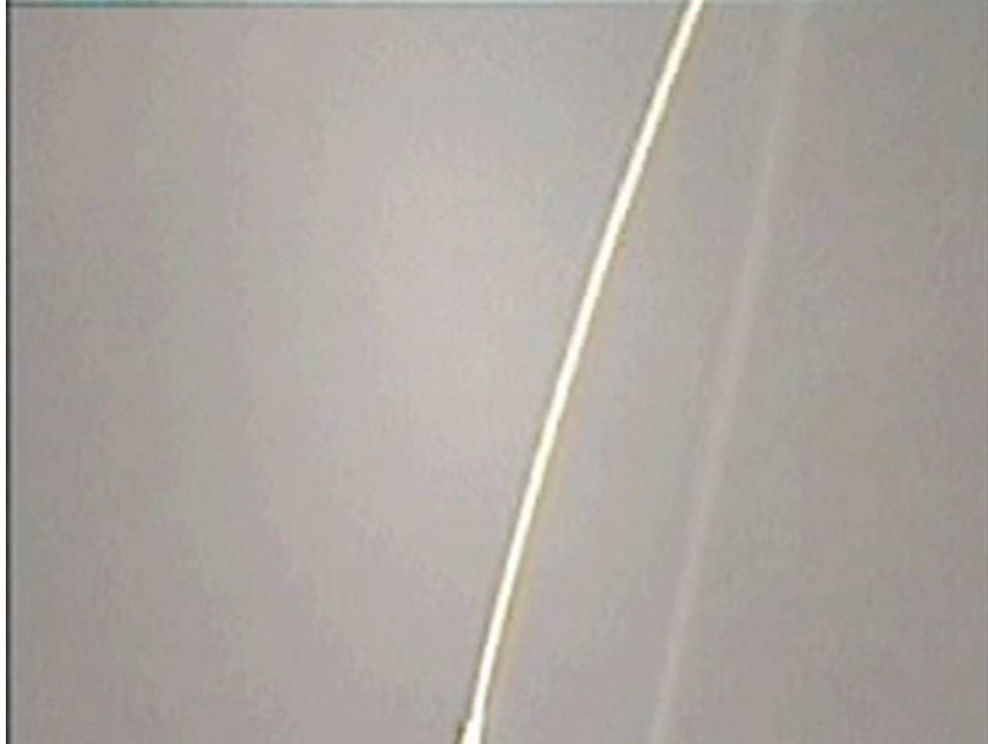
Lorentz force is $\vec{F} = I \int d\vec{\ell} \times \vec{B}$

The tiny wire is like a spring.

A periodic current produces the driving force.

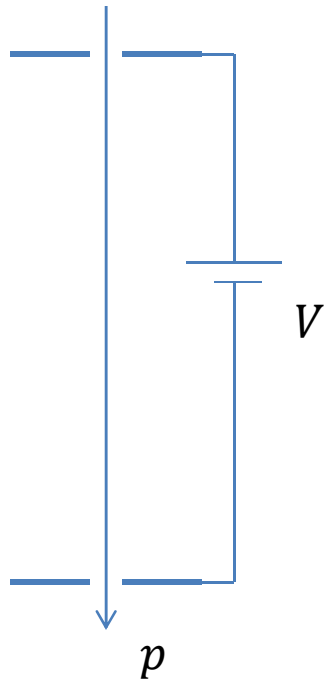


Wire Bond Resonance



Resonance in Nuclear Physics

- A proton accelerated through a potential difference V gains kinetic energy $T = eV$:



Phys. Rev. 75, 246 (1949).

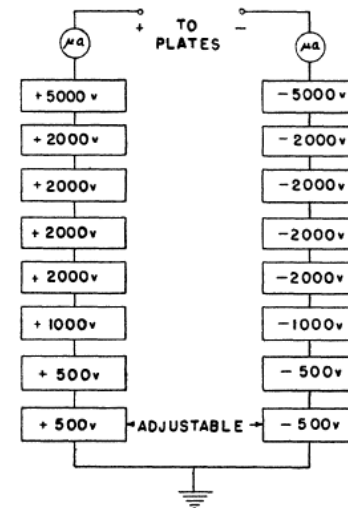


FIG. 1. Block diagram of battery stacks. Adjustable 500-volt boxes were set to any voltage below 500 volts by means of a potentiometer. The polarity of any of the boxes except the adjustable 500-volt box could be selected at will for comparison purposes.

* 5000 volt boxes used Eveready No. 493, 300 volt batteries. All other batteries were of the Burgess XX45, 67½ volt type except for several heavier duty batteries under continuous drain to provide continuous range of adjustments.

** The actual voltage is 504.08 Int. volts and is determined by the resistor divider ratio and the 1.50000 volt setting on the potentiometer.

Resonance in Nuclear Physics

- In quantum mechanics, energy and frequency are proportional:

$$E = \hbar\omega$$

- A given energy corresponds to a driving force with frequency ω .
- When a nucleus resonates at this frequency, the proton energy is easily absorbed.

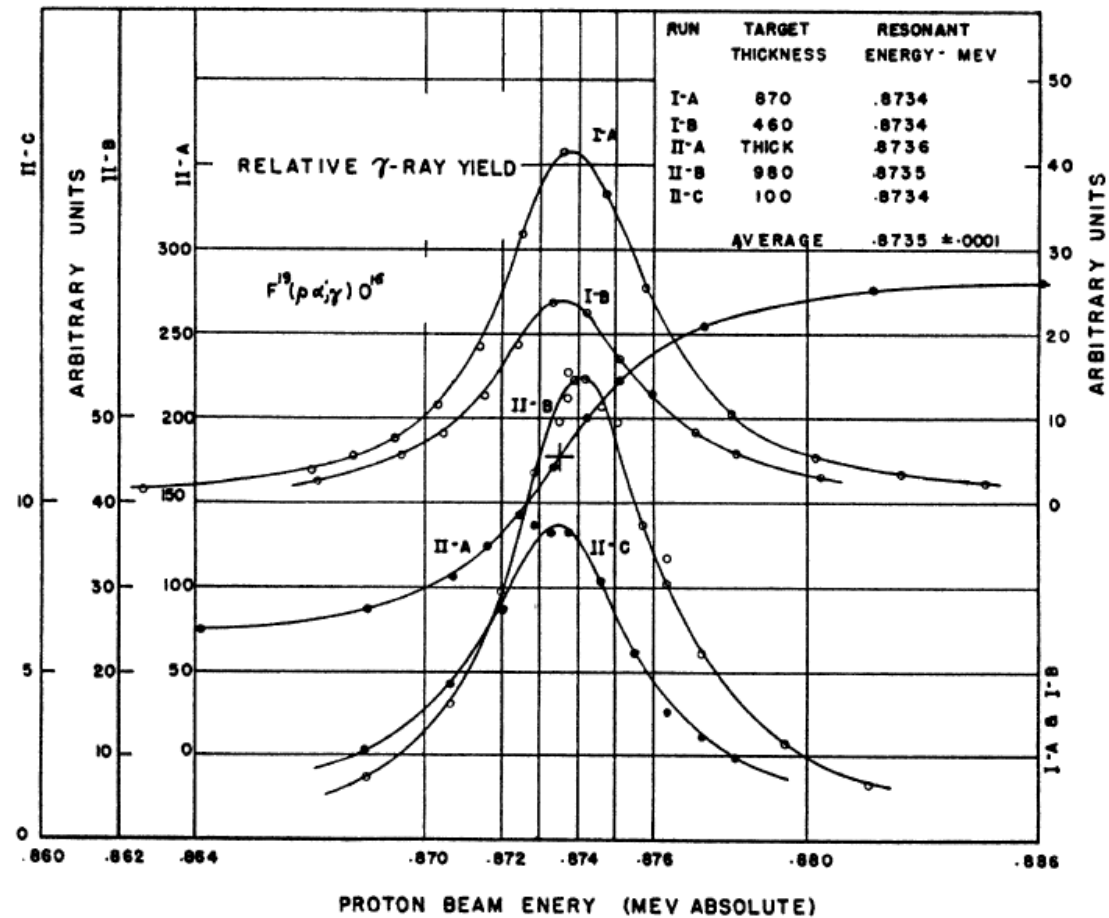
Nuclear Resonance

ABSOLUTE VOLTAGE DETERMINATION

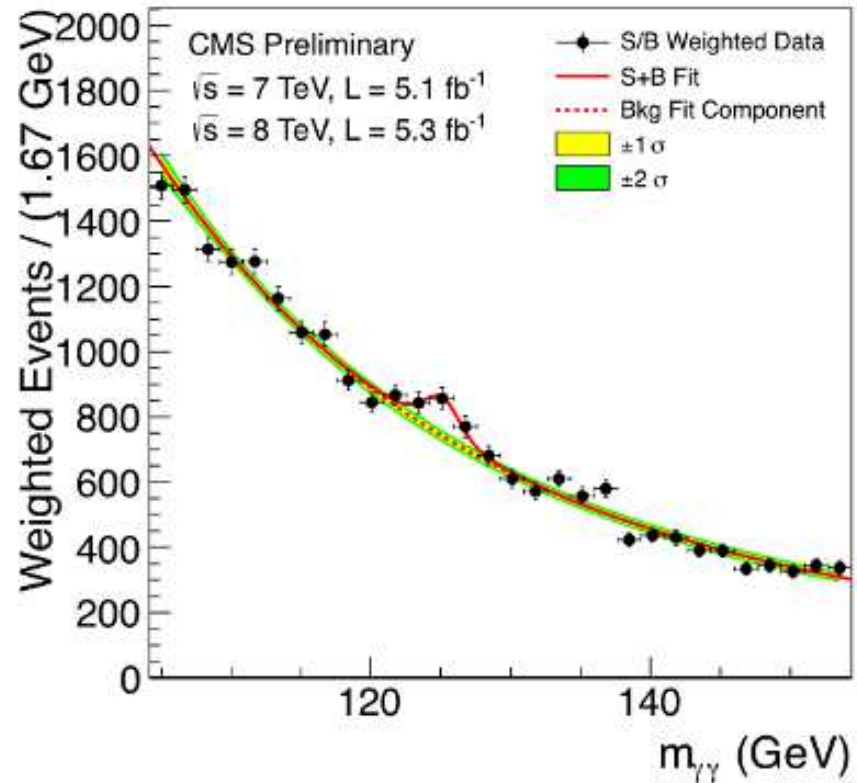
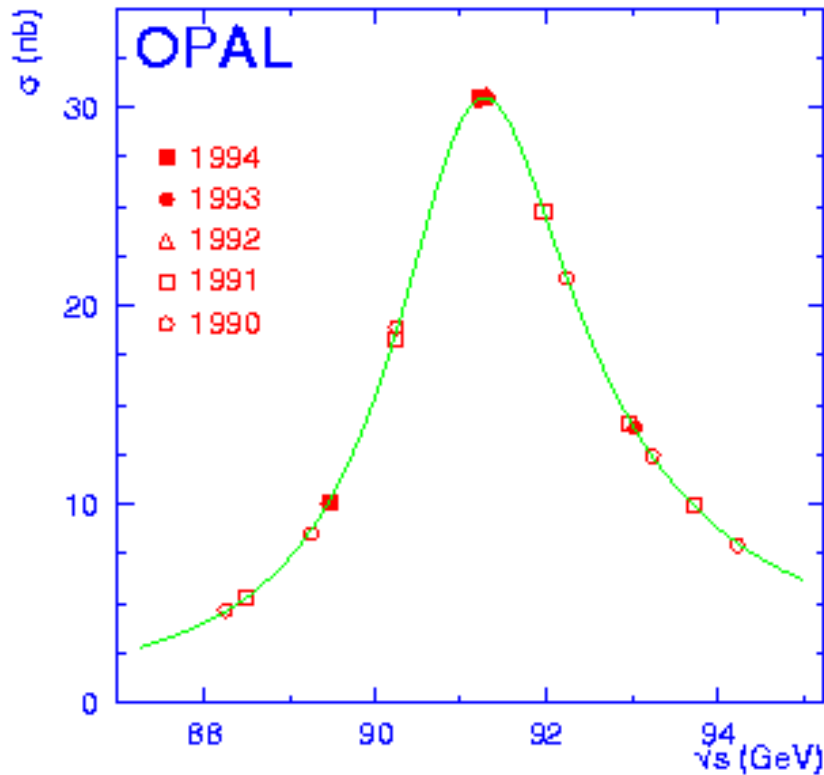
255

FIG. 7. γ -ray yield curves for both series of measurements of $F(p\gamma)$ resonance reaction. Yield values are all on same relative scale.

“Lifetime” is defined in terms of the width of the resonance.



Resonance



- Resonances are the main way we observe fundamental particles.