

Physics 42200

Waves & Oscillations

Lecture 31 – Polarization of Light

Spring 2014 Semester

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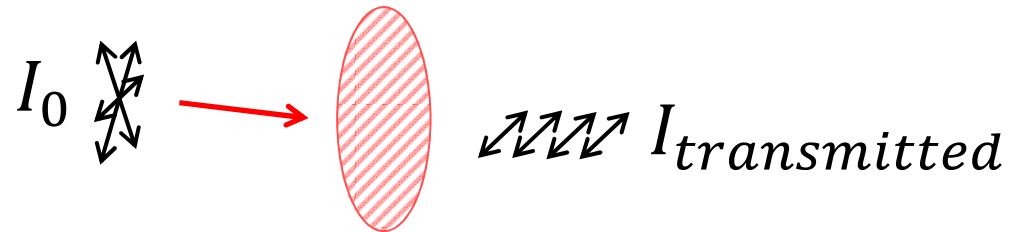
Types of Polarization

- Light propagating through different materials:
 - One polarization component can be absorbed more than the other
 - One polarization component can propagate with a different speed
- What are the properties of the light that emerges?
- Principle of superposition:

$$\vec{E} = \vec{E}_{\parallel} + \vec{E}_{\perp}$$

Linear Polarization

- Electric field component along one axis is completely absorbed



- Components of the transmitted $\vec{E}$ -field are in phase, $\vec{E}$ is aligned with the polarizing axis:

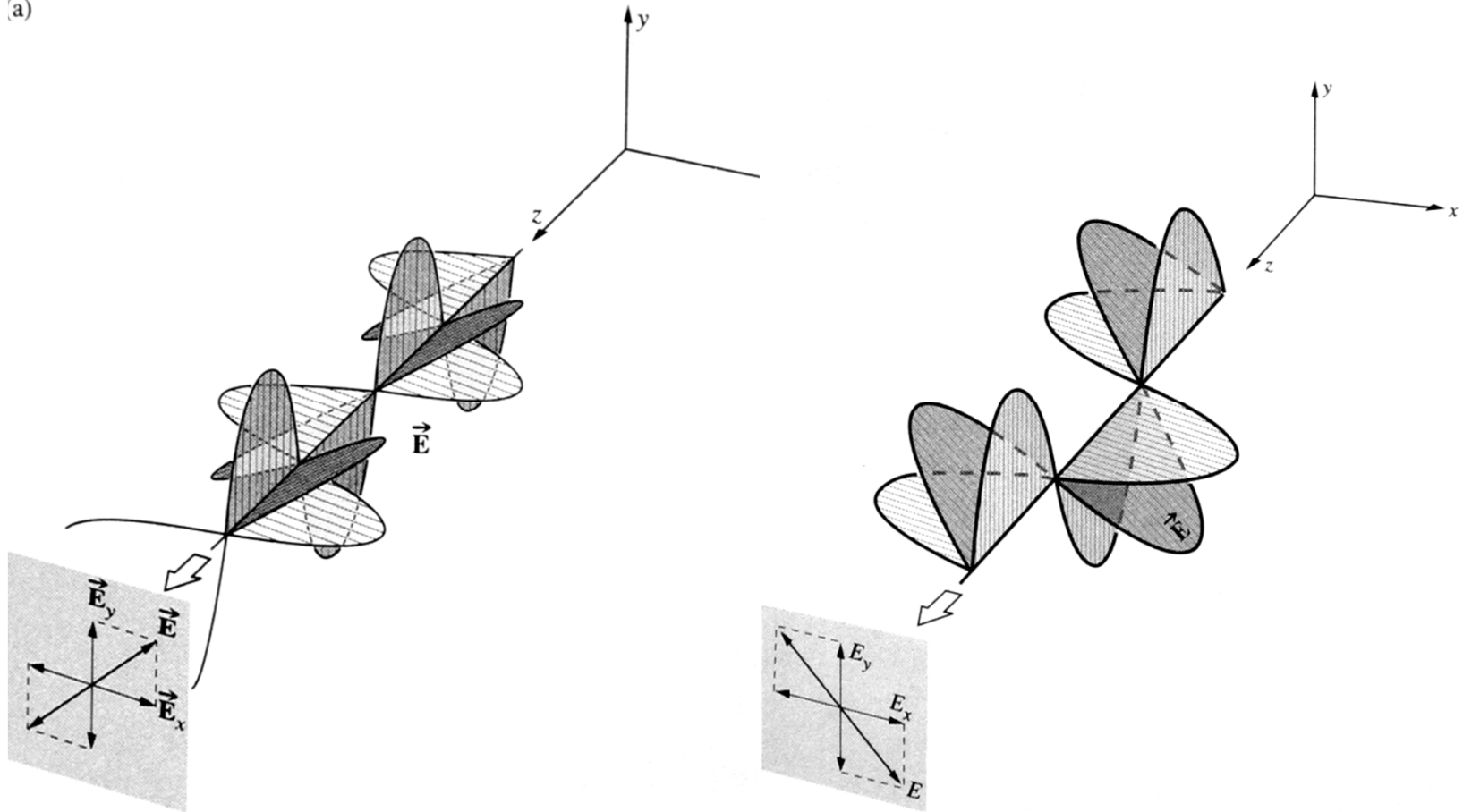
$$\vec{E}(z, t) = (E_x \hat{i} + E_y \hat{j}) \cos(kz - \omega t)$$

- Phases that differ by $\pm\pi, \pm3\pi, \dots$ are still linearly polarized:

$$\text{eg., } \vec{E}(z, t) = (E_x \hat{i} - E_y \hat{j}) \cos(kz - \omega t)$$

Linear Polarization

(a)



- Both components are still in phase
 - Nodes occur at common points on the z-axis

Circular Polarization

- What if the two components had the same amplitude but a different phase?

$$\vec{E}_x(z, t) = E_0 \hat{i} \cos(kz - \omega t)$$

$$\vec{E}_y(z, t) = E_0 \hat{j} \cos(kz - \omega t + \xi)$$

- In particular, what if $\xi = \pm \frac{\pi}{2}$

$$\vec{E}_y(z, t) = E_0 \hat{j} \sin(kz - \omega t)$$

- Resultant electric field:

$$\vec{E}(z, t) = E_0 (\hat{i} \cos(kz - \omega t) + \hat{j} \sin(kz - \omega t))$$

Circular Polarization

$$\vec{E}(z, t) = E_0(\hat{i} \cos(kz - \omega t) + \hat{j} \sin(kz - \omega t))$$

x and y components oscillate:

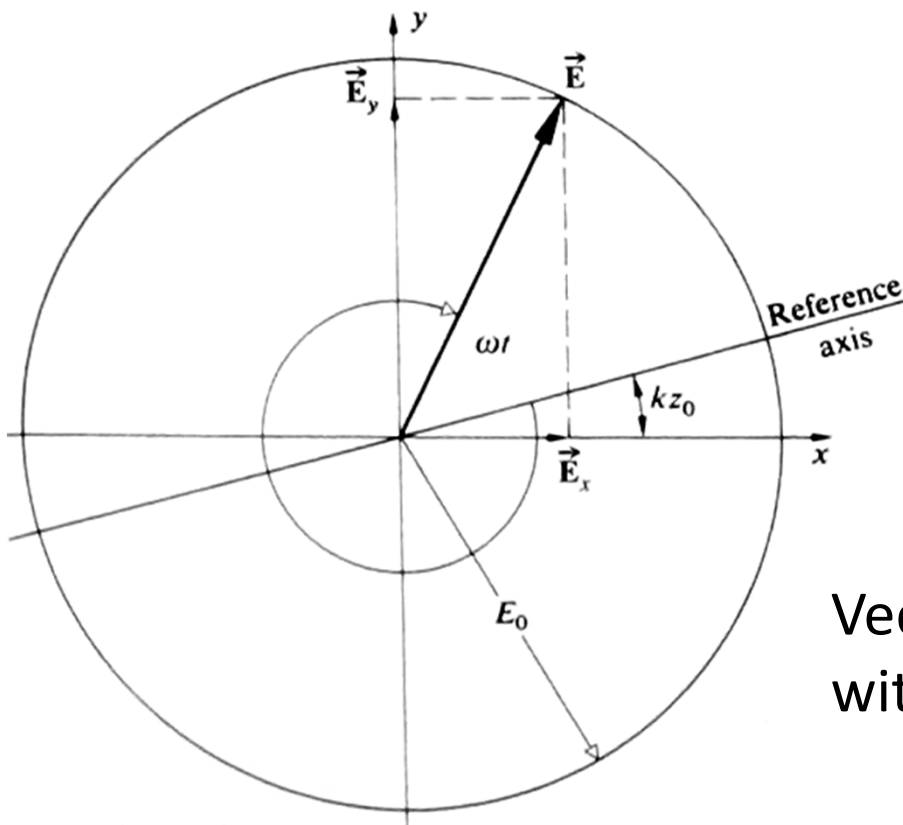
$$E_x = E_0 \cos(kz - \omega t)$$

$$E_y = E_0 \sin(kz - \omega t)$$

$$\text{Angle } \alpha = kz_0 - \omega t$$

Vector E rotates in time
with angular frequency $-\omega$

Vector E rotates in space
with angular spatial speed k



Circular Polarization

$$\vec{E}(z,t) = E_0 [\hat{\mathbf{i}} \cos(kz - \omega t) + \hat{\mathbf{j}} \sin(kz - \omega t)]$$

Right circularly polarized light:

E rotates clockwise as seen by observer

→
Vector makes full turn as wave
advances one wavelength

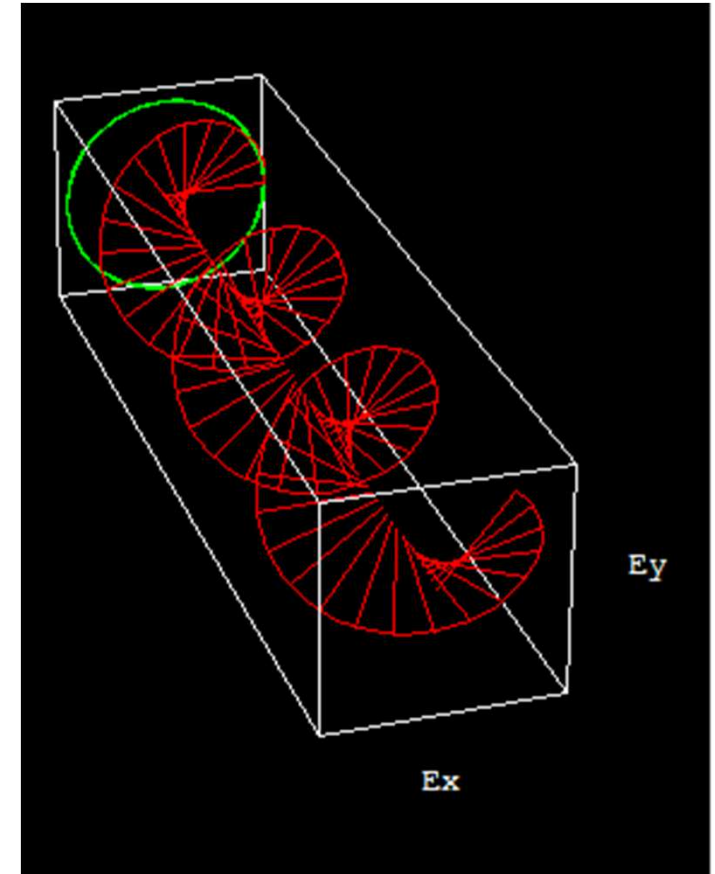
Left circularly polarized light:

E rotates counter clockwise

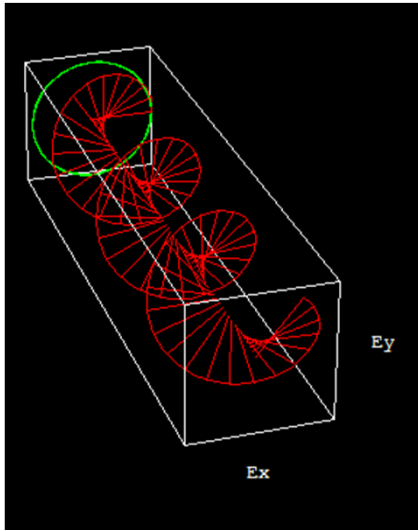
$$\vec{E}(z,t) = E_0 [\hat{\mathbf{i}} \cos(kz - \omega t) - \hat{\mathbf{j}} \sin(kz - \omega t)]$$

What if we have a superposition of left and right circularly polarized light of equal amplitude?

$$\vec{E}(z,t) = 2E_0 \hat{\mathbf{i}} \cos(kz - \omega t) \quad - \text{linearly polarized light}$$



Circular Polarization and Angular Momentum



What would happen with an electron under circularly polarized light?

The rotating electric field would push it in a circle...

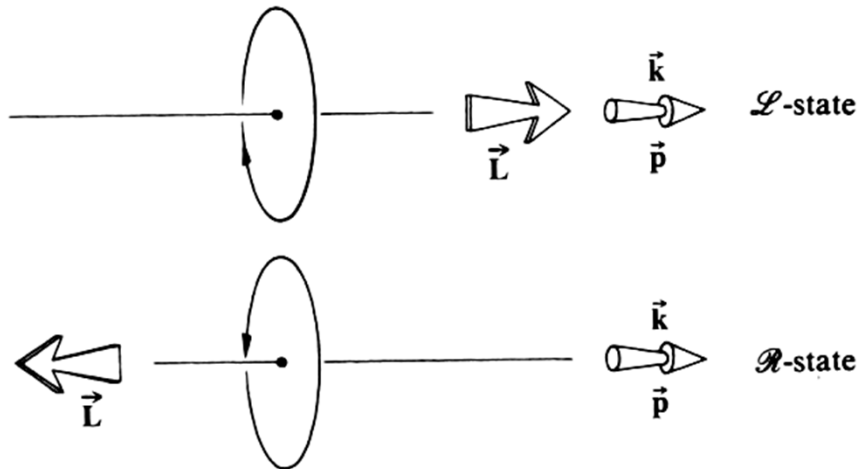
Angular velocity ω - angular momentum L

Light is absorbed, and if it was circularly polarized:

angular momentum $\rightarrow L = \frac{E}{\omega}$

energy of light $\leftarrow E$

ω $\leftarrow$ frequency of light



Photons and Angular Momentum

$$L = \frac{E}{\omega} \quad \text{Photon has energy:} \quad E = h\nu = \frac{h}{2\pi} \omega = \hbar \omega$$

Angular momentum of a photon is independent of its energy:

$$L = \pm \hbar$$

Photon has a *spin*, $+\hbar$ - L-state
 $-\hbar$ - R-state

Whenever a photon is absorbed or emitted by a charged particle, along with the change in its energy it will undergo a change in its angular momentum

First measured in 1935 by Richard Beth

Linearly polarized light: photons exist in either spin state with equal probability

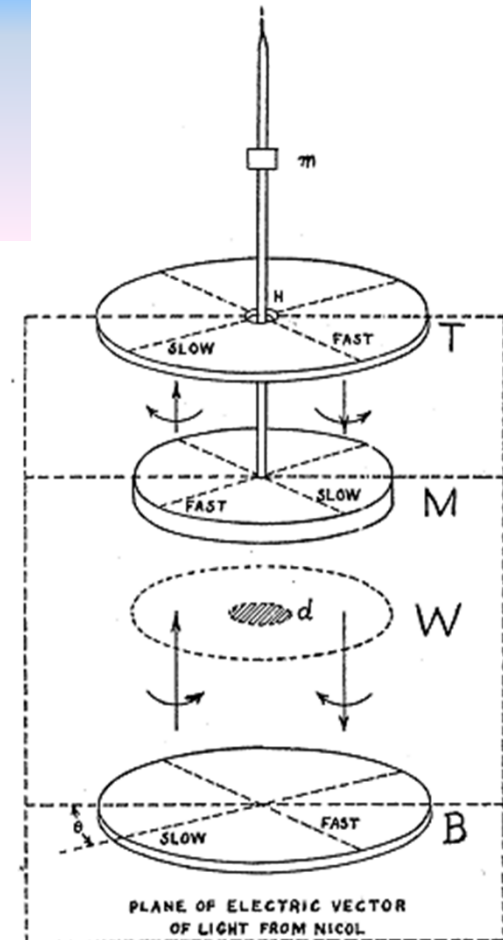
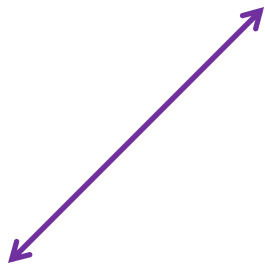


FIG. 3. Wave plate arrangement.

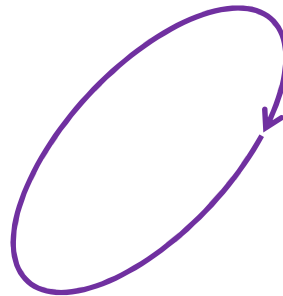
Elliptic Polarization

$$\vec{E}_x(z, t) = E_0 \hat{i} \cos(kz - \omega t)$$
$$\vec{E}_y(z, t) = E_0 \hat{j} \cos(kz - \omega t + \xi)$$

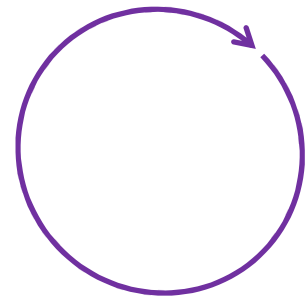
- $\xi = 0, \pm\pi$ corresponds to linear polarization
- $\xi = \pm\frac{\pi}{2}$ corresponds to circular polarization
- What about other values of ξ ?



Linear



Elliptic polarization



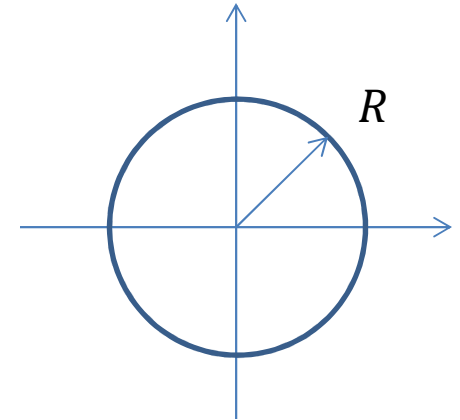
Circular

Geometry Lesson

- Equation for a circle:

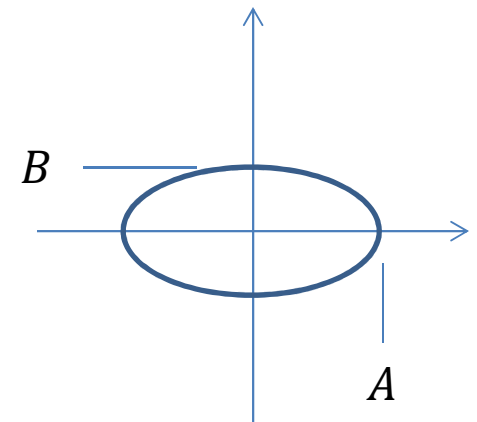
$$x^2 + y^2 = R^2$$

$$\left(\frac{x}{R}\right)^2 + \left(\frac{y}{R}\right)^2 = 1$$



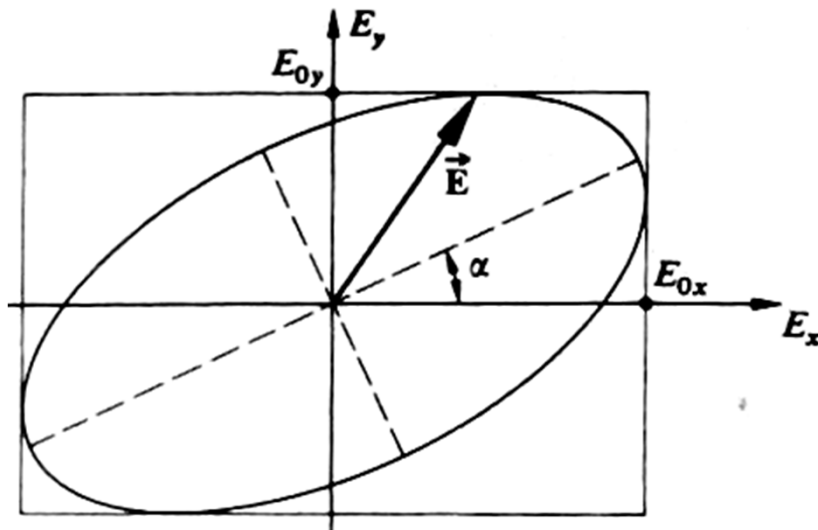
- Equation for an ellipse:

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 = 1$$



Geometry Lesson

- Equation for an ellipse oriented at an angle α with respect to the x-axis:



$$\tan 2\alpha = \frac{2AB \cos \xi}{A^2 - B^2}$$

$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 - 2\left(\frac{x}{A}\right)\left(\frac{y}{B}\right)\cos \xi = \sin^2 \xi$$

Most General Case

$$\vec{E}_x(z, t) = E_{0x} \hat{i} \cos(kz - \omega t)$$

$$\vec{E}_y(z, t) = E_{0y} \hat{j} \cos(kz - \omega t + \xi)$$

- Magnitudes of x - and y -components are different
- Phase difference between x - and y -components
- Algebra:

$$E_x = E_{0x} \cos(kz - \omega t)$$

$$E_y = E_{0y} \cos(kz - \omega t + \xi)$$

$$\frac{E_y}{E_{0y}} = \cos(kz - \omega t) \cos \xi - \sin(kz - \omega t) \sin \xi$$

$$\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \cos \xi = -\sin(kz - \omega t) \sin \xi$$

$$\left(\frac{E_x}{E_{0x}} \right)^2 = \cos^2(kz - \omega t) = 1 - \sin^2(kz - \omega t)$$

$$\sin^2(kz - \omega t) = 1 - \left(\frac{E_x}{E_{0x}} \right)^2$$

Most General Case

$$\left(\frac{E_y}{E_{0y}} - \frac{E_x}{E_{0x}} \cos \xi \right)^2 = \sin^2(kz - \omega t) \sin^2 \xi$$
$$= \left[1 - \left(\frac{E_x}{E_{0x}} \right)^2 \right] \sin^2 \xi$$

$$\left(\frac{E_y}{E_{0y}} \right)^2 - 2 \left(\frac{E_y}{E_{0y}} \right) \left(\frac{E_x}{E_{0x}} \right) \cos \xi + \left(\frac{E_x}{E_{0x}} \right)^2 (\sin^2 \xi + \cos^2 \xi) = \sin^2 \xi$$

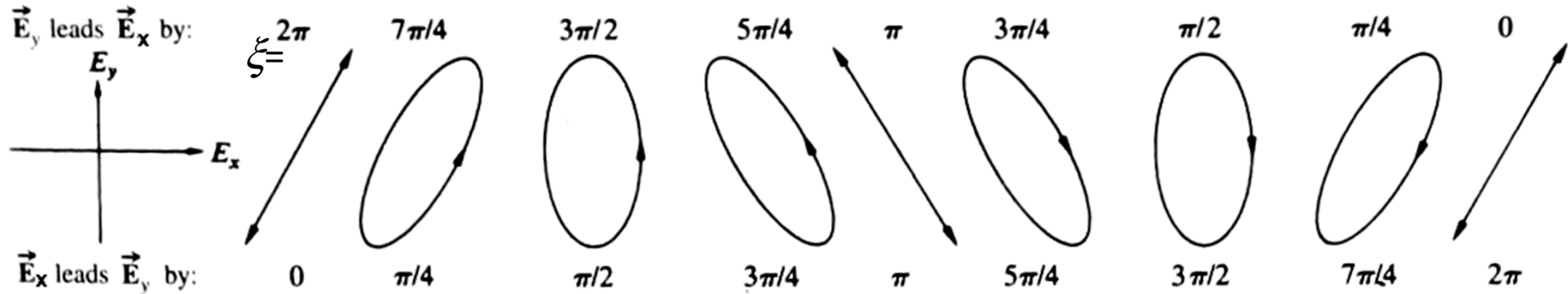
$$\left(\frac{\mathbf{E}_y}{\mathbf{E}_{0y}} \right)^2 - 2 \left(\frac{\mathbf{E}_y}{\mathbf{E}_{0y}} \right) \left(\frac{\mathbf{E}_x}{\mathbf{E}_{0x}} \right) \mathbf{cos} \xi + \left(\frac{\mathbf{E}_x}{\mathbf{E}_{0x}} \right)^2 = \mathbf{sin}^2 \xi$$

This is the equation for an ellipse.

Elliptic Polarization

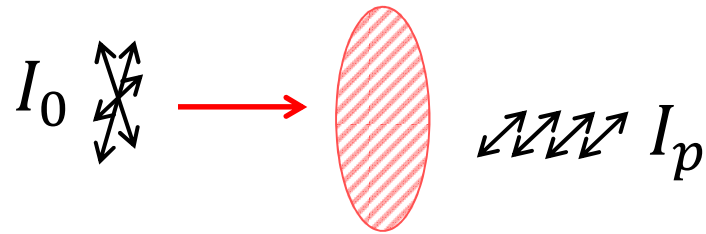
$$\left(\frac{E_y}{E_{0y}}\right)^2 + \left(\frac{E_x}{E_{0x}}\right)^2 - 2\left(\frac{E_y}{E_{0y}}\right)\left(\frac{E_x}{E_{0x}}\right)\cos\xi = \sin^2\xi$$

When $E_y \approx 2E_x$:



Changing Polarization

- How can we turn unpolarized light into linearly polarized light?



- Pass it through a polarizer
- How do we get equal magnitudes of the x - and y -components?
 - Rotate the polarizer so that it is 45° with respect to the x -axis.

Changing Polarization

- How do we turn linearly polarized light into light with circular polarization?

$$\vec{E}(z, t) = E_0(\hat{i} \cos(kz - \omega t) + \hat{j} \sin(kz - \omega t))$$

- Shift the phase of one component by $\pm\pi/2$
- But how?
- Remember the birefringent crystal?
 - Light polarized along the optic axis travels at a different speed compared with light polarized perpendicular to the optic axis
 - Make the thickness such that light emerges with the desired phase shift.
 - This device is called a “quarter-wave plate” because it shifts one component by one quarter of a wavelength.

Quarter Wave Plates

- Consider a birefringent crystal with ordinary and extraordinary indices of refraction n_o and n_e
- How thick should the crystal be to retard one component by $\Delta\lambda/\lambda = 2m\pi + \pi/2 = (4m + 1)\pi/2$?
- Ordinary component: $E_o(z, t) = E \cos(k_o z - \omega t)$
- Extraordinary component: $E_e(z, t) = E \cos(k_e z - \omega t)$

$$k_o = \frac{\omega}{v_o} = \frac{\omega n_o}{c} = 2\pi \frac{n_o}{\lambda_0}$$

$$k_e = \frac{\omega}{v_e} = \frac{\omega n_e}{c} = 2\pi \frac{n_e}{\lambda_0}$$

- After propagating a distance, d , we want

$$(k_o - k_e)d = (4m + 1)\pi/2$$

$$(n_o - n_e)d = \lambda_0(4m + 1)/4$$

$$d = \frac{\lambda_0}{n_o - n_e} (4m + 1)/4$$

Quarter Wave Plates

- Consider calcite: $(n_e - n_o) = -0.172$
- The thickness at normal incidence depends on the wavelength of light.

– Suppose $\lambda_0 = 560 \text{ nm}$ (green) and pick $m = 1$

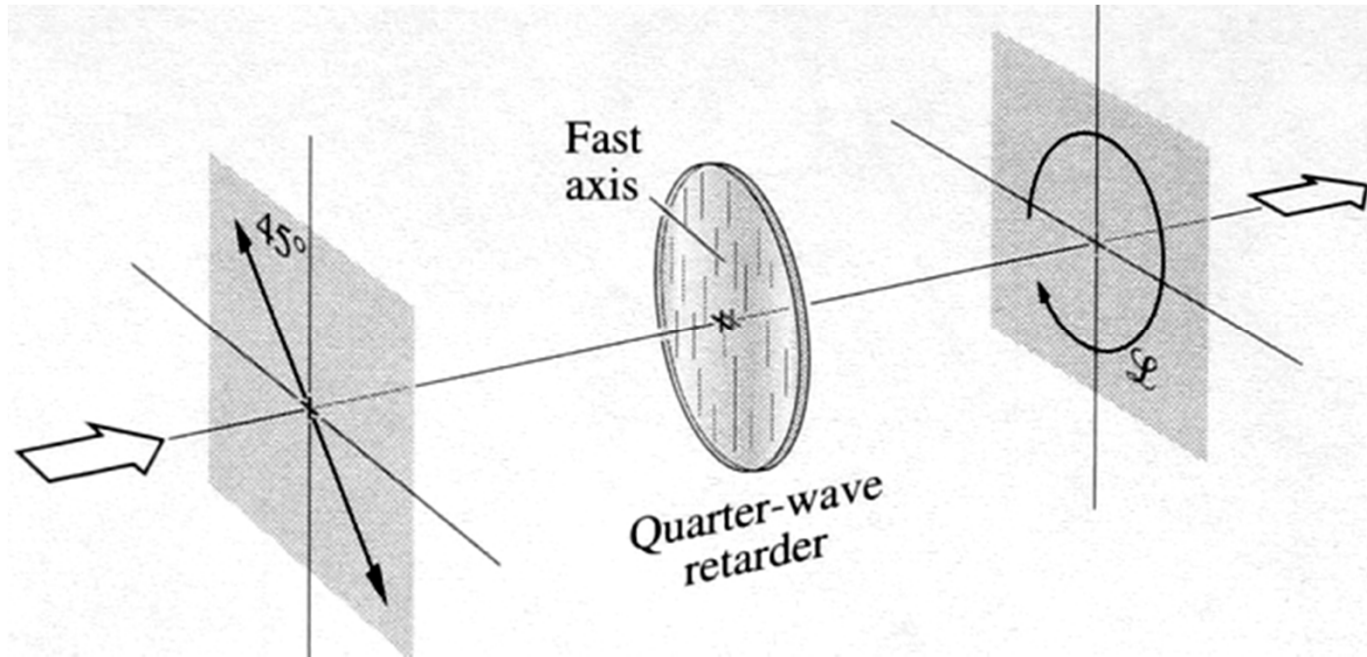
$$\begin{aligned} d &= \frac{\lambda_0}{n_o - n_e} (4m + 1)/4 \\ &= \left(\frac{560 \text{ nm}}{0.172} \right) \frac{1}{4} = 814 \text{ nm} \end{aligned}$$

- This is very thin... if the thickness were to be $500 \mu\text{m}$ then $m \approx 150$

$$d = \left(\frac{560 \text{ nm}}{0.172} \right) \frac{1}{2} (4 \times 150 + 1) = 489.186 \mu\text{m}$$

- Less fragile but the path length would be significantly different if light was not at precisely normal incidence.

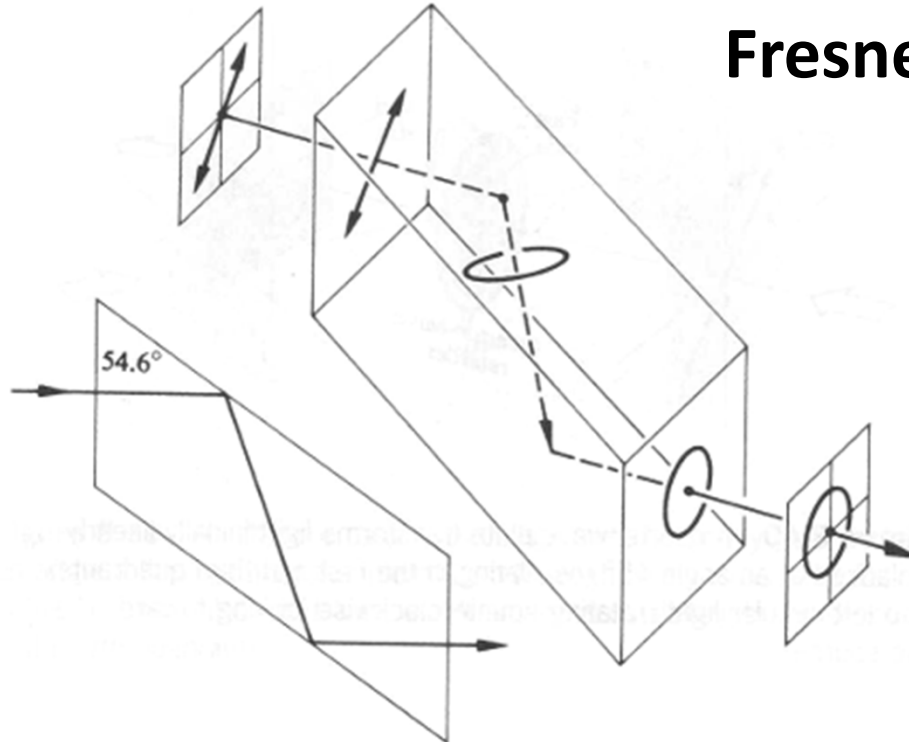
Quarter Wave Plate



- Additional quarter-wave plates introduce additional phase shifts:
 - Two quarter-wave plates produces linear polarization rotated by 90° with respect to the polarization axis of the incident light.

Reflective Retarders

Total internal reflection: phase shift between the two components.
Glass - $n=1.51$, and 45° shift occurs at incidence angle 54.6° .



Fresnel rhomb

Unlike devices that use birefringent materials this is achromatic.
(works for all wavelengths)

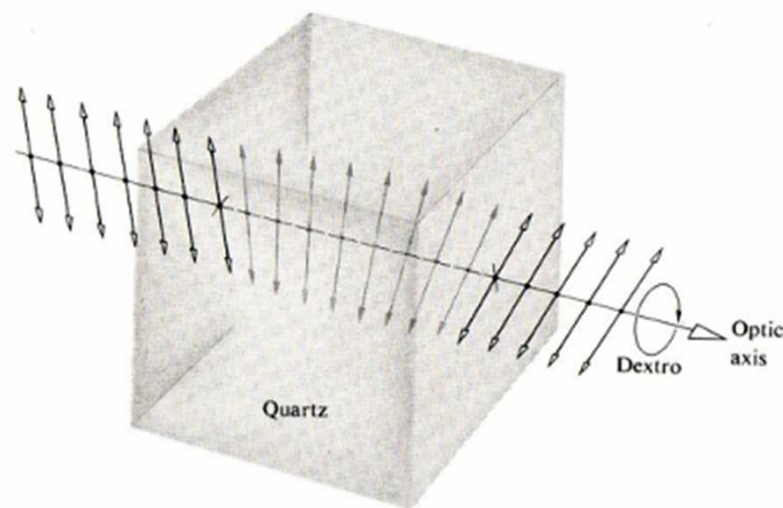


François Arago
(1786-1853)

Optical Activity

Discovered in 1811, Dominique F. J. Arago

Optically active material: any substance that cause a plane of polarization to appear to rotate

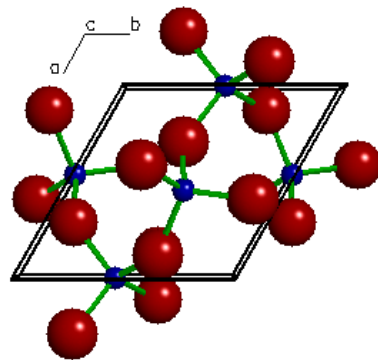
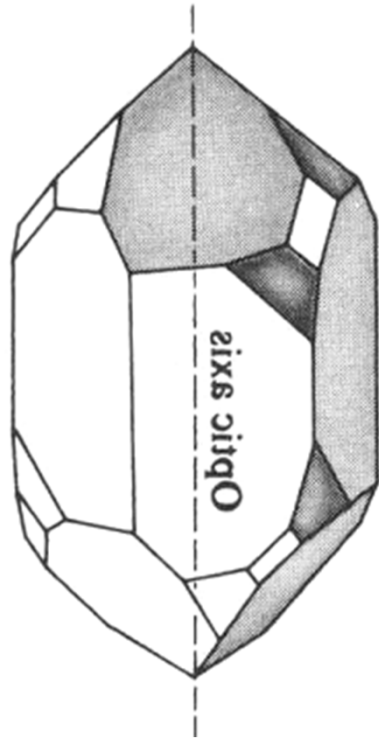
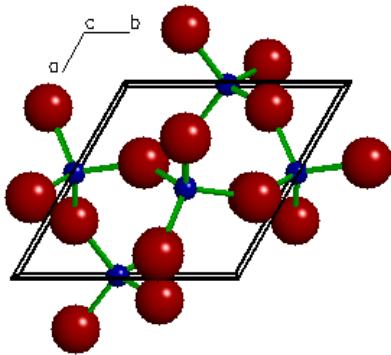
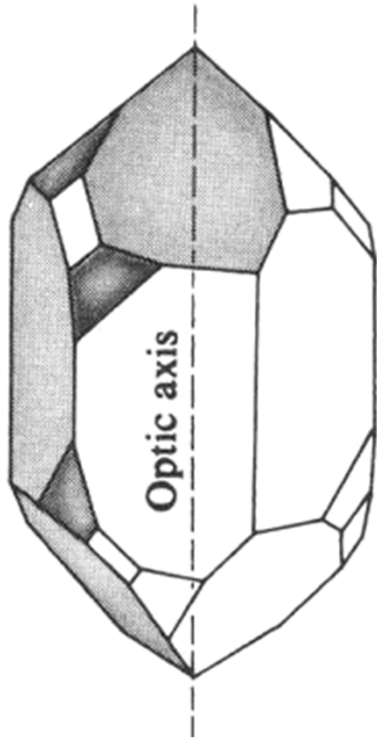


Dextrorotatory (d-rotatory) - polarization rotates right (CW)

Levorotatory (l-rotatory) - polarization rotates left (CCW)

Latin: dextro - right
levo - left

Optical Activity



Many materials have molecules or crystal structures that are non mirror-symmetric.

These usually interact with left- and right-circular polarized light in different ways.

Circular birefringence:
different indices of refraction for L- or R-polarized light.

Optical Activity

- Fresnel described linearly polarized light as the superposition of L- and R-polarization:

$$\vec{E}_R = \frac{E_0}{2} [\hat{i} \cos(k_R z - \omega t) + \hat{j} \sin(k_R z - \omega t)]$$

$$\vec{E}_L = \frac{E_0}{2} [\hat{i} \cos(k_L z - \omega t) - \hat{j} \sin(k_L z - \omega t)]$$


- Add them together:

$$\vec{E} = E_0 \cos\left(\frac{k_R + k_L}{2} z - \omega t\right) \left[\hat{i} \cos\left(\frac{k_R - k_L}{2} z\right) + \hat{j} \sin\left(\frac{k_R - k_L}{2} z\right) \right]$$

- Linear polarization: direction at a fixed point in z is constant.

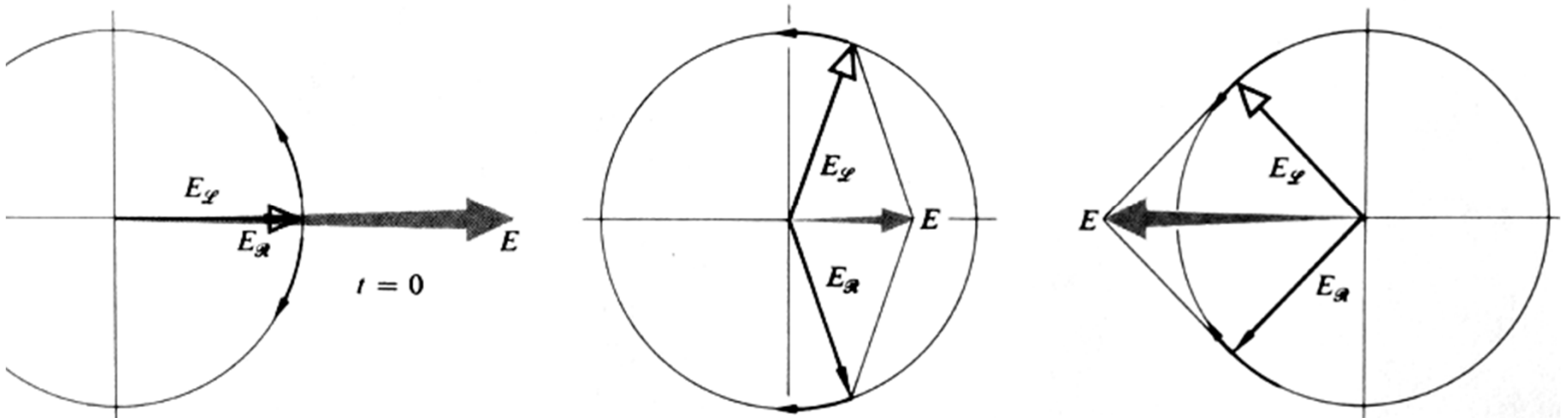
Optical Activity

$$\vec{E}_R = \frac{E_0}{2} [\hat{i} \cos(k_R z - \omega t) + \hat{j} \sin(k_R z - \omega t)]$$

$$\vec{E}_L = \frac{E_0}{2} [\hat{i} \cos(k_L z - \omega t) - \hat{j} \sin(k_L z - \omega t)]$$

$$\vec{E} = E_0 \cos\left(\frac{k_R + k_L}{2} z - \omega t\right) \left[\hat{i} \cos\left(\frac{k_R - k_L}{2} z\right) + \hat{j} \sin\left(\frac{k_R - k_L}{2} z\right) \right]$$

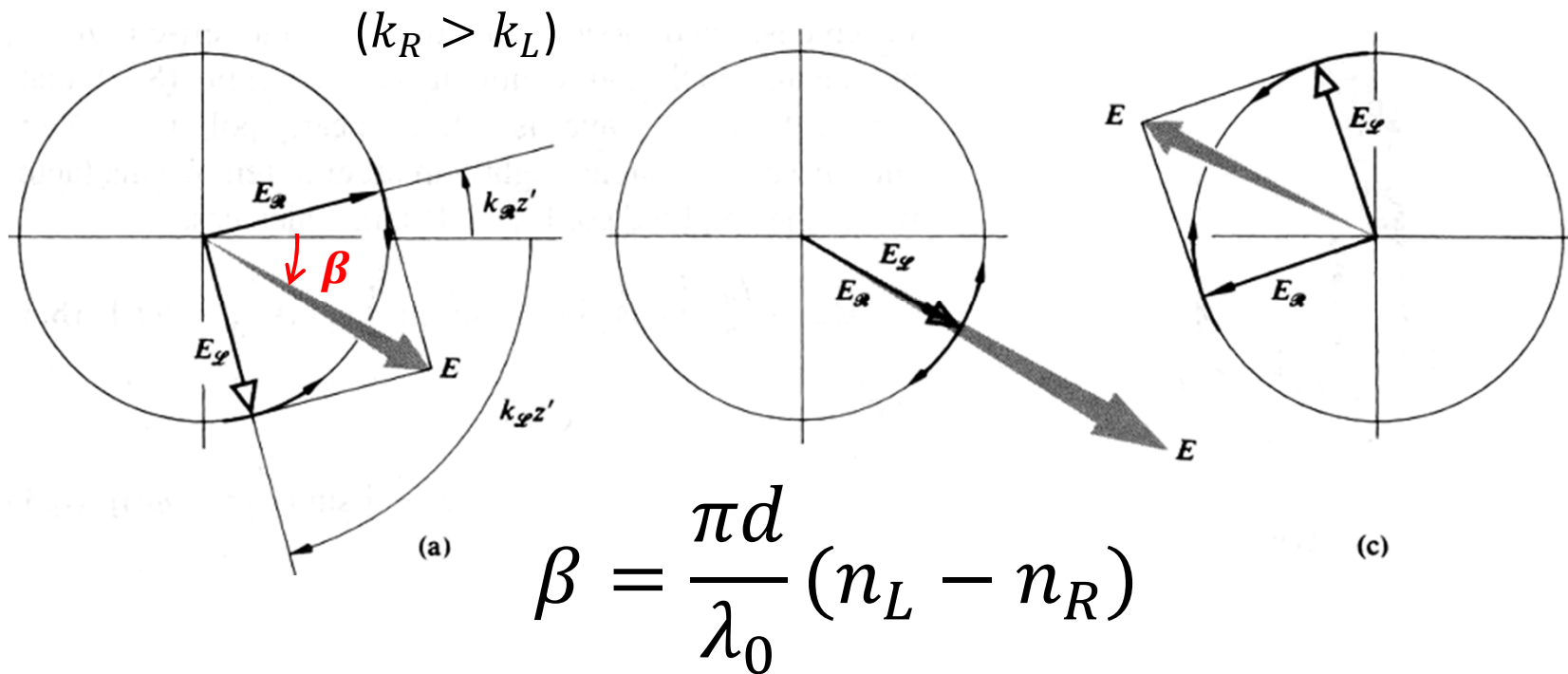
Time dependence of the $\vec{E}$ vector at $z = 0$: $\vec{E} = E_0 \hat{i} \cos(\omega t)$



Optical Activity

$$\vec{E} = E_0 \cos\left(\frac{k_R + k_L}{2}z - \omega t\right) \left[\hat{i} \cos\left(\frac{k_R - k_L}{2}z\right) + \hat{j} \sin\left(\frac{k_R - k_L}{2}z\right) \right]$$

- After propagating a distance $z = d$ through an optically active material, $\vec{E} = E_0[\hat{i} \cos \beta + \hat{j} \sin \beta] \cos(\omega t)$



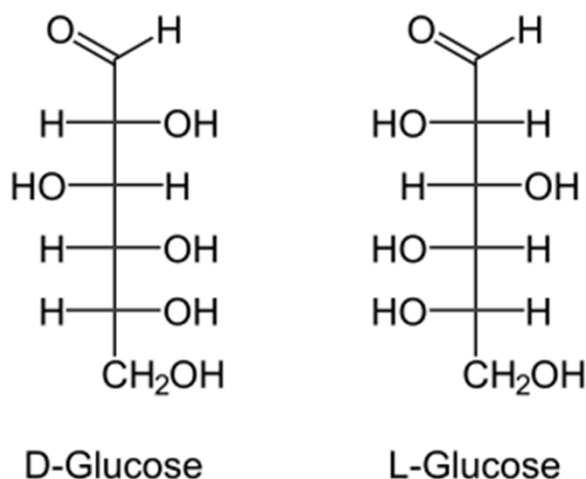
Optical Activity

- Application in organic chemistry:
- Experimental setup:
 - Sample concentration: 1 gram per millileter
 - Path length: 10 cm
 - Wavelength is 589 nm (Sodium “D-line”)
 - The “*specific rotation*” is the angle α that linearly polarized light is rotated
- Examples:

Substance	Specific rotation
Sucrose	+66.47°
Lactose	+52.3°
Levulose (D-Fructose)	−92.4°
Dextrose (D-Glucose)	+52.5°
L-Glucose	−52°

Stereoisomers

- Same chemical formula but mirror-symmetric structure:



- Measurements of optical rotation provide information about sample purity and composition.
- Useful for analytic chemistry, regardless of the underlying physics.