Purdue University Department of Physics

Physics 42200

Waves & Oscillations

Lecture 34 – Interference

Spring 2014 Semester

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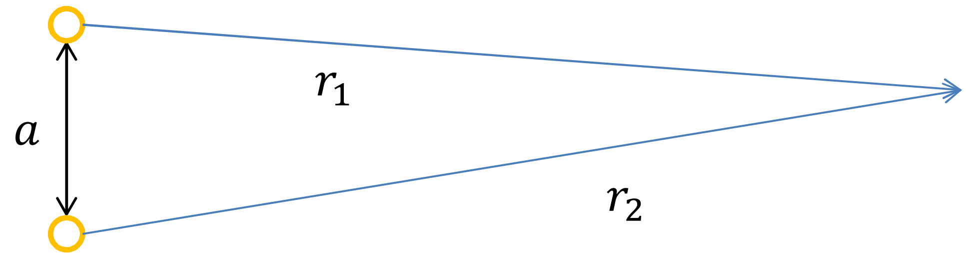
Interference



- Two electric fields, identical frequency ω:

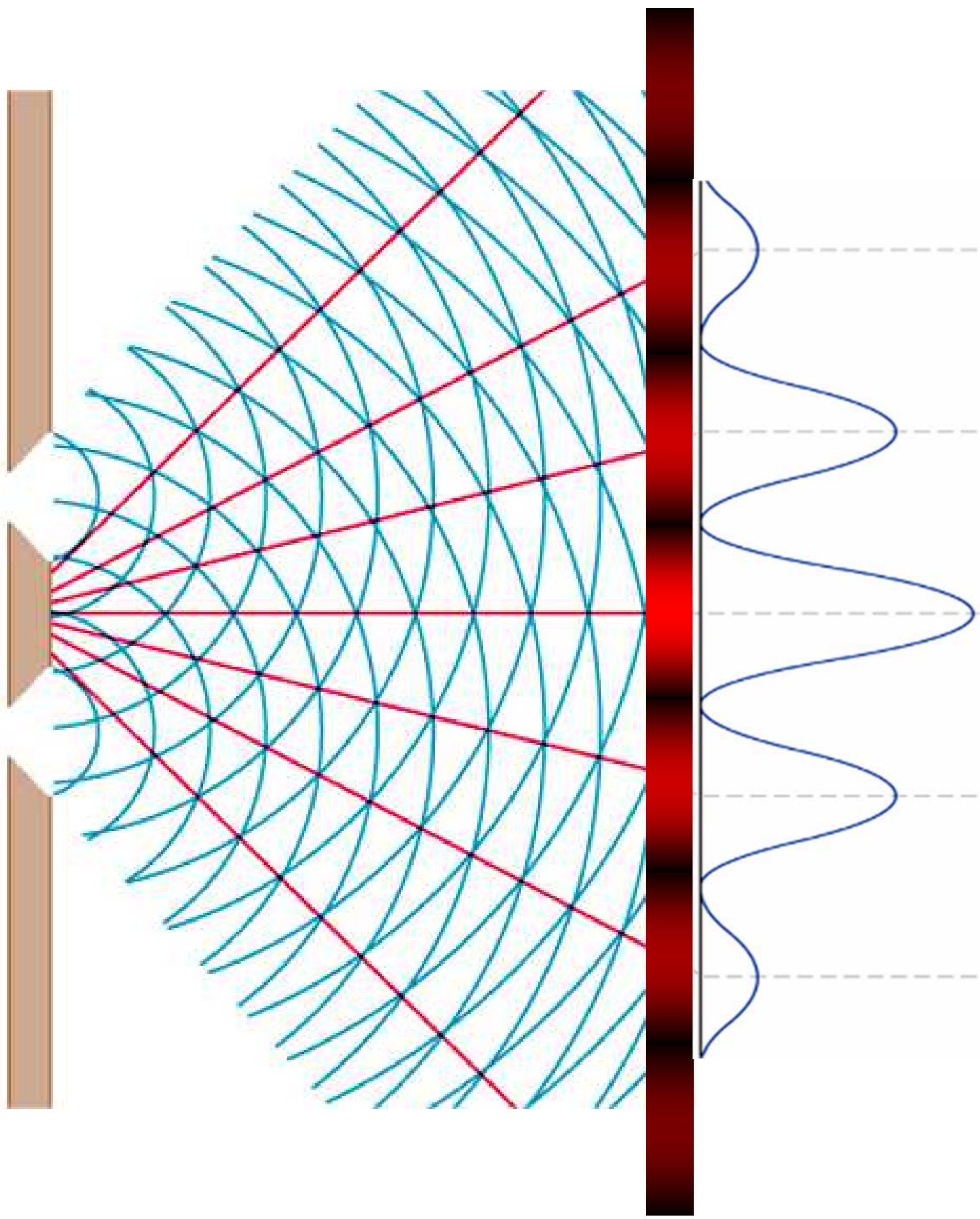
$$\vec{E}_1(\vec{x},t) = \vec{E}_{10}\cos(\vec{k}\cdot\vec{x} - \omega t + \xi_1)$$
$$\vec{E}_2(\vec{x},t) = \vec{E}_{20}\cos(\vec{k}\cdot\vec{x} - \omega t + \xi_2)$$

- When $\vec{E}_{01} = \vec{E}_{02}$ and $\xi_1 = \xi_2$:

$$I = 2I_0(1+\cos\delta) = 4I_0\cos^2\frac{\delta}{2}$$
$$\delta = k(r_1 - r_2)$$

- Constructive interference when $\delta = 2n\pi$
- Destructive interference when $\delta = (2n+1)\pi$

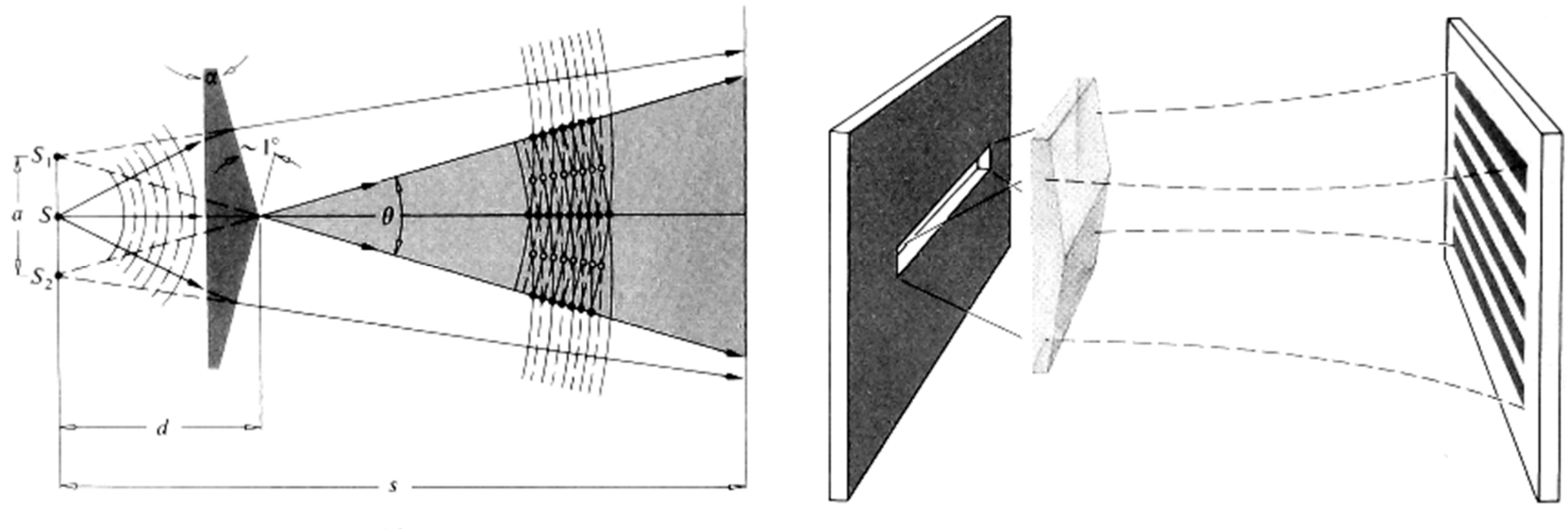
Young's Double Slit Experiment



The two slits act as two sources of coherent light.

Today we consider several other ways that light can produce interference effects.

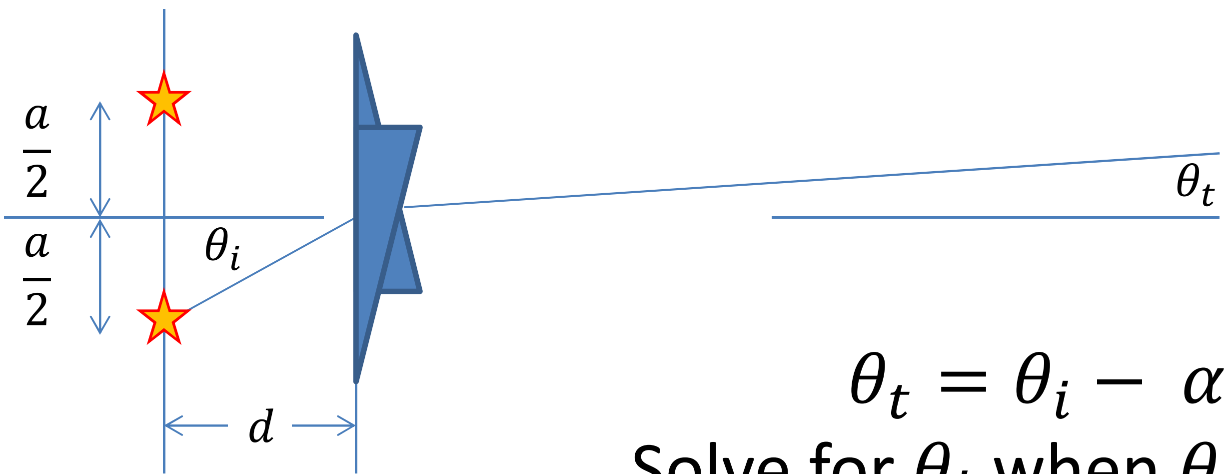
Other Interference Experiments: Fresnel's Double Prism Interferometer



- The general approach with many interference problems is to figure out how a particular system is equivalent to a double-slit experiment.

Fresnel's Double Prism Interferometer

- First, what is the spacing between the two equivalent light sources?
  - Where is the image of the light source?

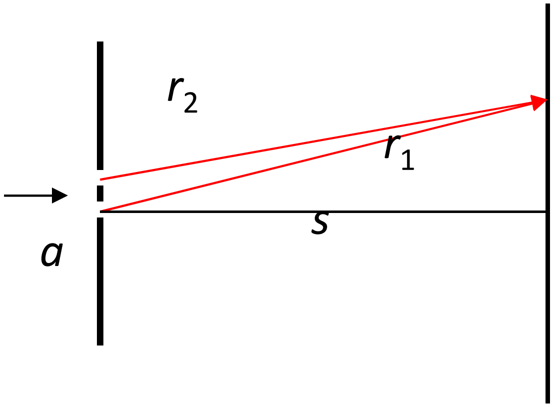


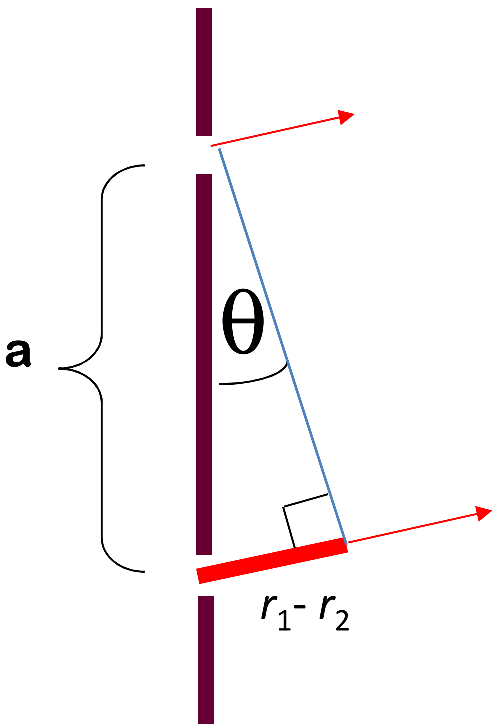
$$\theta_t = \theta_i - \alpha(n-1)$$

Solve for θ_i when $\theta_t = 0°$...

$$\frac{a}{2} = d\,\theta = d\alpha(n-1)$$

Young's Double Slit Experiment





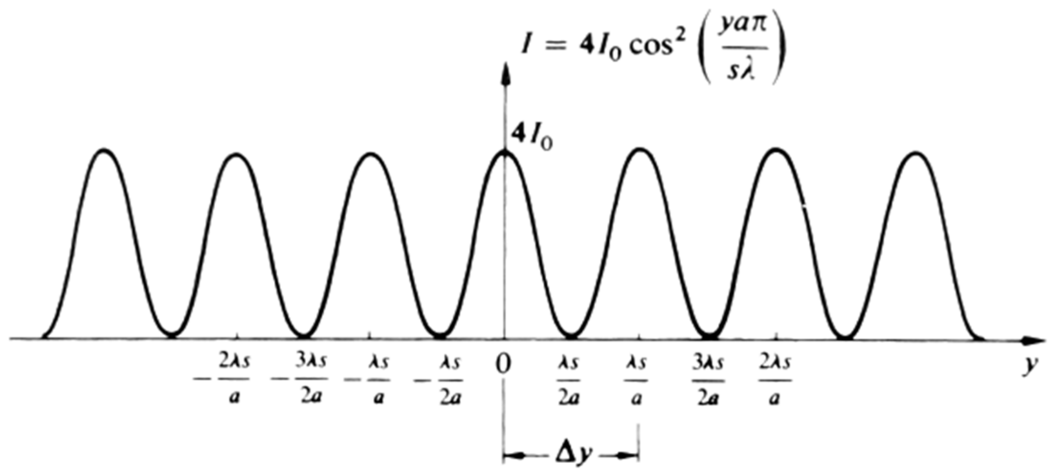
Far from the source, $s \gg a$,

$$I = 4I_0 \cos^2 \frac{\delta}{2}$$

$$= 4I_0 \cos^2 \left(\frac{k(r_1 - r_2)}{2} \right)$$

$$r_1 - r_2 = a \sin\theta \approx a \tan\theta \approx \frac{ay}{s}$$

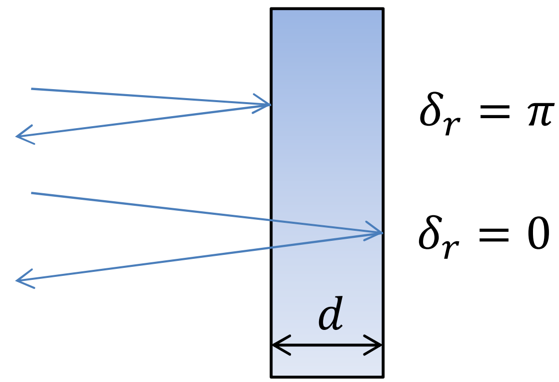
$$I \approx 4I_0 \cos^2 \frac{kay}{2s} = 4I_0 \cos^2 \frac{\pi a y}{s\lambda}$$



Interference From Thin Films

- Important result:

$$\left(\frac{E_r}{E_i}\right)_\perp = \frac{n_1 - n_2}{n_1 + n_2}$$



  – external reflection introduces a phase shift of π

- Wavelength in a material with index of refraction n:

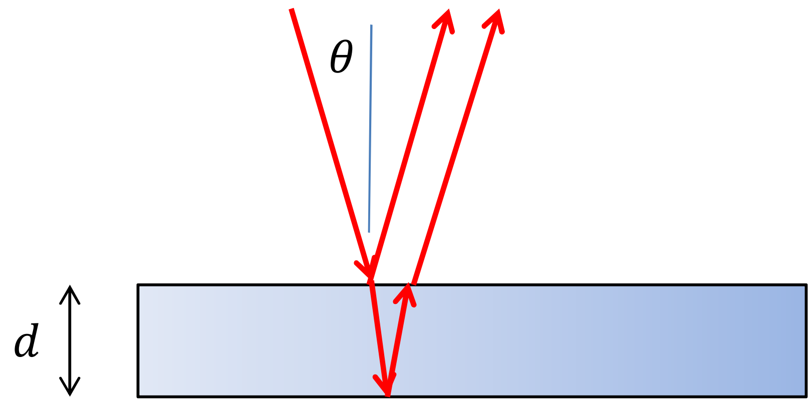
$$\lambda = \lambda_0 / n$$

- Number of wavelengths in thickness $2d$:

$$N = \frac{2dn}{\lambda_0}$$

- Phase difference: $\delta = 2\pi\left(N + \frac{1}{2}\right)$

Interference from Thin Films



- Phase difference for normal incidence:

$$\delta = 2\pi\left(\frac{2nd}{\lambda_0} + \frac{1}{2}\right)$$

- Phase difference when angle of incidence is θ:

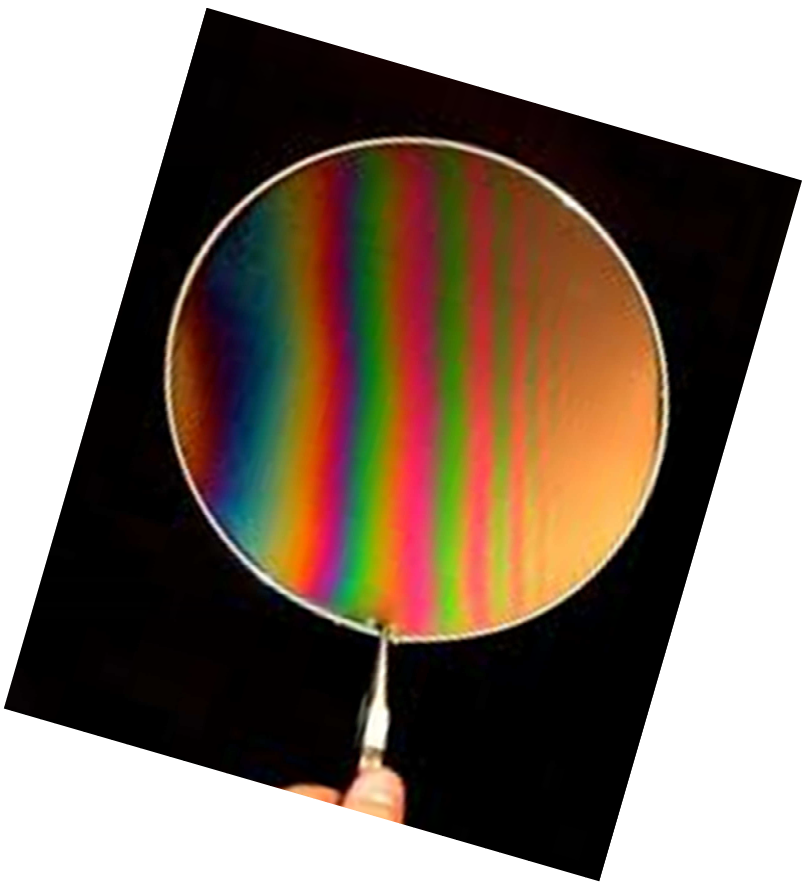
$$\delta = 2\pi\left(\frac{2nd}{\lambda_0 \cos\theta} + \frac{1}{2}\right)$$

- For monochromatic light, bright fringes have $\delta = 2\pi m$ and are located at

$$\cos\theta = \frac{nd}{\pi\lambda_0\left(m - \frac{1}{2}\right)}$$

Interference from Thin Films

$$\delta = 2\pi\left(\frac{2nd}{\lambda_0 \cos\theta} + \frac{1}{2}\right)$$

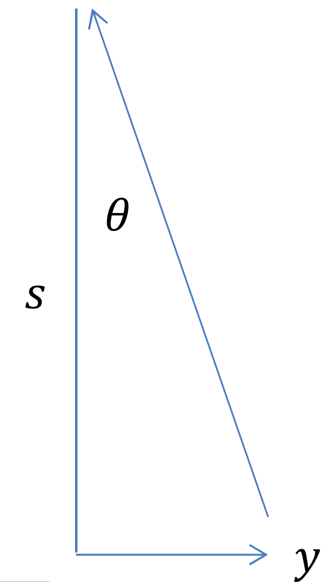


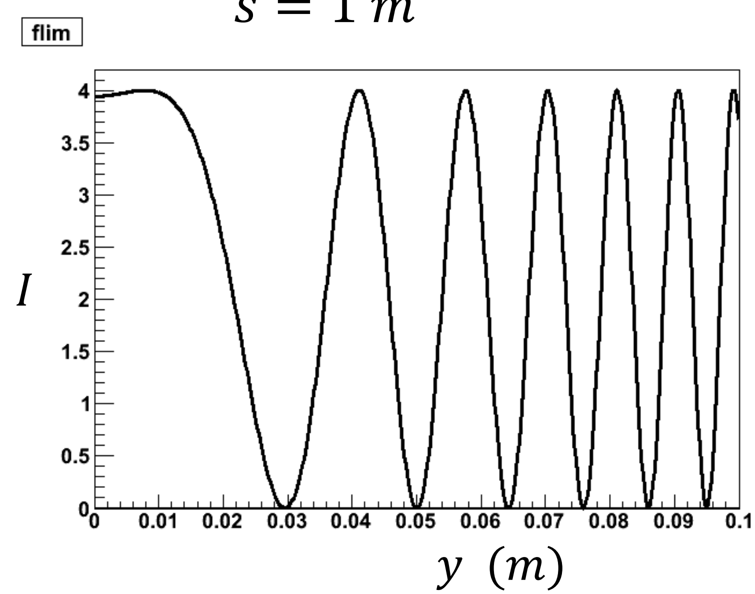
$\lambda_0 = 650\ nm$ (red light)

$d = 0.3\ mm$

$n = 1.333$

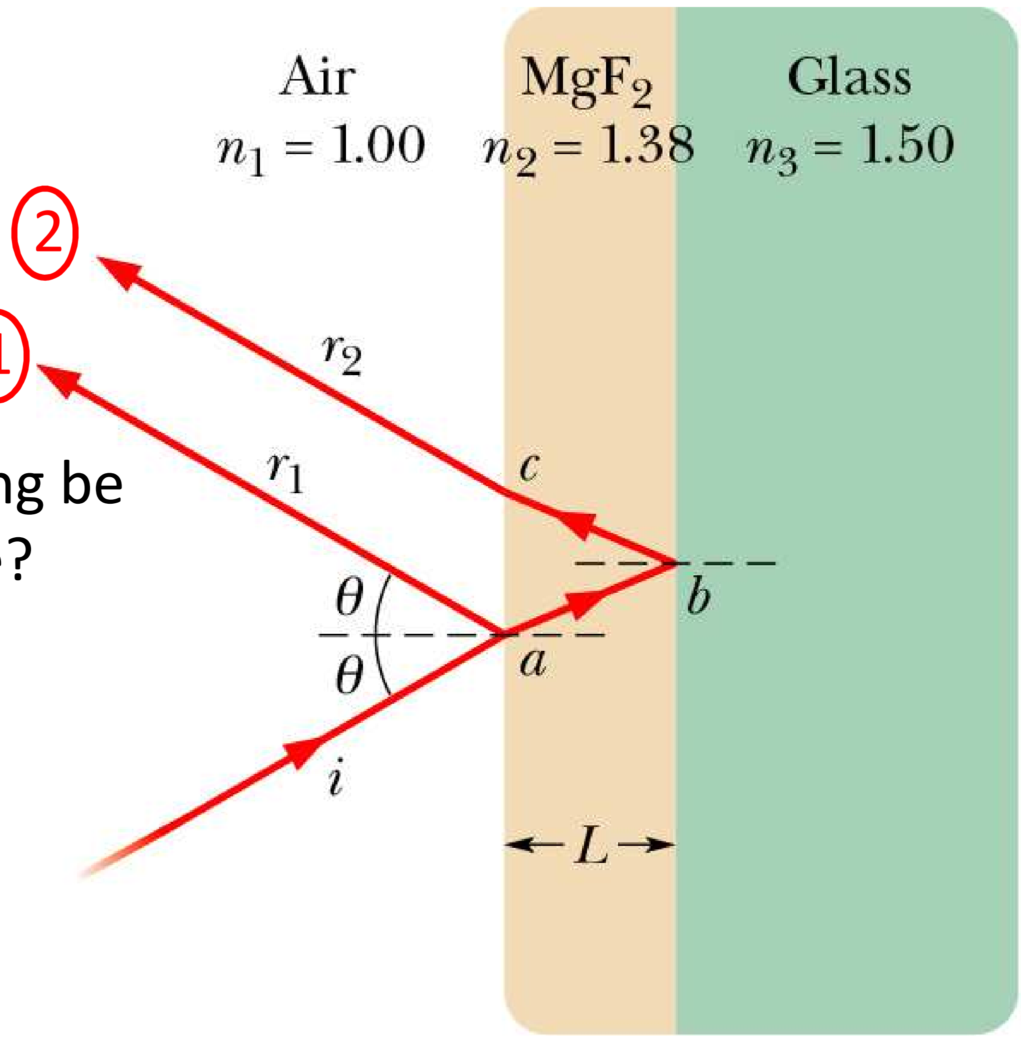
$s = 1\ m$





Coating a Glass Lens to Suppress Reflections:

180^0 phase change at both *a* and *b* since reflection is off a more optically dense medium



How thick should the coating be for destructive interference?

$$2t = \lambda'/_2$$

$$t = \lambda'/_4 = \lambda/_{4n_2}$$

What frequency to use?

Visible light: 400-700 nm

Coating a Glass Lens to Suppress Reflections:

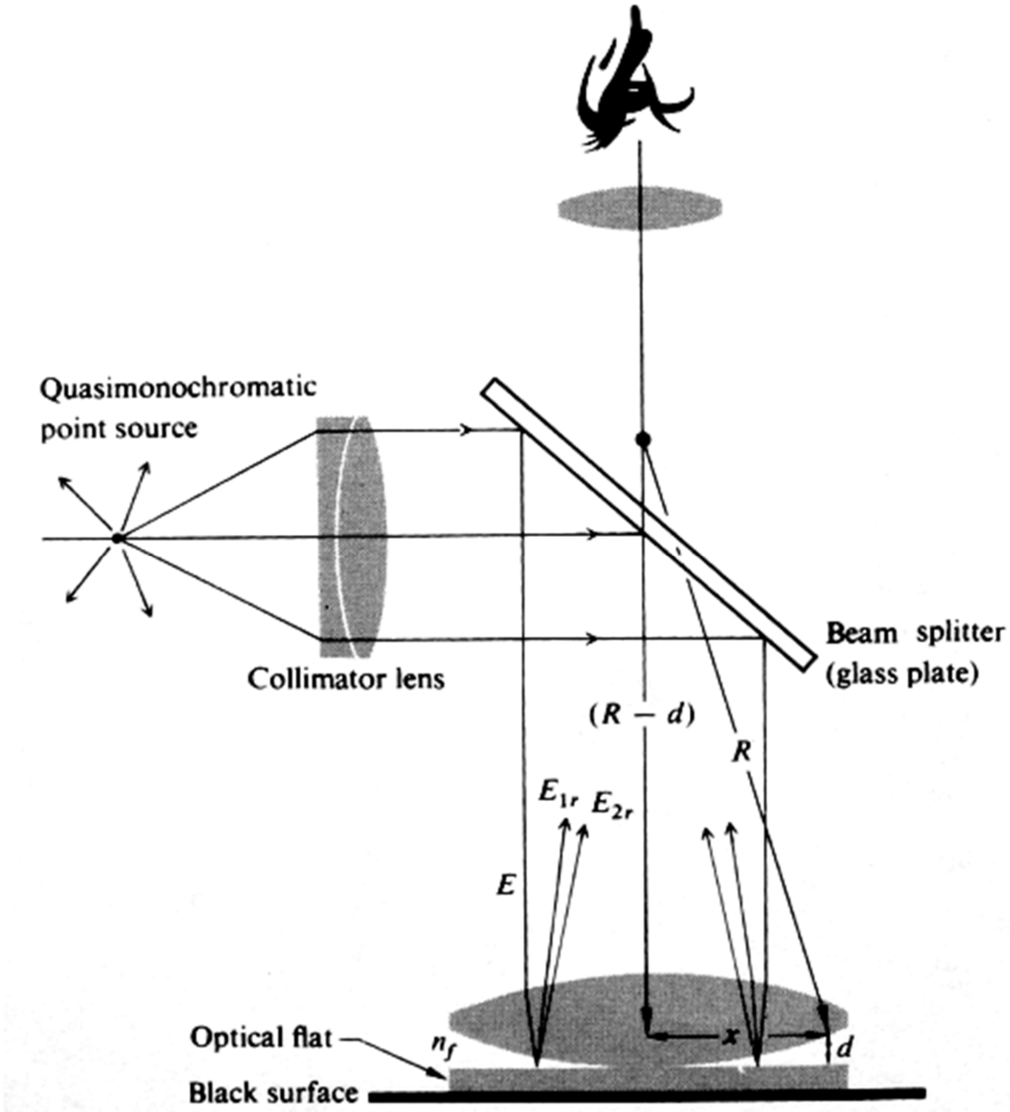
For λ= 550 nm and least thickness (m=1)

$$t = \frac{\lambda}{4n}$$

$$= \frac{550\ \text{nm}}{4 \times 1.38} = 99.6\ \text{nm}$$

- Note that the thickness needs to be different for different wavelengths.
- If the light reflected off the front and back surfaces interferes destructively, then all the energy must be transmitted

Newton's Rings



Why is center dark?

$$x^2 + (R-d)^2 = R^2$$

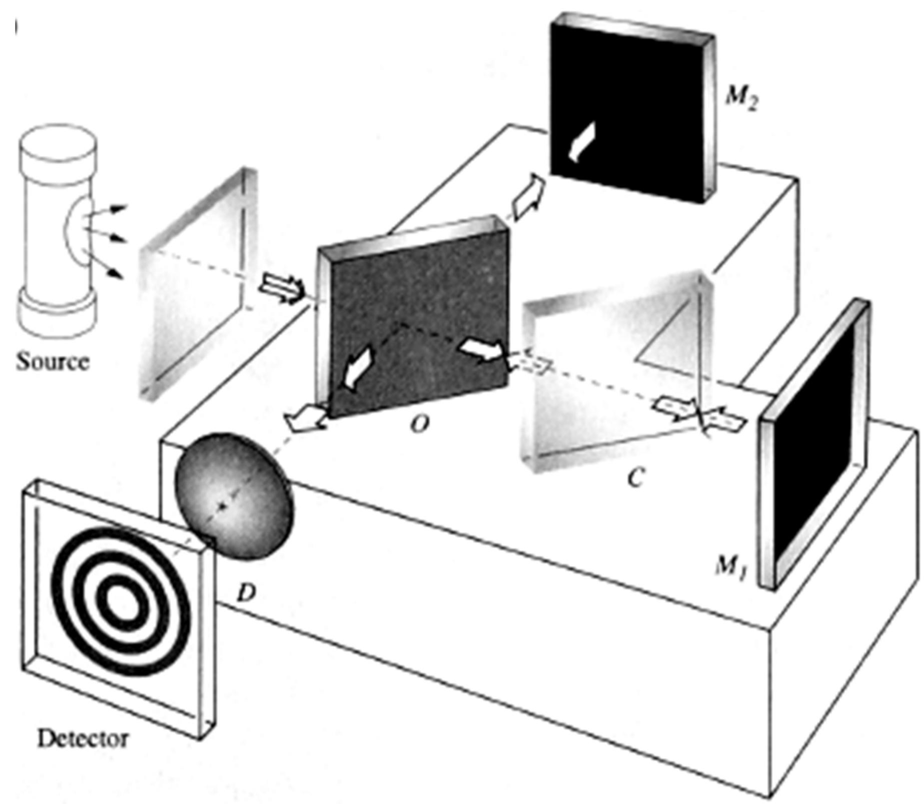
$$\downarrow$$

$$x^2 = 2Rd$$

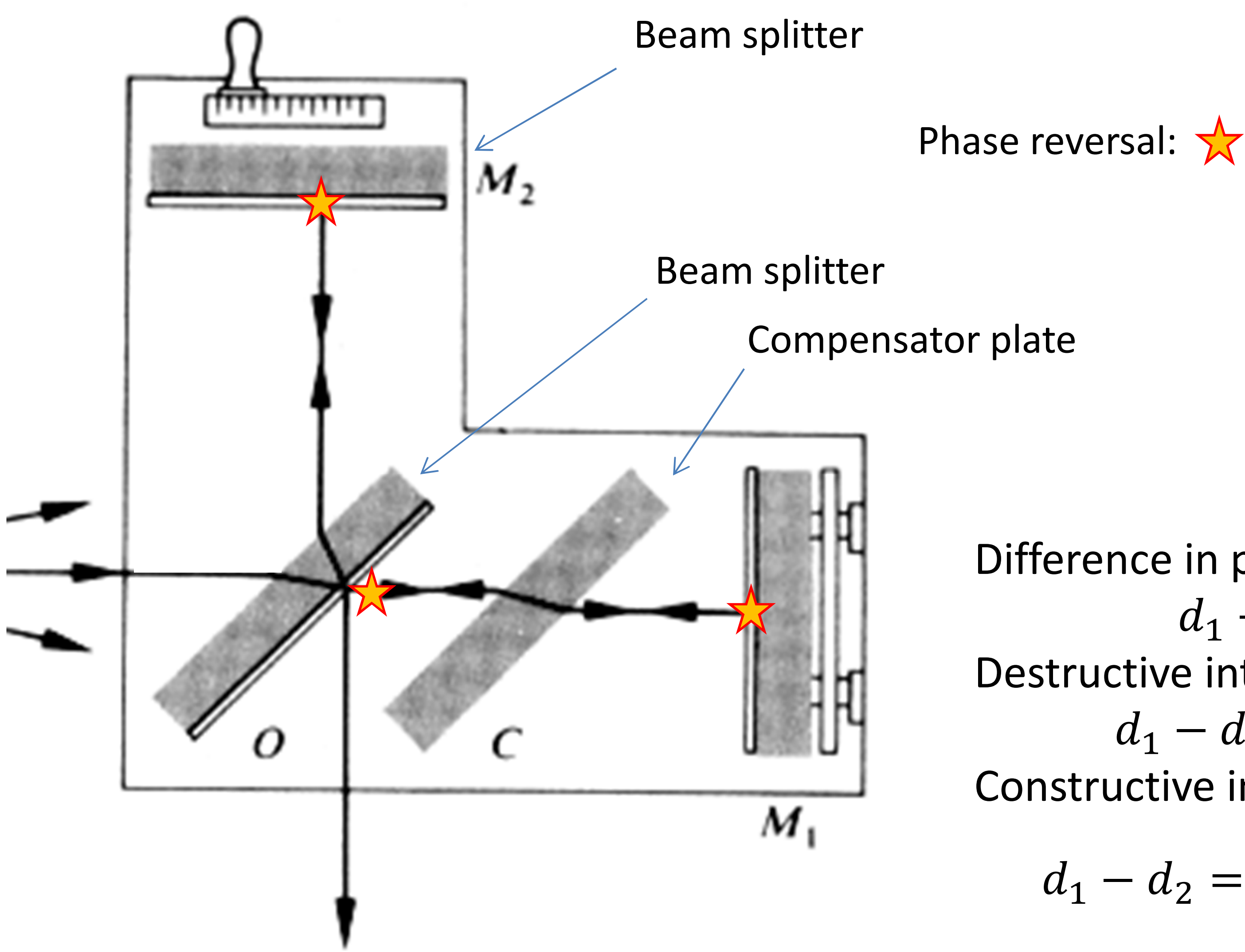
maxima: $2d = (m+½)\lambda$

$$x^2 = \left(m + \frac{1}{2}\right) R\lambda$$

Michelson Interferometer



Michelson Interferometer



Difference in path length:

$$d_1 - d_2$$

Destructive interference:

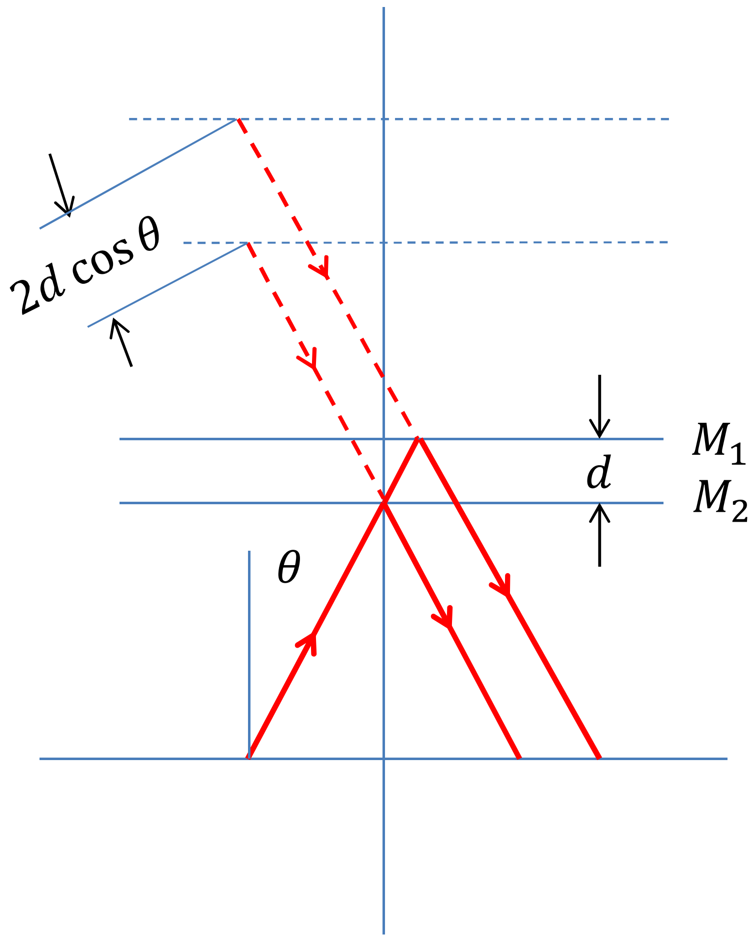
$$d_1 - d_2 = m\lambda$$

Constructive interference:

$$d_1 - d_2 = \left(m + \frac{1}{2}\right)\lambda$$

Michelson Interferometer

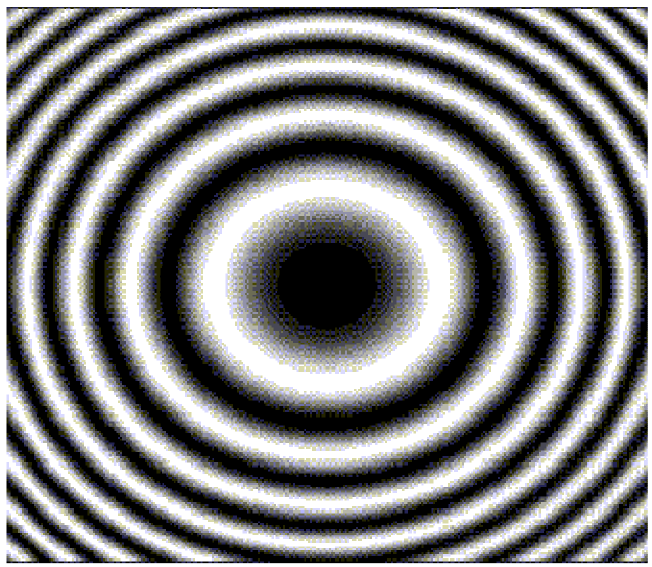
- Equivalent optics:



Michelson Interferometer

- Bright fringes occur when

$$\delta = 2\pi\left(\frac{2d}{\lambda\cos\theta} + \frac{1}{2}\right) = 2\pi m$$



Michelson Interferometer

- How does the position of a fringe change when the path length changes?

$$\frac{2d}{\lambda\cos\theta} = m + \frac{1}{2}$$

$$2d = \lambda\cos\theta\left(m + \frac{1}{2}\right)$$

$$2\Delta d = -\lambda\sin\theta\left(m + \frac{1}{2}\right)\Delta\theta$$

$$\frac{\Delta\theta}{\Delta d} = -\frac{2}{\left(m + \frac{1}{2}\right)\lambda\sin\theta}$$

Michelson Interferometer

- Application: Consider two closely spaced wavelengths, λ and λ'
- Bright fringes from one wavelength occur when

$$\frac{2d}{\lambda} = m$$

- Bright fringes from the other wavelength occur when

$$\frac{2d}{\lambda'} = m'$$

- The two fringes will coincide when

$$\frac{2d}{\lambda} = \frac{2d}{\lambda'} + N$$

Michelson Interferometer

- Adjust the position of the movable mirror so that the next set of fringes coincide

$$\frac{2d'}{\lambda} = \frac{2d'}{\lambda'} + N + 1$$

- Subtract these:

$$\frac{2d'}{\lambda} - \frac{2d}{\lambda} = \frac{2d'}{\lambda'} - \frac{2d}{\lambda'} + 1$$

$$\lambda' - \lambda = \frac{\lambda\lambda'}{2\Delta d} \approx \frac{\lambda^2}{2\Delta d}$$

- For the yellow sodium line,

$$\left.\begin{aligned} \lambda &= 588.991\,nm \\ \lambda' &= 589.595\,nm \end{aligned}\right\} \Delta\lambda = 0.604\,nm$$

$$\Delta d = \lambda^2/2\Delta\lambda = 287{,}472\,nm = 0.287\,mm$$

Michelson Interferometer

Michelson-Morley Experiment

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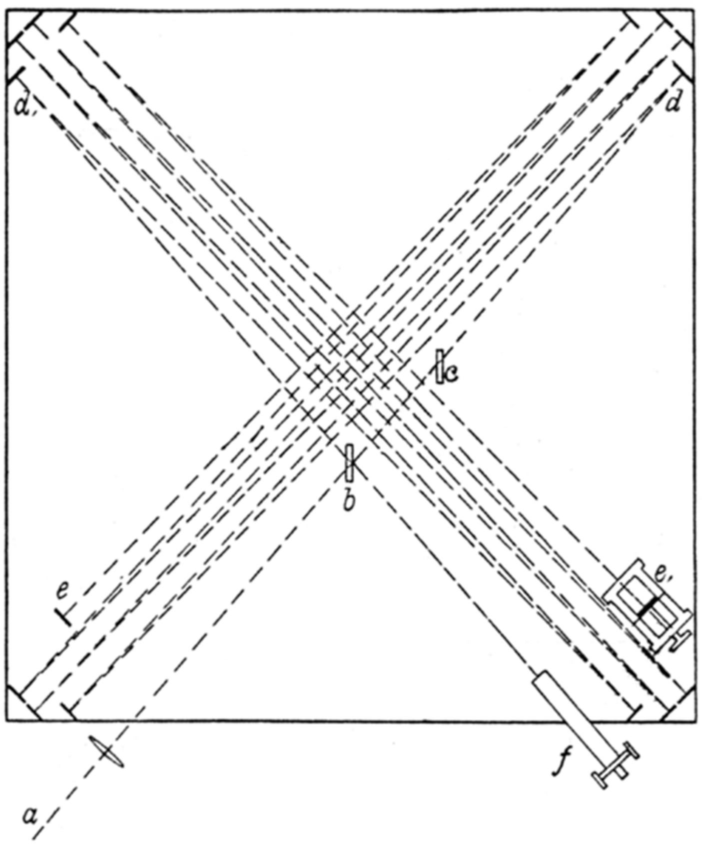
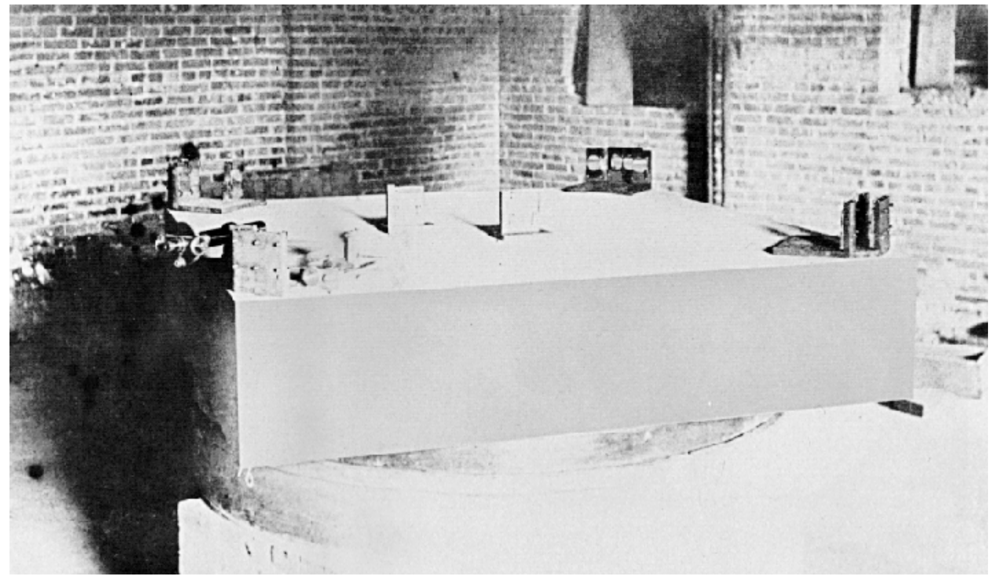




Time in the direction of the ether:

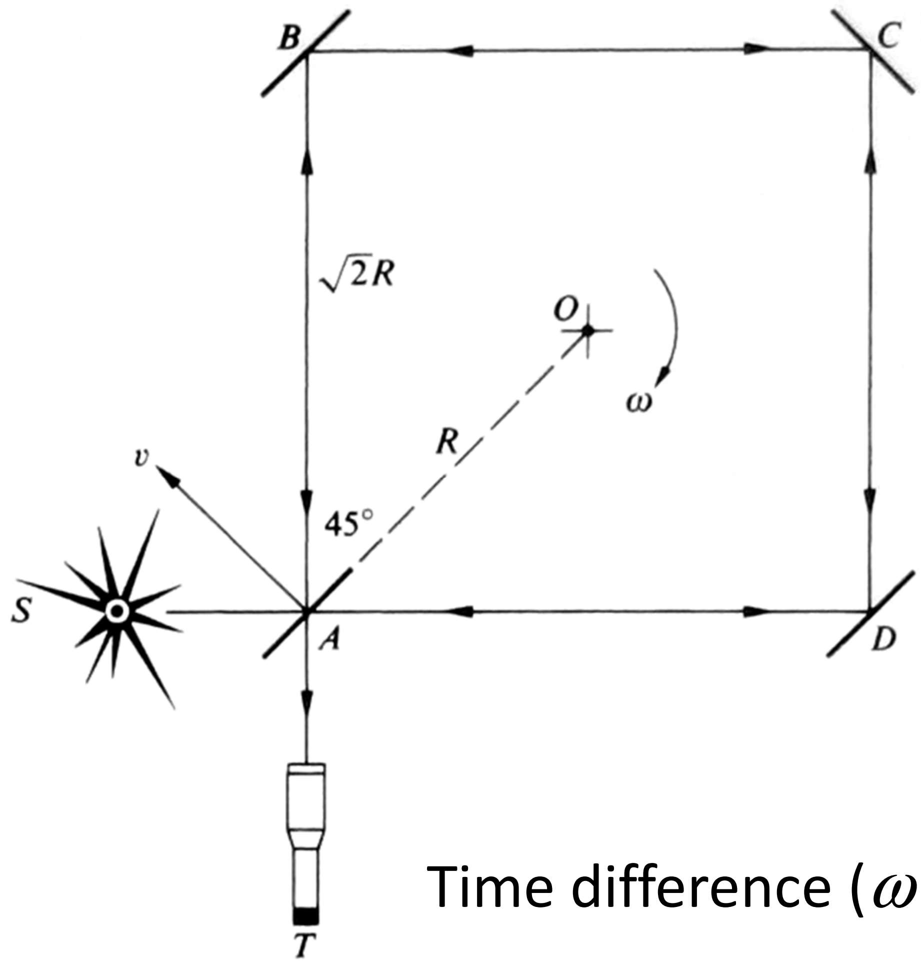
$$\Delta t = \frac{2w}{c}\left(1 + \frac{v^2}{c^2}\right)$$

Time perpendicular to the direction of the ether:

$$\Delta t = \frac{2w}{c}\left(1 + \frac{v^2}{2c^2}\right)$$

No interference observed ➔ No ether

Rotating Sagnac Interferometer



Interferometer rotates with angular velocity ω

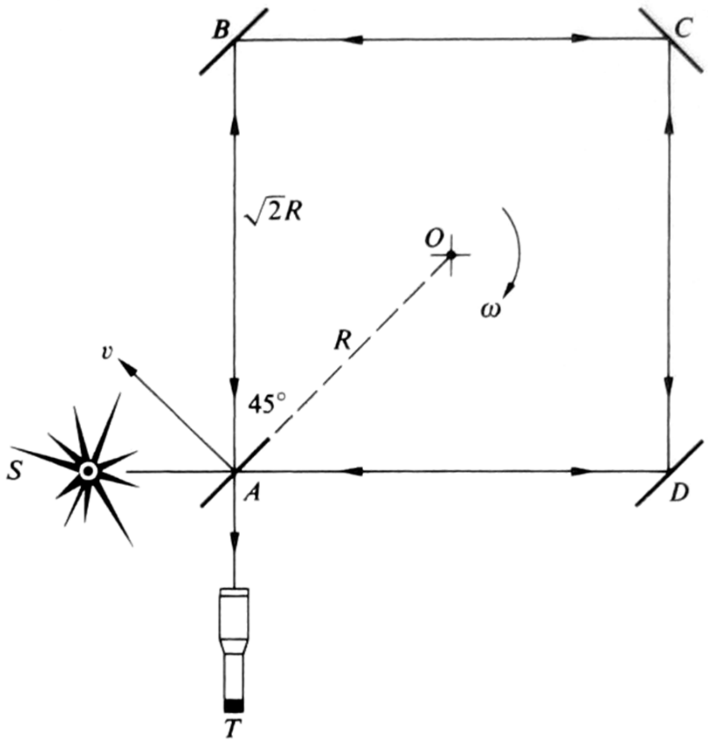
Travel AB: $$t_{AB} = \frac{R\sqrt{2}}{c - v/\sqrt{2}}$$

$$t_{AB} = \frac{2R}{\sqrt{2}c - \omega R}$$

Travel AD: $$t_{AD} = \frac{2R}{\sqrt{2}c + \omega R}$$

Time difference ($\omega R << c$): $$\Delta t \approx \frac{8R^2\omega}{c^2} = \frac{4A\omega}{c^2}$$

Rotating Sagnac Interferometer: Example



Michelson and Gale, 1925

Rotation of earth: $\omega = 2\pi/24$ hours

$\omega = 7.27\times10^{-5}\ \mathrm{s}^{-1}$

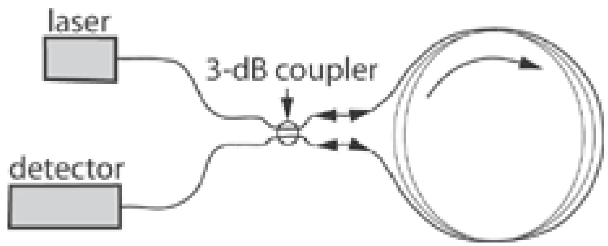
A=(500 m)^2

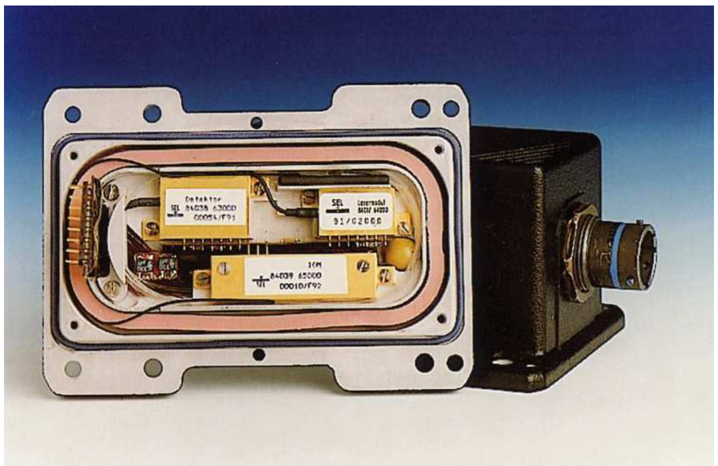
$$\Delta t \approx \frac{4A\omega}{c^2} = \frac{4\cdot(500\,\mathrm{m})^2\cdot\left(7.27\times10^{-5}\mathrm{s}^{-1}\right)}{\left(3\times10^{8}\,\mathrm{m/s}\right)^2}$$

$$\Delta t \approx 8.1\times10^{-16}\ \mathrm{s}$$

One period of light wave: λ/c = (500 nm)/(3×10^8 m/s)=1.7×10^{-15} s

Sagnac Interferometer: Gyroscope





- Typical applications: navigation, avionics, mining, drilling, industrial robots