Purdue University Department of Physics

Physics 42200

Waves & Oscillations

Lecture 37 – Diffraction

Spring 2014 Semester

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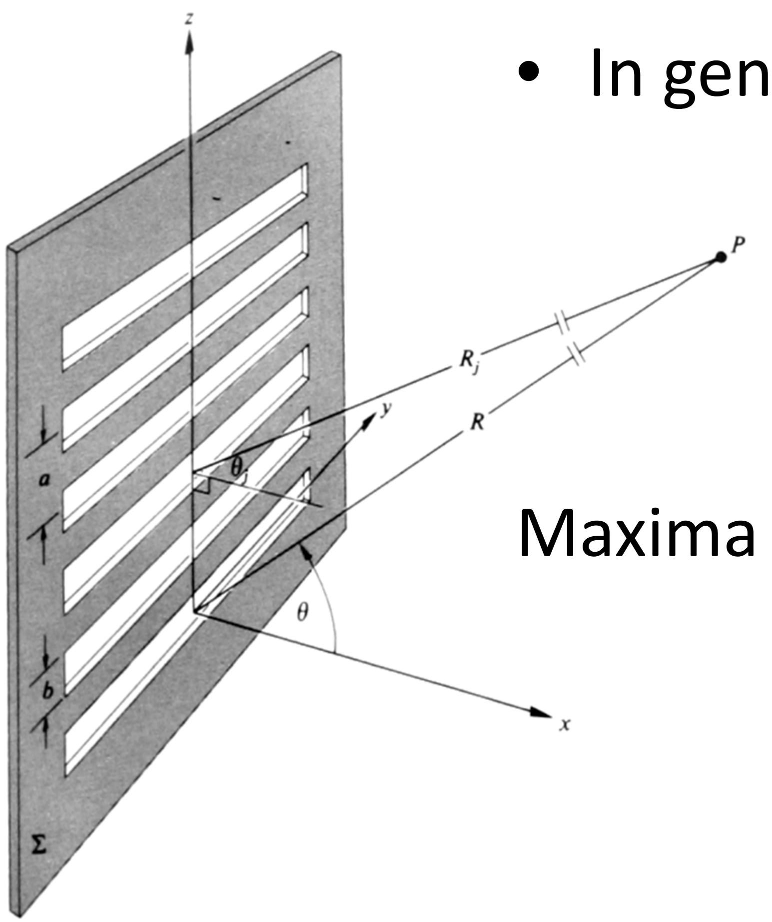
Three-Slit Fraunhofer Diffraction

- Light intensity: $I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin 3\alpha}{\sin\alpha}\right)^2$



- In general, when there are N slits:

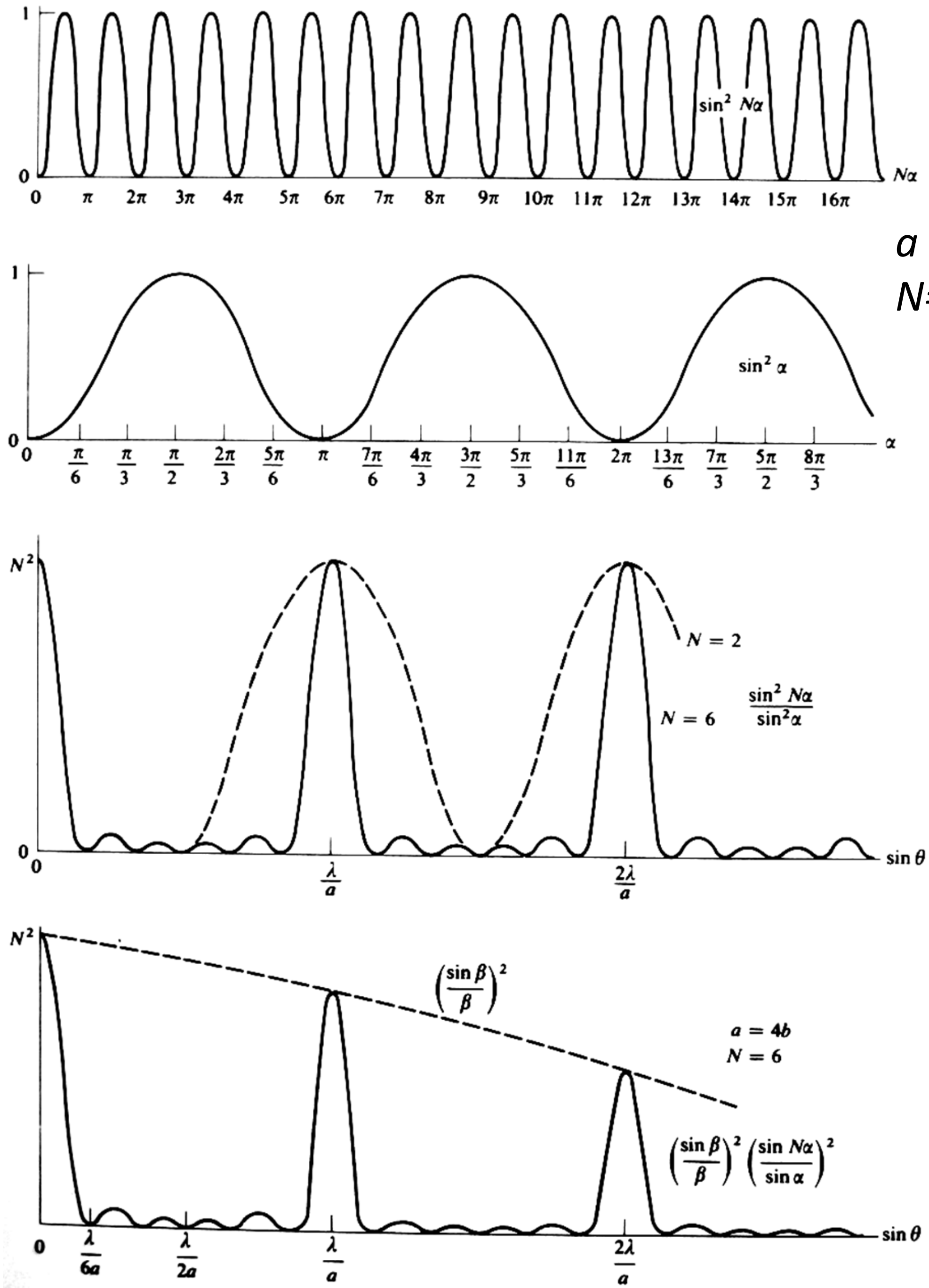
$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

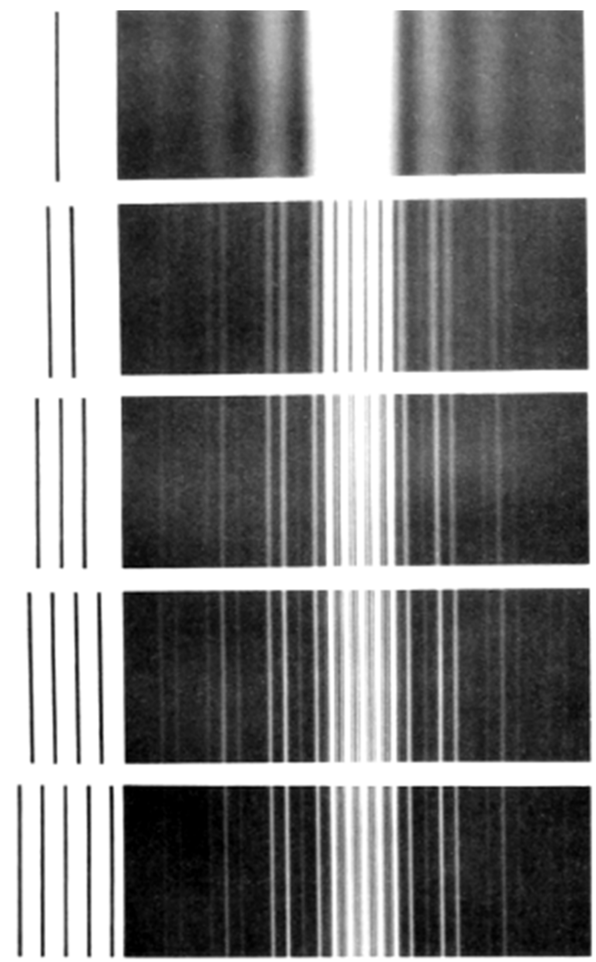
Maxima occur when $\alpha = \frac{1}{2}ka\sin\theta = m\pi$

$$a\sin\theta = m\lambda$$

$$I(\theta) = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \left(\frac{\sin N\alpha}{\sin \alpha} \right)^2$$

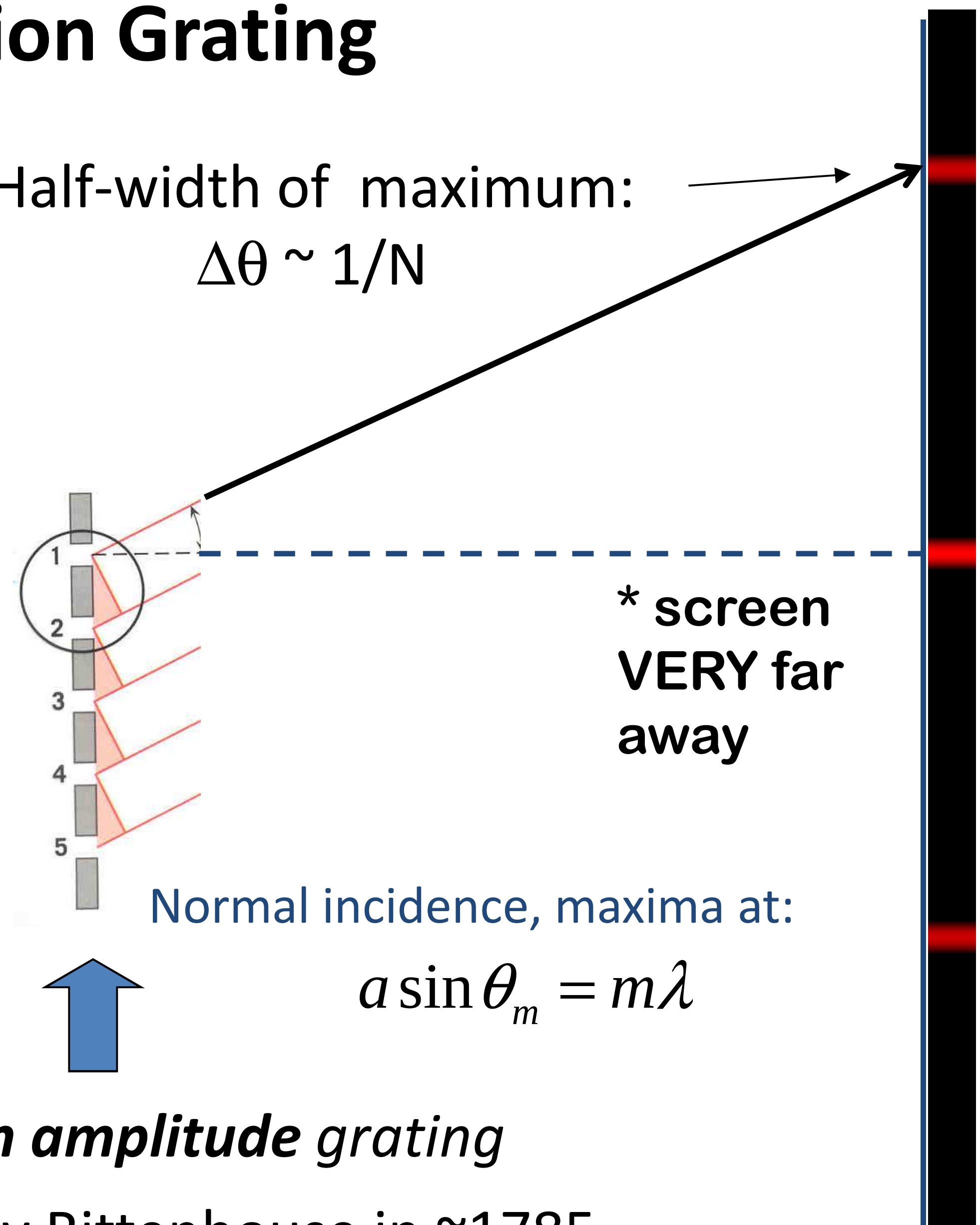
a = 4b
N=6





Diffraction Grating

Usually gratings have thousands of slits and are characterized by the number of slits per cm (for example: 6000 cm^{-1})

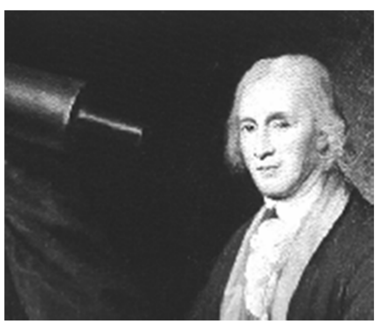


Normal incidence, maxima at:

$$a \sin\theta_m = m\lambda$$

Transmission amplitude *grating*

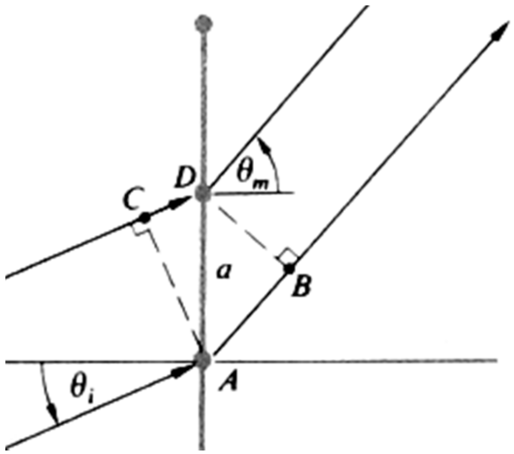
Introduced by Rittenhouse in ~1785

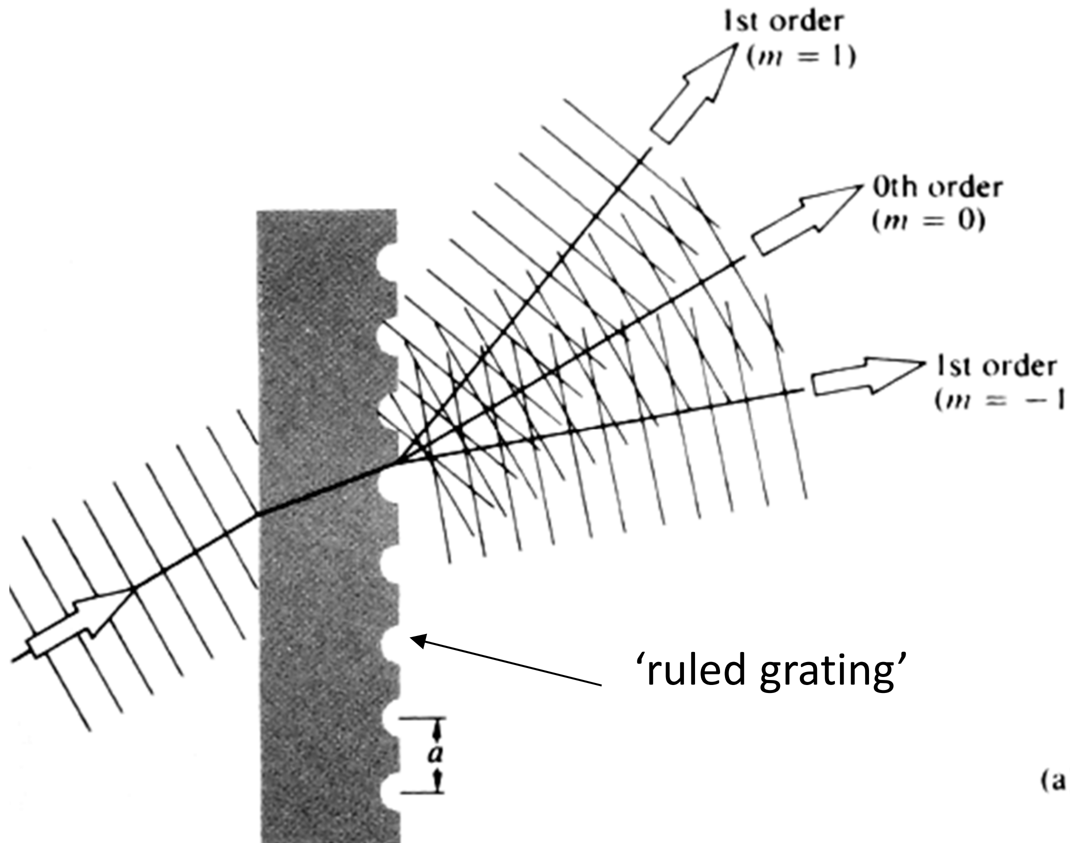


David Rittenhouse
1732 - 1796

Transmission Phase Grating

General case:
Incidence angle $\theta_i \neq 0$

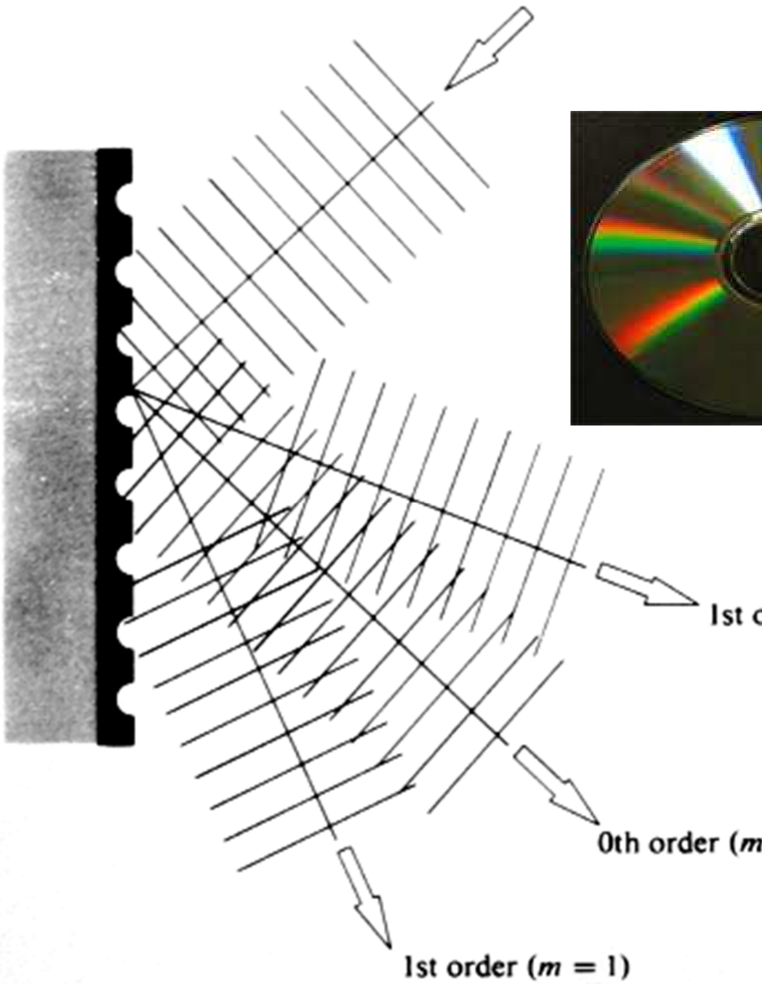




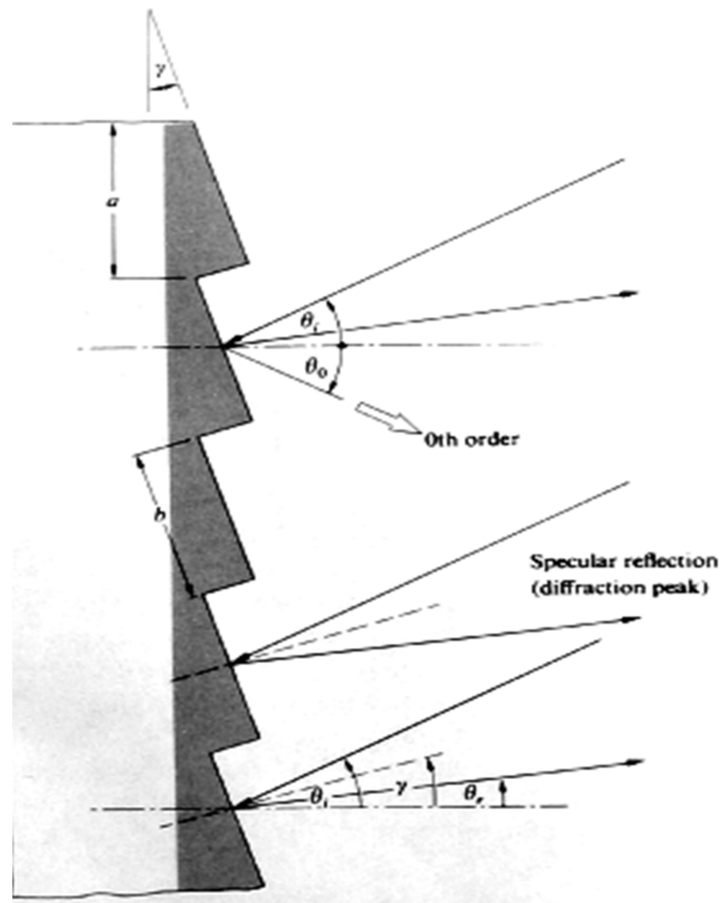
$$\overline{AB} - \overline{CD} = a\sin\theta_m - a\sin\theta_i$$

Maxima: $a(\sin\theta_m - \sin\theta_i) = m\lambda$ for arbitrary incidence angle

Reflection Phase Grating



Examples: CD disk
Finely machined surfaces

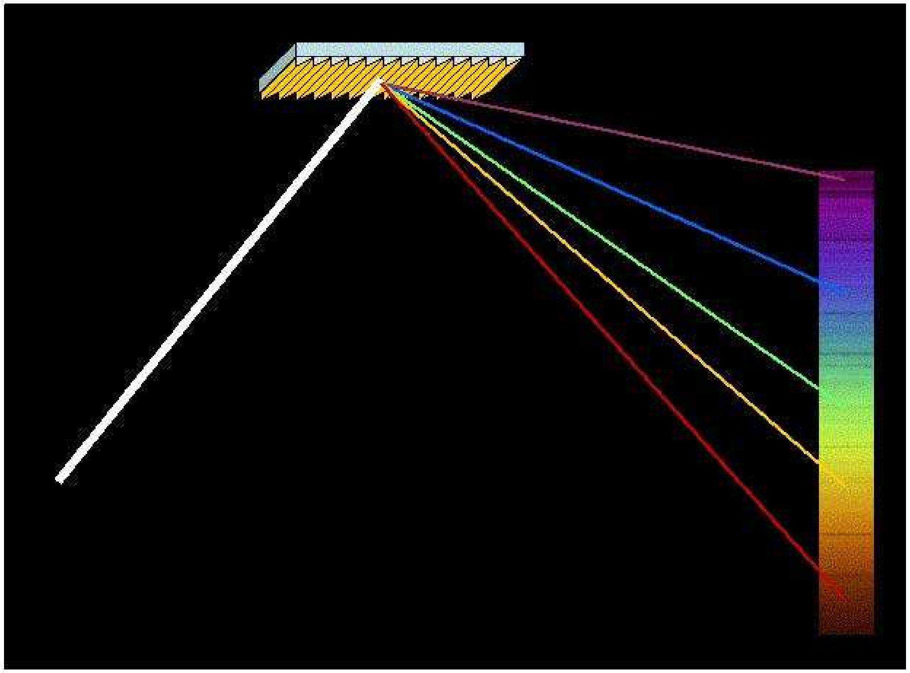


Diffraction Grating Spectrometers

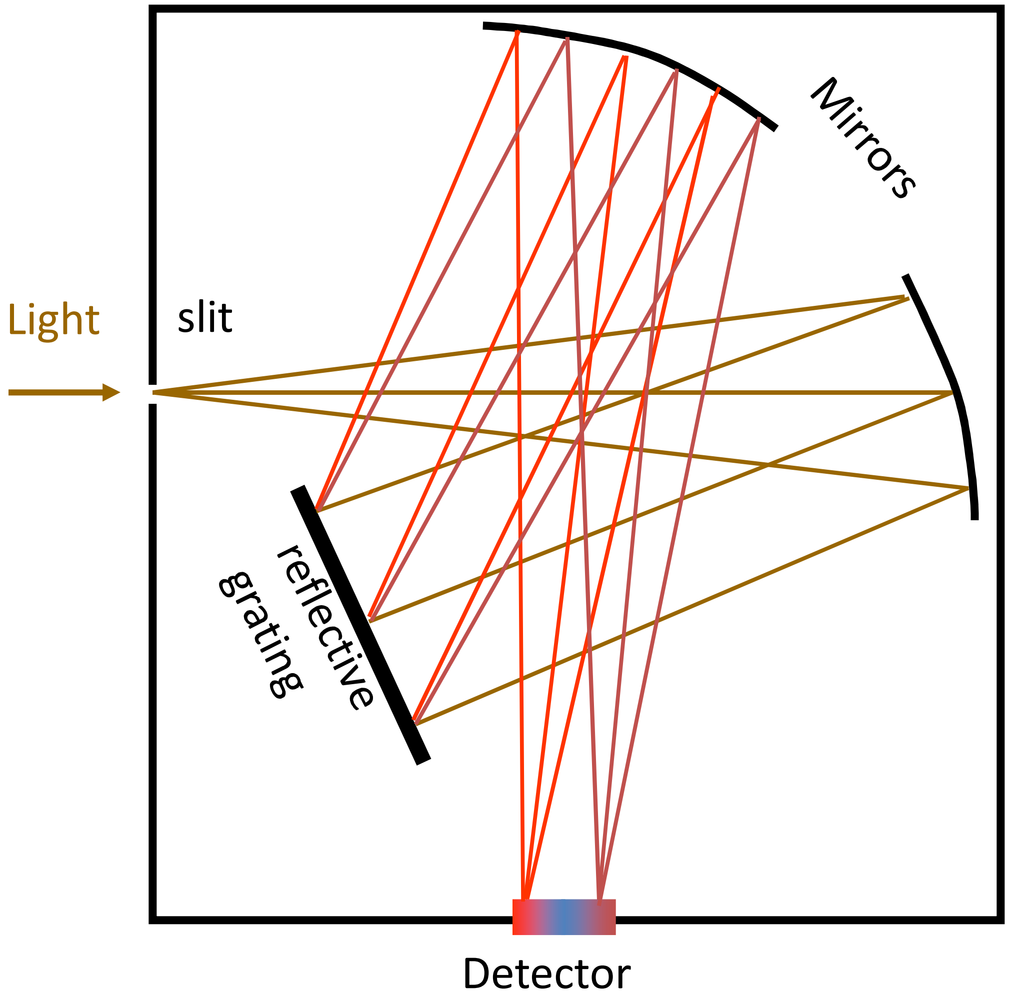
- Angle of maximum intensity depends on wavelength:

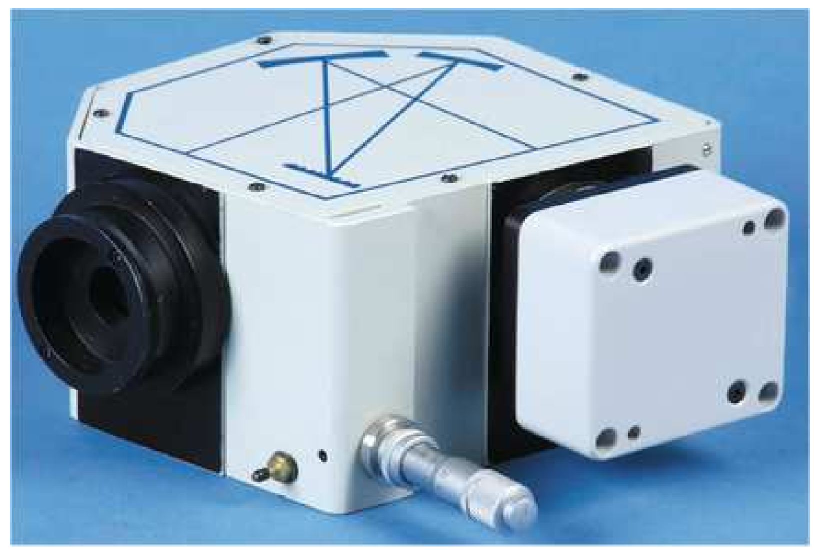
$$\sin\theta = \frac{m\lambda}{a}$$

- Diffraction gratings are used to separate and analyze the spectrum of light:

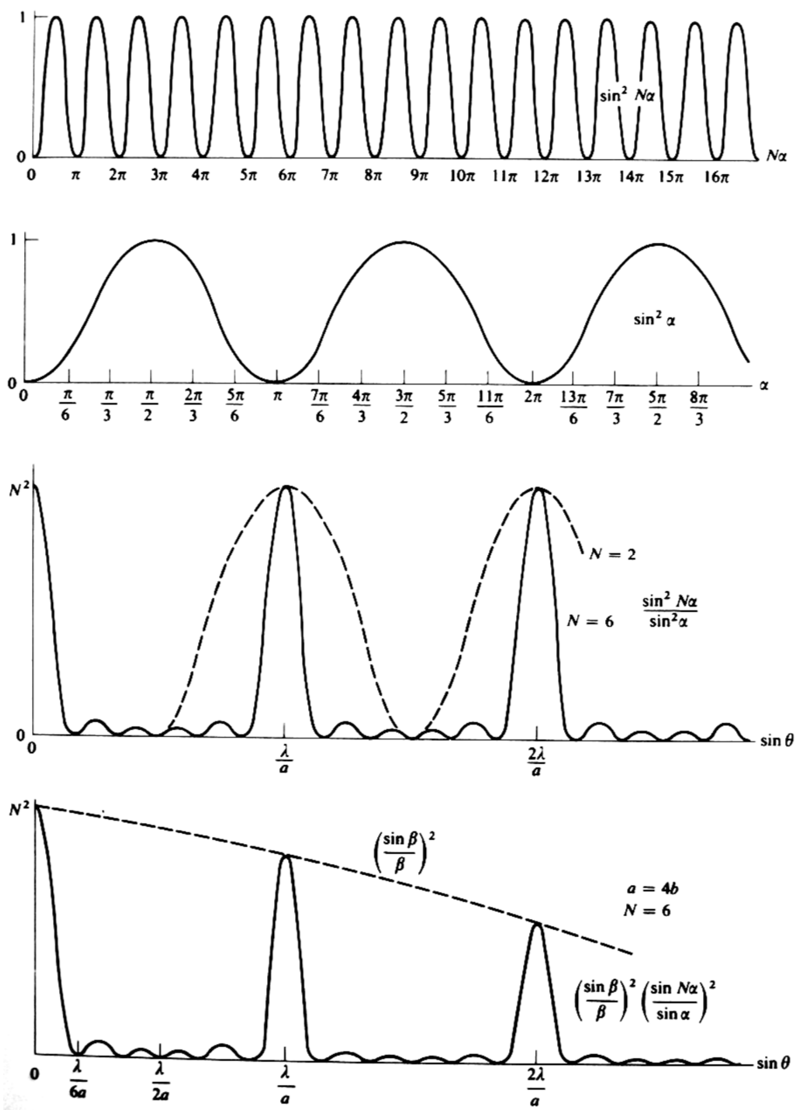


Diffraction Grating Spectrograph





Width of Spectral Lines



$$I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

- Maxima occur when

$$\alpha = m\pi$$

- Otherwise, zeros occur when

$$N\alpha = m'\pi$$

- Zeros on either side of a peak

$$N\alpha_{\pm} = (Nm \pm 1)\pi$$

- Width of peak:

$$\Delta\alpha = \alpha_+ - \alpha_- = \frac{2\pi}{N}$$

Width of Spectral Lines

$$\Delta\alpha = \frac{2\pi}{N}$$

$$\alpha = \frac{1}{2}ak\sin\theta = \frac{\pi a}{\lambda}\sin\theta$$

$$\Delta\alpha = \frac{\pi a}{\lambda}\cos\theta\,\Delta\theta$$

- Angular resolution:

$$\Delta\theta = \frac{2\lambda}{Na\cos\theta}$$

$$(\Delta\theta)_{min} = \frac{1}{2}\Delta\theta = \frac{\lambda}{Na\cos\theta}$$

Angular Dispersion

- The angle depends on the wavelength:

$$a \sin\theta = m\lambda$$

$$a \cos\theta \, \Delta\theta = m\Delta\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{m}{a \cos\theta}$$

- Chromatic resolving power is defined:

$$\mathcal{R} \equiv \frac{\lambda}{(\Delta\lambda)_{min}}$$

$$(\Delta\lambda)_{min} = \frac{a\cos\theta}{m}(\Delta\theta)_{min} = \frac{a\cos\theta}{m}\,\frac{2\lambda}{Na\cos\theta} = \frac{\lambda}{Nm}$$

$$\mathcal{R} = Nm = \frac{Na\sin\theta}{\lambda}$$

Resolving Power

- The chromatic resolving power is proportional to Na
- Example: 6000 lines per cm, 15 cm width

$$N = (6000\, lines/cm) \times (15\, cm) = 90{,}000$$

$$a = 1/(6000\, lines/cm) = 1.667\, \mu m$$

$$\left.\begin{aligned} \lambda &= 588.991\, nm \\ \lambda' &= 589.595\, nm \end{aligned}\right\} \Delta\lambda = 0.604\, nm$$

$m = 2$ (second order)

$$\sin\theta = \frac{m\lambda}{a} =$$

$$2 \times (589 \times 10^{-7} cm) \times (6000\, lines/cm)$$

$$= 0.707$$

$$\mathcal{R} = mN = 2 \times 90{,}000 = 180{,}000$$

$$(\Delta\lambda)_{min} = \frac{\lambda}{\mathcal{R}} = \frac{(589\, nm)}{180{,}000} = 0.00327\, nm$$

Overlapping Orders

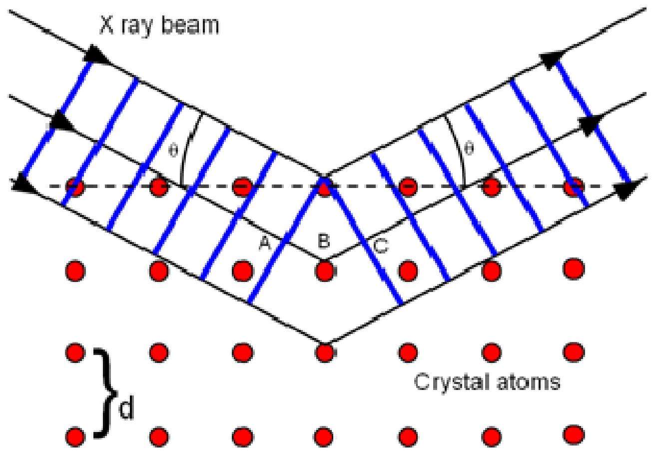
- Confusion can arise when a spectral line at one order overlaps with a different spectral line at a different order:

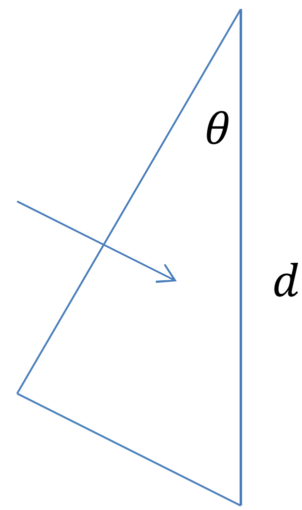
$$\sin\theta = \frac{(m+1)\lambda}{a} = \frac{m\lambda'}{a} = \frac{m(\lambda+\Delta\lambda)}{a}$$

$$\Delta\lambda = \frac{\lambda}{m} \equiv (\Delta\lambda)_{fsr} \text{ (free spectral range)}$$

X-ray Diffraction

- With short enough wavelengths, the atoms in a crystal lattice form the diffraction grating:



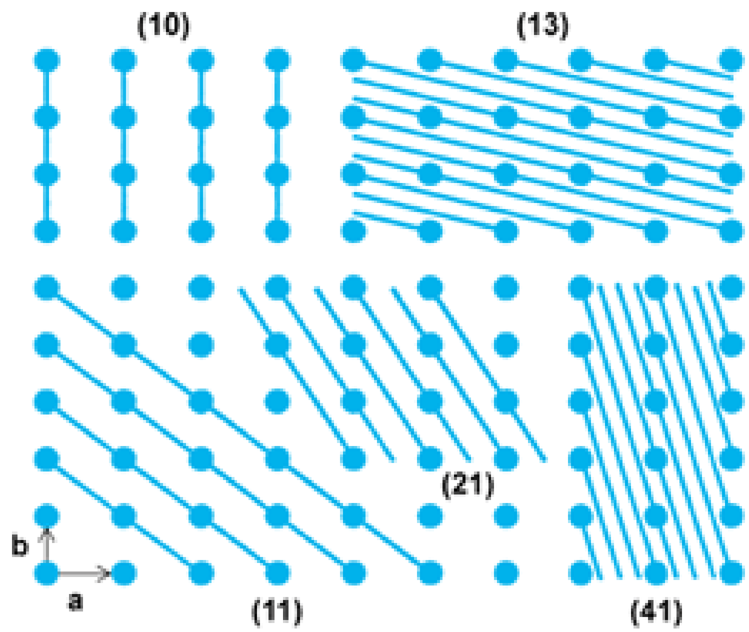


$$2d\sin\theta = n\lambda$$

(Bragg's Law)

X-ray Diffraction

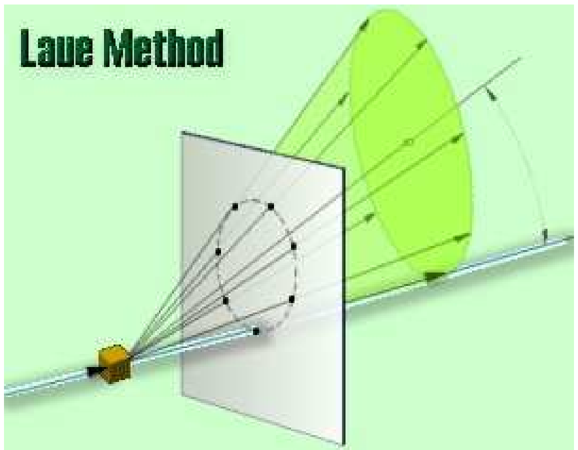
- Regular crystal lattices have many "planes":

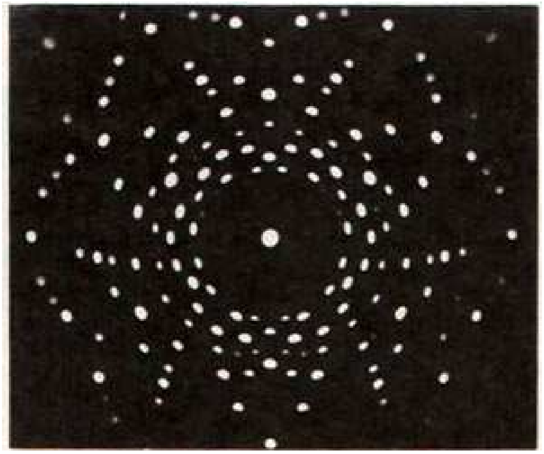


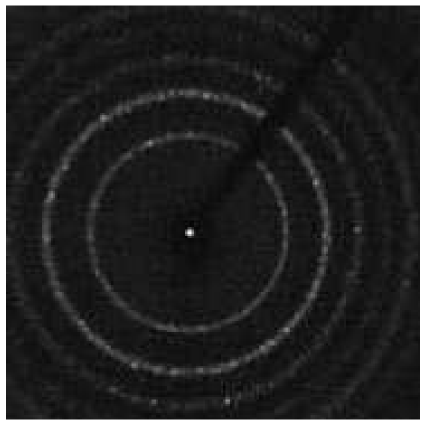
$$2d_{hkl} \sin \theta_{hkl} = n\lambda$$

X-ray Diffraction

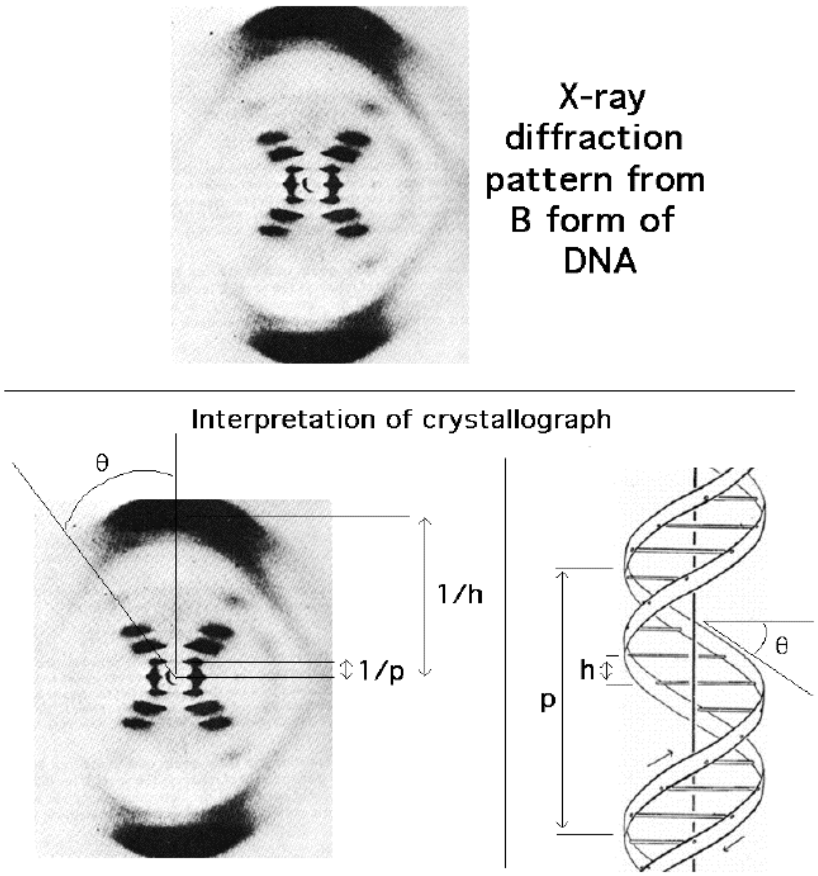
- Max von Laue exposed crystals to a continuous x-ray spectrum:







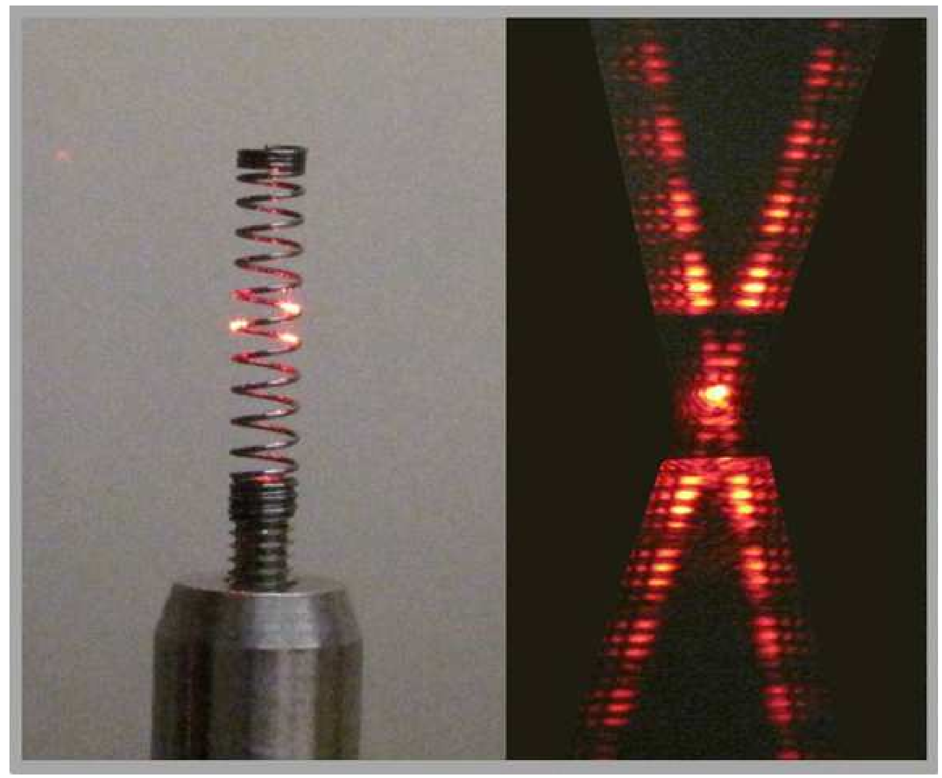
X-ray Diffraction



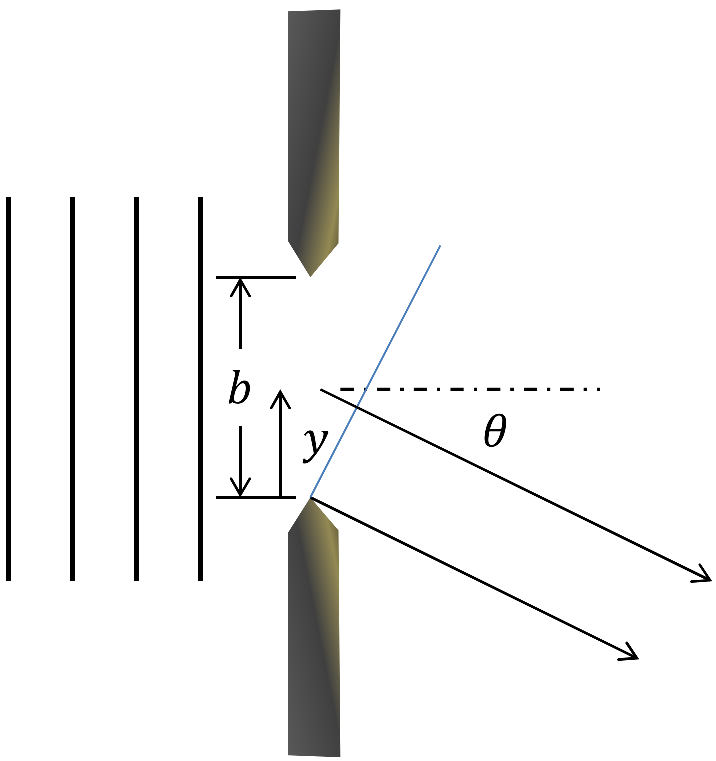
θ - tilt of helix (angle from perpendicular to long axis)

h = 3.4 Å (Distance between bases)

p = 34 Å (Distance for one complete turn of helix; Repeat unit of the helix)



Fraunhofer Diffraction

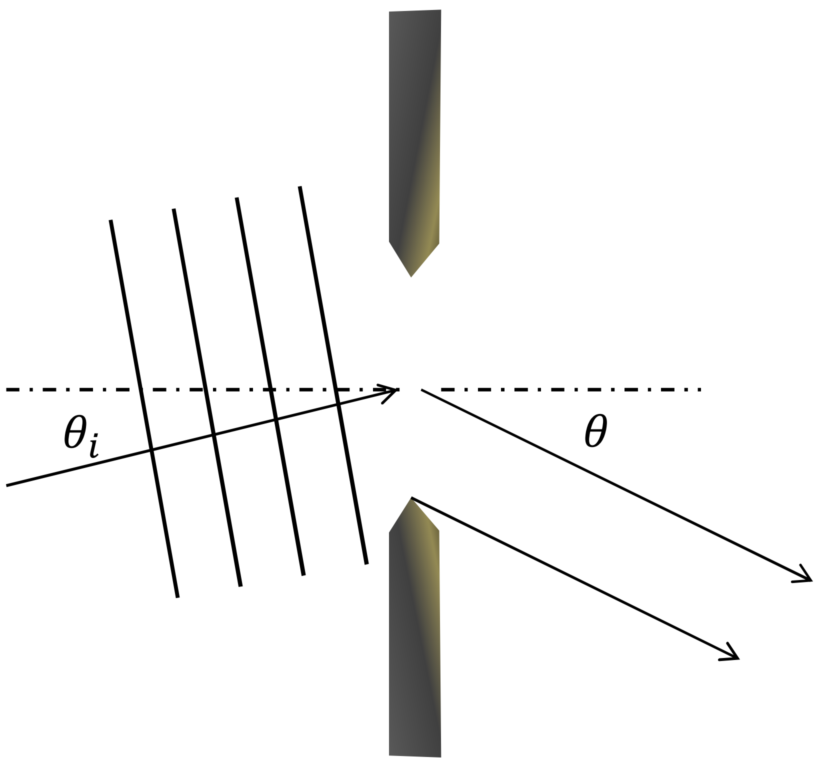


Fraunhofer diffraction:

- The phase varies linearly across the aperture
- The intensity of light arriving from each part of the aperture is equal

$$dE = \frac{\mathcal{E}_L e^{ik\mathbf{y}\sin\theta} dy}{\mathbf{R}}$$

Fraunhofer Diffraction



Fraunhofer diffraction:

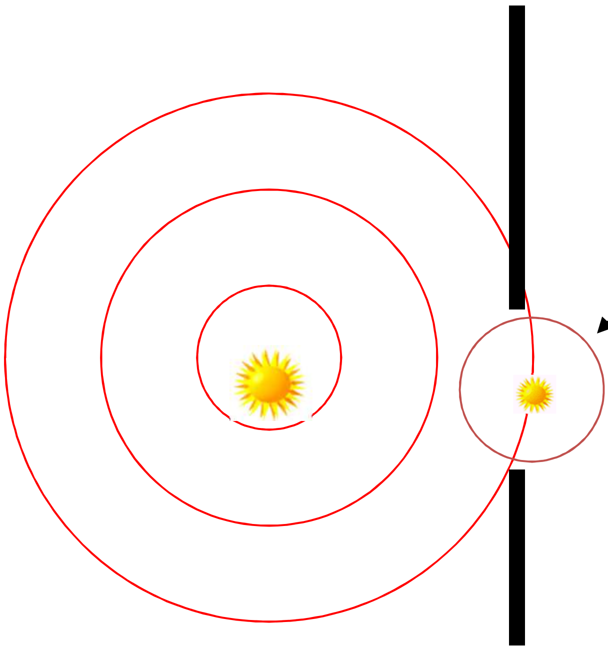
- The phase varies linearly across the aperture
- The intensity of light arriving from each part of the aperture is equal

$$dE = \frac{\mathcal{E}_L e^{ik\mathbf{y}(\sin\theta - \sin\theta_i)} dy}{\mathbf{R}}$$

Huygens-Fresnel Principle

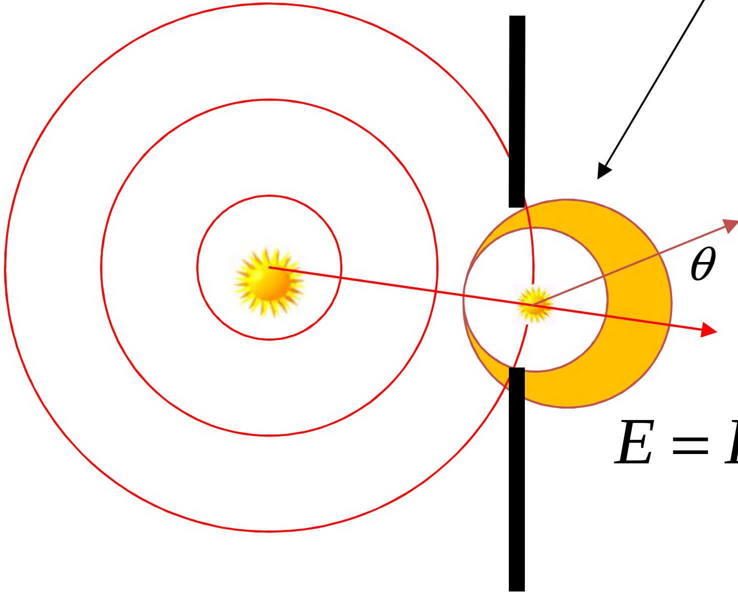
- Each point on a wave front is a source of spherical waves that are in phase with the incident wave.
- The light at any point in the direction of propagation is the sum of all such spherical waves, taking into account their relative phases and path lengths.
- *The secondary spherical waves are preferentially emitted in the forward direction.*
- Fresnel presented a very different way of thinking about the propagation and diffraction of light.
  - The details might be the subject of extensive debate
  - It relies completely on the wave nature of light
  - The predictions were confirmed by experiment

Huygens-Fresnel Principle



Huygens: source of spherical wave

Problem: it must also go backwards, but that is not observed in experiment



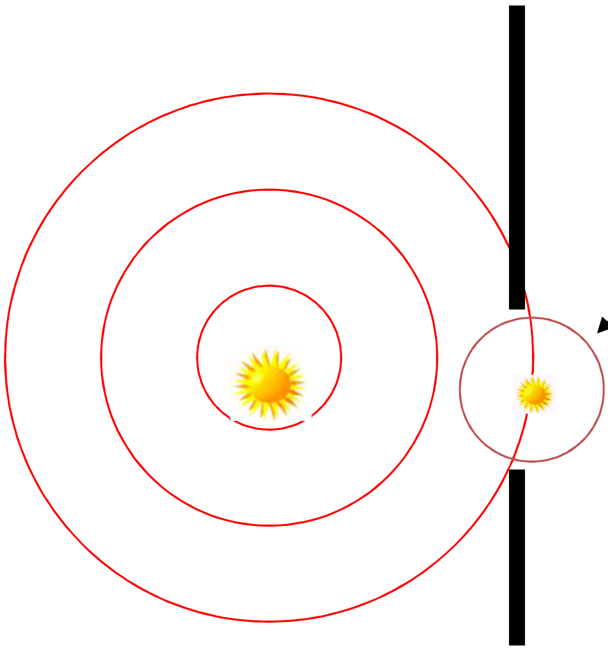
Intensity of the spherical wavelet depends on direction. Fresnel supposed that…

$$K(\theta) \to 0 \text{ as } \theta \to \frac{\pi}{2}$$

inclination factor

$$E = K(\theta)\frac{E}{r}\cos(\omega t - kr + \xi)$$

Huygens-Fresnel Principle



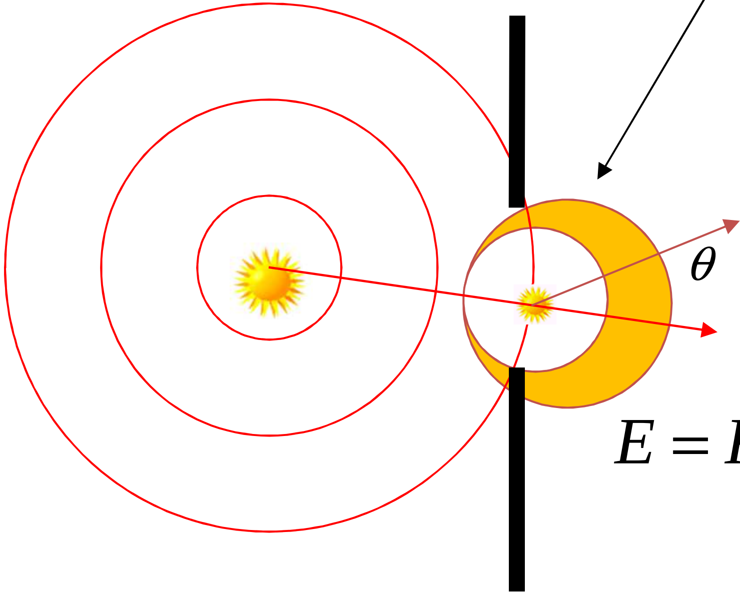
Huygens: source of spherical wave

Problem: it must also go backwards, but that is not observed in experiment

Intensity of the spherical wavelet depends on direction…

$$K(\theta) = \frac{1}{2}(1 + \cos\theta)$$

inclination factor

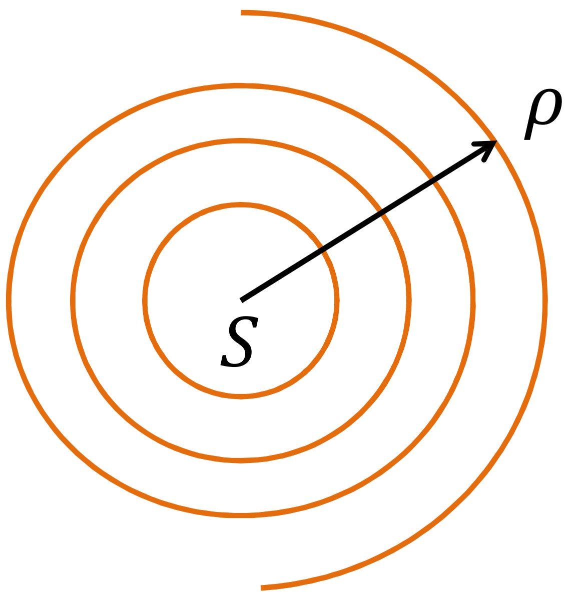


$$E = K(\theta)\frac{E}{r}\cos(\omega t - kr + \xi)$$

Propagation of Spherical Waves

- Consider a spherical wave emitted from a source S at time $t = 0$.

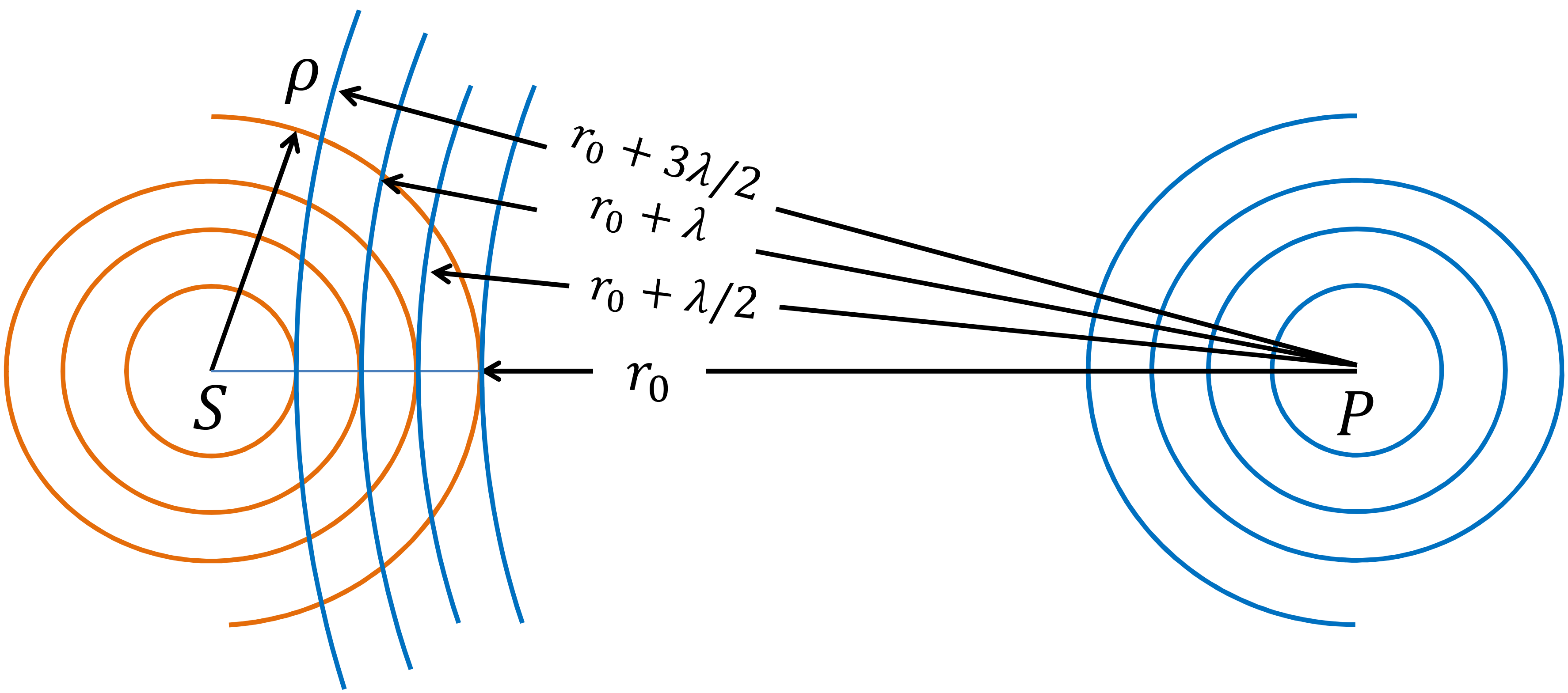
$$E(\rho, t') = \frac{\mathcal{E}_0}{\rho} \cos(\omega t' - k\rho)$$



- These spherical waves expand outward from S

Propagation of Spherical Waves

- Consider a series of concentric spheres around another point P with radii $r_0, r_0 + \lambda/2, r_0 + \lambda, \cdots$



Propagation of Spherical Waves

- Consider a series of concentric spheres around another point P with radii $r_0, r_0 + \lambda/2, r_0 + \lambda, \cdots$