PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 42200

Waves & Oscillations

Lecture 39 – Review

Spring 2014 Semester

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Final Exam

Date: Thursday, May 8th

Time: 3:30 to 5:30 pm

Room: Phys 112

You can bring two double-sided pages of notes/formulas.

Bring something to write with.

You shouldn't need a calculator.

Today’s Review of Optics

- Polarization
  - Reflection and transmission
  - Linear and circular polarization
  - Stokes parameters/Jones calculus
- Geometric optics
  - Laws of reflection and refraction
  - Spherical mirrors
  - Refraction from one spherical surface
  - Thin lenses
  - Optical systems (multiple thin lenses, apertures, stops)
  - Thick lenses, ray tracing, transfer matrix
  - Aberrations

Wednesday’s Review of Optics

- Interference
  - Double-slit experiments
  - Fresnel’s double-mirror, double-prism, Lloyd’s mirror
  - Thin films, Michelson interferometer
  - Multiple beam interferometry, Fabry-Perot interferometer
- Diffraction
  - Fraunhofer diffraction
  - Single-slit, multiple-slit diffraction
  - Diffraction gratings
  - Fresnel diffraction

Polarization

- General representation:

$$\vec{E}(z,t) = \vec{E}_0 \cos(kz - \omega t + \xi)$$

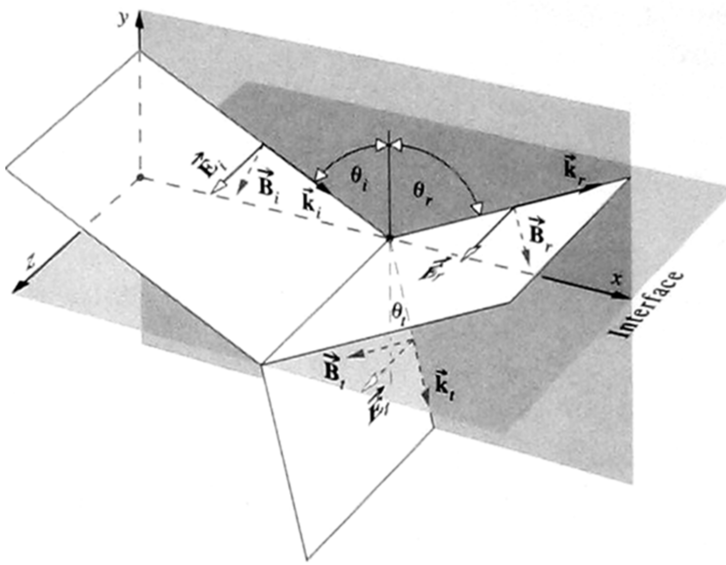
- Light intensity: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$, $I = \langle \vec{S} \rangle = \epsilon_0\ c\langle E^2 \rangle = \frac{\epsilon_0 c}{2} |E_0|^2$
- Linear polarization:
  - $\vec{E}_0$ is constant
- Circular polarization
  - Orthogonal components out of phase by $\xi = \pm\frac{\pi}{2}$

$$\vec{E}(z,t) = E_0(\hat{\imath}\cos(kz - \omega t) \pm \hat{\jmath}\sin(kz - \omega t))$$

    - Right circular polarization: **+**
    - Left circular polarization: **-**
- Degree of polarization: $V = I_p/(I_p + I_n)$ or $V = \frac{I_h - I_v}{I_h + I_v}$

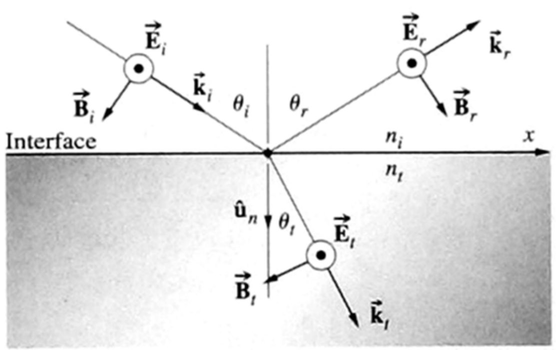
Polarization

- Make sure you understand the following geometry:



$E_{\perp}$ is the component of $\vec{E}$ that is perpendicular to plane of incidence

$E_{\parallel}$ is the component of $\vec{E}$ that is parallel to plane of reflection



In this example, $\vec{E}$ only has an $E_{\perp}$ component ($E_{\parallel} = 0$) while $\vec{B}$ only has a $B_{\parallel}$ component ($B_{\perp} = 0$).

Polarization

- Fresnel equations:

$$\left(\frac{E_r}{E_i}\right)_\perp = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

$$\left(\frac{E_r}{E_i}\right)_\parallel = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\left(\frac{E_t}{E_i}\right)_\perp = \frac{(n_1/n_2)\sin(2\theta_i)}{\sin(\theta_i+\theta_t)}$$

$$\left(\frac{E_t}{E_i}\right)_\parallel = \frac{2\cos(\theta_i)\sin(\theta_t)}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)}$$

- You have to calculate θ_t using Snell's law.

Polarization

- Brewster's angle, θ_B: $\theta_i + \theta_t = 90°$
  - $r_{\parallel} \to 0$ when $\theta_i \to \theta_B$ (polarization by reflection)
- When light is at normal incidence,

$$\theta_i = \theta_t = 0 \text{ and } E_{\parallel} = 0\ldots$$

$$r_{\perp} = \left(\frac{E_r}{E_i}\right)_{\perp} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t_{\perp} = \left(\frac{E_t}{E_i}\right)_{\perp} = \frac{2n_1}{n_1 + n_2}$$

- When $n_1 < n_2$, $r_{\perp} = -1$ (phase changes by π).

Polarization

- Remember that the reflected and transmitted intensities depend on the square of r and t.
- Also remember that intensity depends on v

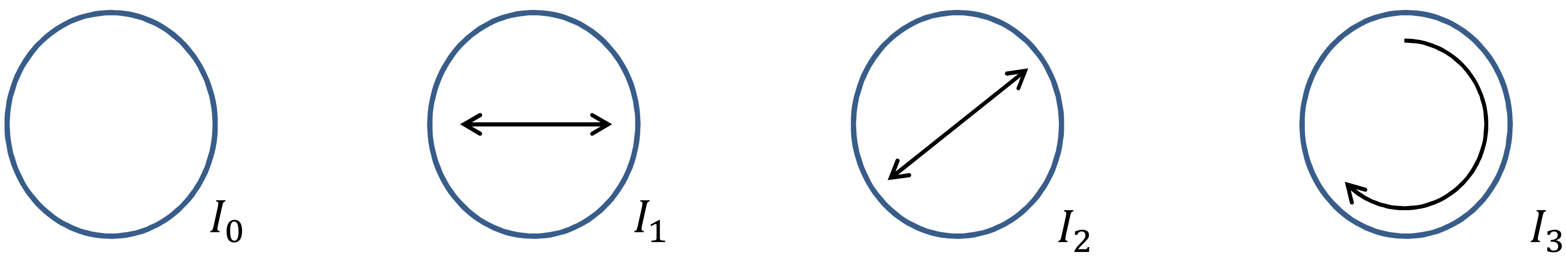
$$I = \langle \vec{S} \rangle = \epsilon_0 \, v \langle E^2 \rangle = \frac{\epsilon_0 c}{n} \langle E^2 \rangle$$

- Energy conservation:

$$R = \frac{I_r}{I_i} = r^2 \qquad T = \frac{I_t \cos\theta_t}{I_i \cos\theta_i} = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} \, t^2$$

Stokes Parameters

- Stokes selected four filters
- Each filter transmits exactly half the intensity of unpolarized light



Unpolarized: filters out ½ the intensity of any incident light.

Linear: transmits only horizontal component

Linear: transmits only light polarized at 45°

Circular: transmits only R-polarized light

$$\left.\begin{aligned} S_0 &= 2I_0 \\ S_1 &= 2I_1 - 2I_0 \\ S_2 &= 2I_2 - 2I_0 \\ S_3 &= 2I_3 - 2I_0 \end{aligned}\right\}$$

Stokes parameters

Stokes Parameters

- The Stokes parameters that describe a mixture of polarized light components is the weighted sum of the Stokes parameters of each component.
- Example: Two components
  – 40% has vertical linear polarization
  – 60% has right circular polarization
- Calculate Stokes parameters:

$$S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = 0.4 \times \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + 0.6 \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.4 \\ 0 \\ 0.6 \end{bmatrix}$$

- Degree of polarization:

$$V = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} = \sqrt{(0.4)^2 + (0.6^2)} = 0.72$$

Mueller Matrices

$$S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \Rightarrow S' = MS$$

Example: right circular polarizer

- Incident unpolarized light

$$S_0 = 1, S_1 = 0, S_2 = 0, S_3 = 0$$

- Emerging circular polarization

$$S_0 = \frac{1}{2}, S_1 = 0, S_2 = 0, S_3 = \frac{1}{2}$$

- Mueller matrix:

$$M = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Jones Calculus

- Applies to $\vec{E}$, not intensity
  - Light must be coherent
- Electric field vectors:

$$\vec{E}_x(z,t) = E_{0x}\hat{\imath}\cos(kz - \omega t + \varphi_x)$$
$$\vec{E}_y(z,t) = E_{0y}\hat{\jmath}\cos\left(kz - \omega t + \varphi_y\right)$$

- Jones vector:

$$\tilde{E} = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix} \Rightarrow \vec{E} = \frac{1}{\sqrt{E_{0x}^2 + E_{0y}^2}} \begin{bmatrix} E_{0x} \\ E_{0y}e^{i\xi} \end{bmatrix}$$

Jones Calculus

- Examples:
  - Horizontal linear polarization: $\vec{E}_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
  - Vertical linear polarization: $\vec{E}_y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
  - Linear polarization at 45°: $\vec{E}_{45^\circ} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
  - Right circular polarization: $\vec{E}_R = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ e^{-i\pi/2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$
  - Left circular polarization: $\vec{E}_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ e^{+i\pi/2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$

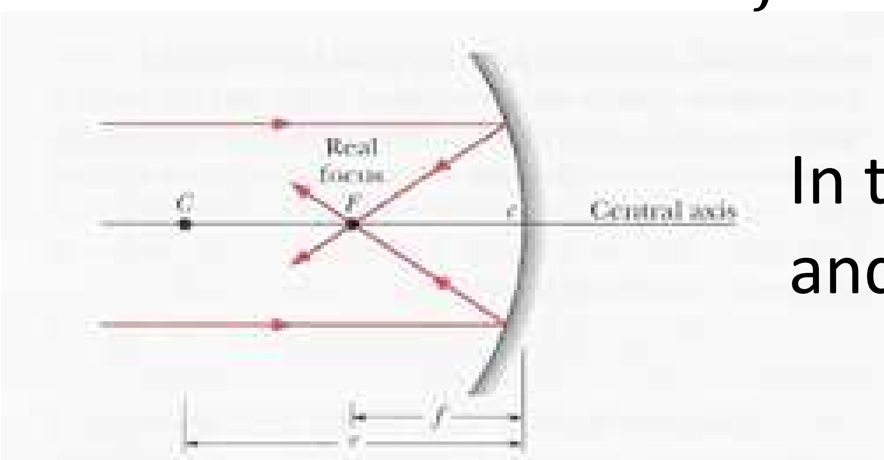
Jones Calculus

- $\overrightarrow{E'}$ and $\vec{E}$ are related by a 2x2 matrix (the Jones matrix):
$$\overrightarrow{E'} = A\,\vec{E}$$
- If light passes through several optical elements, then
$$\overrightarrow{E'} = A_n \cdots A_2 A_1 \vec{E}$$
- Examples:
  - Transmission through an optically inactive material:
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
  - Rotation of the plane of linear polarization (eg, propagation through a sugar solution)
$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Geometric Optics

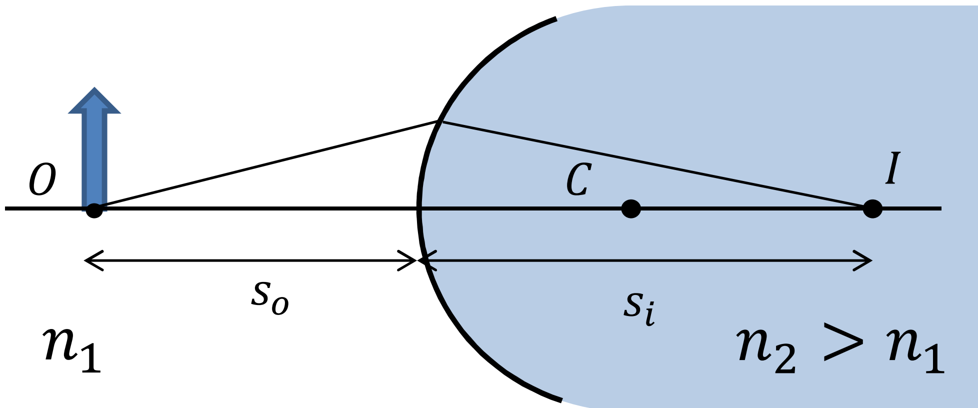
- Law of reflection: $\theta_1' = \theta_1$
- Law of refraction (Snell's law): $n_1 \sin\theta_1 = n_2 \sin\theta_2$
- Make sure you understand the geometry and sign conventions for lenses and mirrors:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \text{where } f = \frac{r}{2}$$



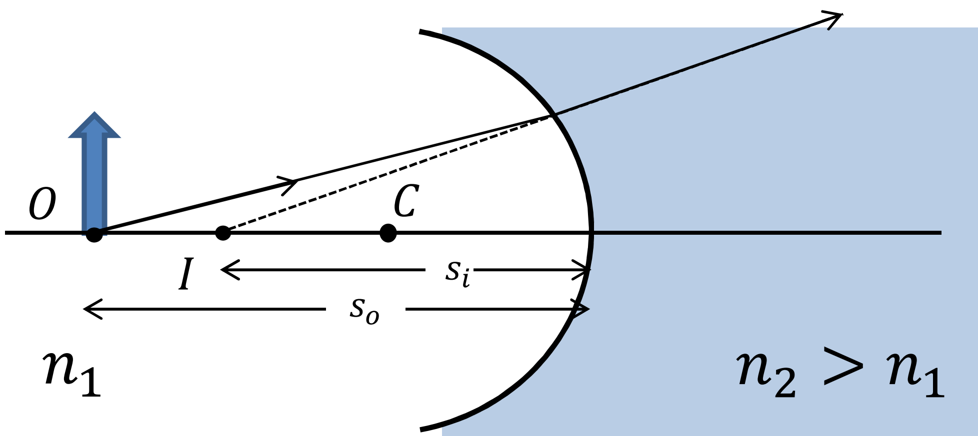
In this diagram, f, r, s and s' are all positive.

Spherical Refracting Surfaces



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

All quantities are positive here.



$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

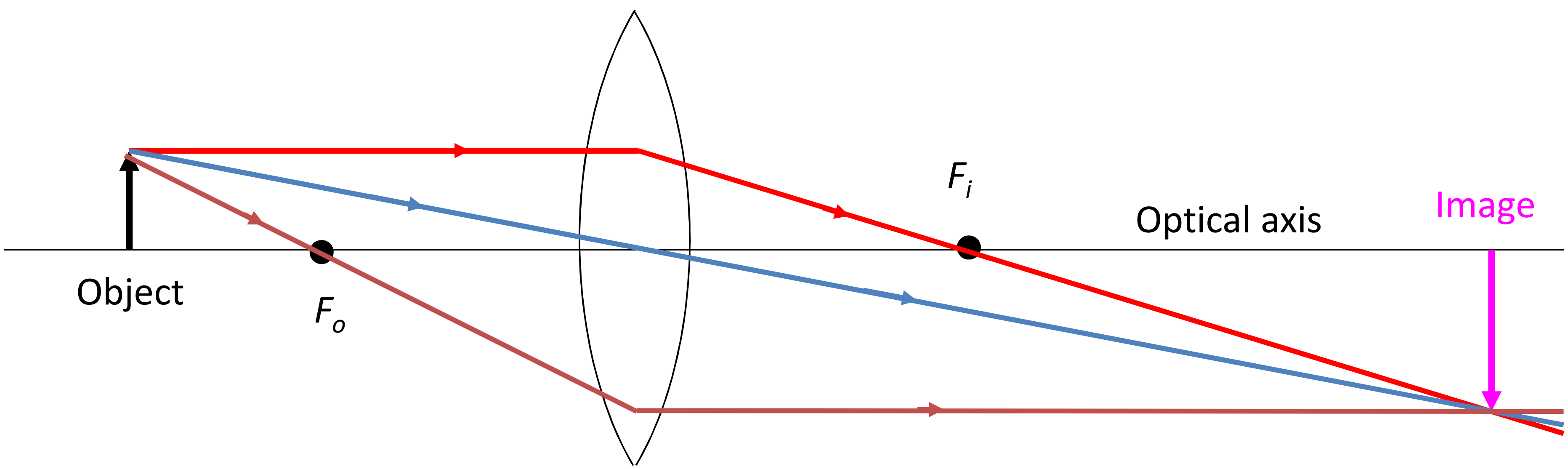
(same formula but now R<0)

Thin Lenses

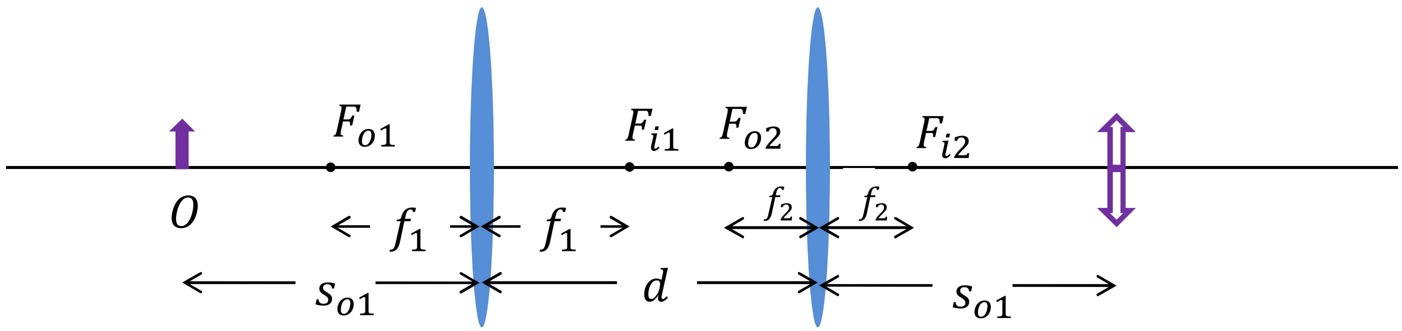
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

$m_T = -s_i / s_o$ (transverse magnification)

- You should be able to calculate image positions and draw ray diagrams:



Multiple Thin Lenses



- Two techniques:
  - Calculate position of intermediate image formed by first lens, then the final image formed by second lens
  - Use front/back focal lengths:
- Front focal length:

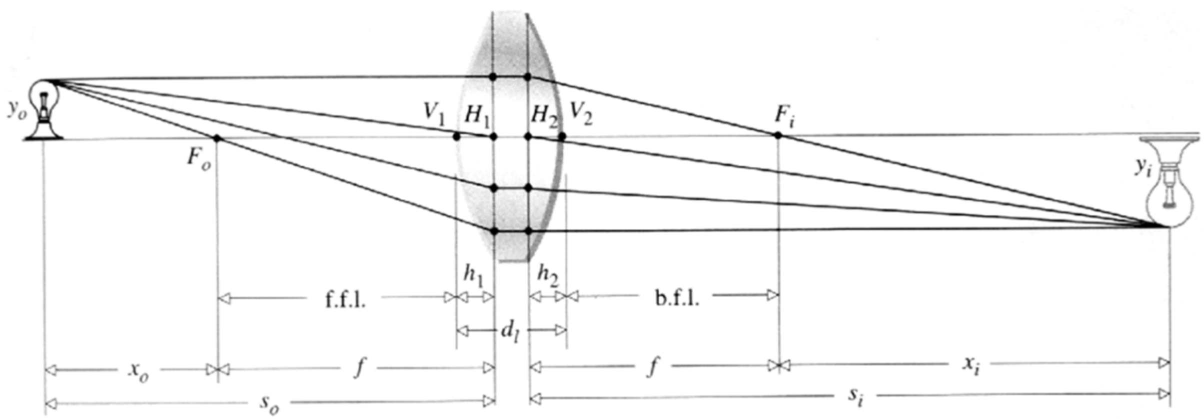
$$\text{f.f.l.} = \frac{f_1(d-f_2)}{d-(f_1+f_2)}$$

- Back focal length:

$$\text{b.f.l.} = \frac{f_2(d-f_1)}{d-(f_1+f_2)}$$

Thick Lenses

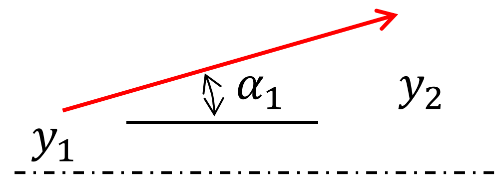
- Two approaches:
  - Calculate positions of intermediate images formed by each spherical refracting surface
  - Calculate positions of principle planes:



Focal length: $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$

Principal planes: $h_1 = -\frac{f(n-1)d}{nR_2}, h_2 = -\frac{f(n-1)d}{nR_1}$

Ray Tracing



- Ray vector: $\vec{r_i} = \begin{bmatrix} n_i \alpha_i \\ y_i \end{bmatrix}$
- Transfer matrix: $T = \begin{bmatrix} 1 & 0 \\ d/n & 1 \end{bmatrix}$
- Refraction matrix: $R = \begin{bmatrix} 1 & -D \\ 0 & 1 \end{bmatrix}, D = \frac{n_t - n_i}{R}$
- Mirror matrix: $M = \begin{bmatrix} -1 & -2n/R \\ 0 & 1 \end{bmatrix}$

Aberrations

- Images usually suffer from some degree of distortion
  - Seidel’s primary aberrations
    - Spherical
    - Coma
    - Astigmatism
    - Field curvature
    - Distortion
  - Chromatic aberrations

Typical Exam Questions

- An optical system consists of a horizontal linear polarizer, a solution of dextrorotary sugar, which rotates the plane of polarized light by $+45°$, followed by a left circular polarizing filter.

(a) Write the Mueller matrices for each component

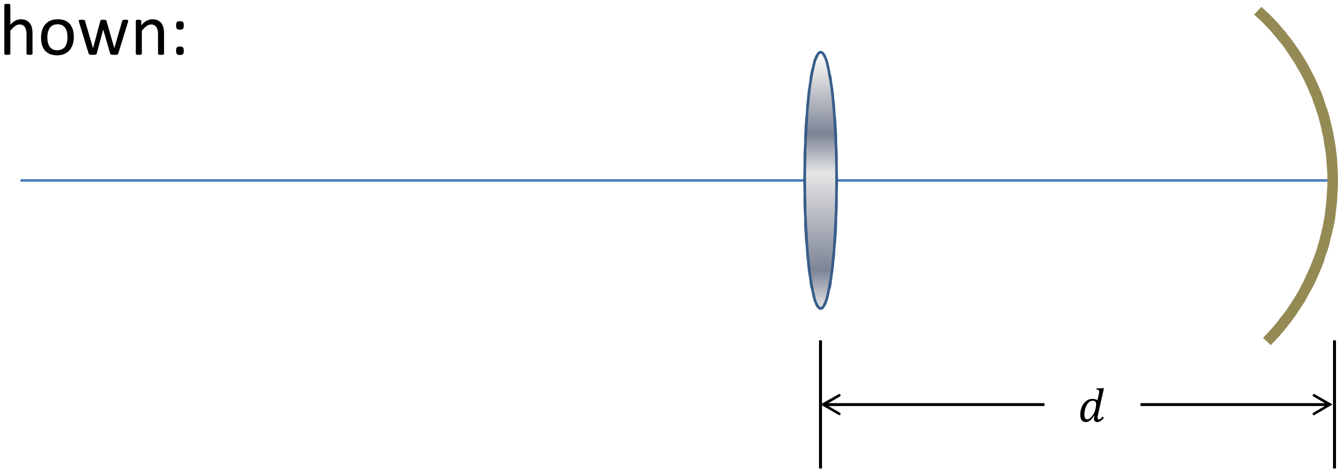
(b) Calculate the intensity of transmitted light if the incident light is unpolarized

(c) Calculate the intensity of transmitted light if the incident light is left circular polarized

(d) Is the system symmetric? That is, is the intensity of transmitted light the same if the paths of all light rays are reversed?

Typical Exam Questions

- An optical system consists of a thin lens with focal length f and a concave spherical mirror with radius R as shown:



(a) Where is the effective focal point of this combination of optical elements?

(b) What is the transverse magnification?

Typical Exam Questions

- Answer one, or the other, but not both…

(a) Describe two types of optical aberration and techniques that can be used to minimize them.

(b) Compare and contrast the assumptions and main features of Fraunhofer and Fresnel diffraction. Under what circumstances is Fraunhofer diffraction a special case of Fresnel diffraction?