Purdue University Department of Physics

Physics 42200

Waves & Oscillations

Lecture 29 – Geometric Optics

Spring 2014 Semester

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Aberrations

- We have continued to make approximations:
  - Paraxial rays
  - Spherical lenses
  - Index of refraction independent of wavelength
- How do these approximations affect images?
  - There are several ways…
  - Sometimes one particular effect dominates the performance of an optical system
  - Useful to understand their source in order to introduce the most appropriate corrective optics
- How can these problems be reduced or corrected?

Aberrations

- Limitations of paraxial rays:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$$

- Paraxial approximation:

$$\sin\theta \approx \theta$$

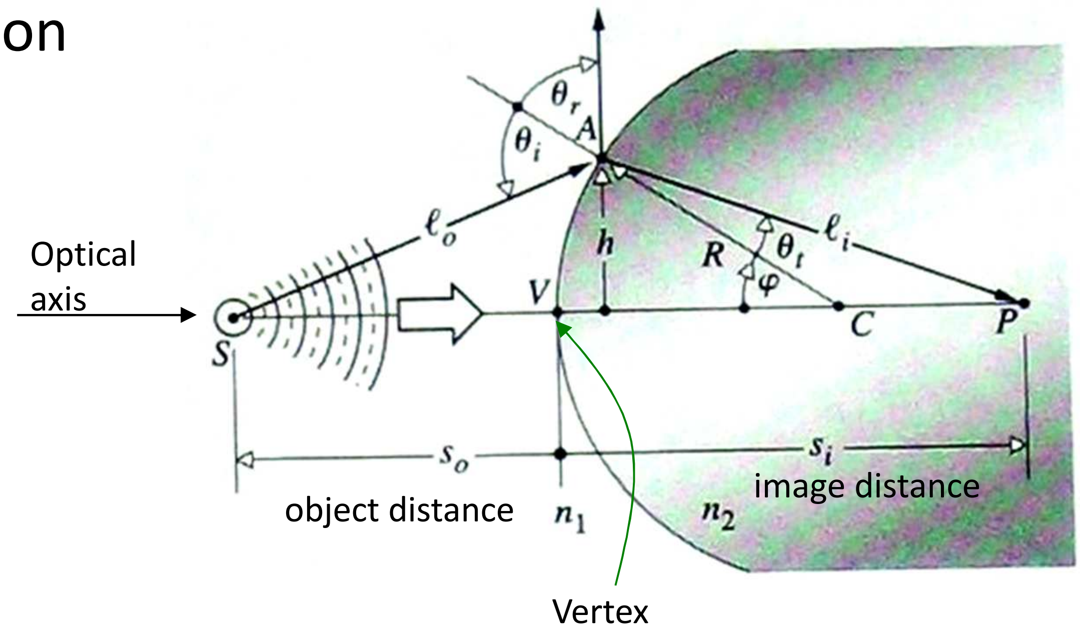
- Third-order approximation:

$$\sin\theta \approx \theta - \frac{\theta^3}{3!}$$

- The optical equations are now non-linear
  - The lens equations are only approximations
  - Perfect images might not even be possible!
  - Deviations from perfect images are called aberrations
  - Several different types are classified and their origins identified.

Aberrations

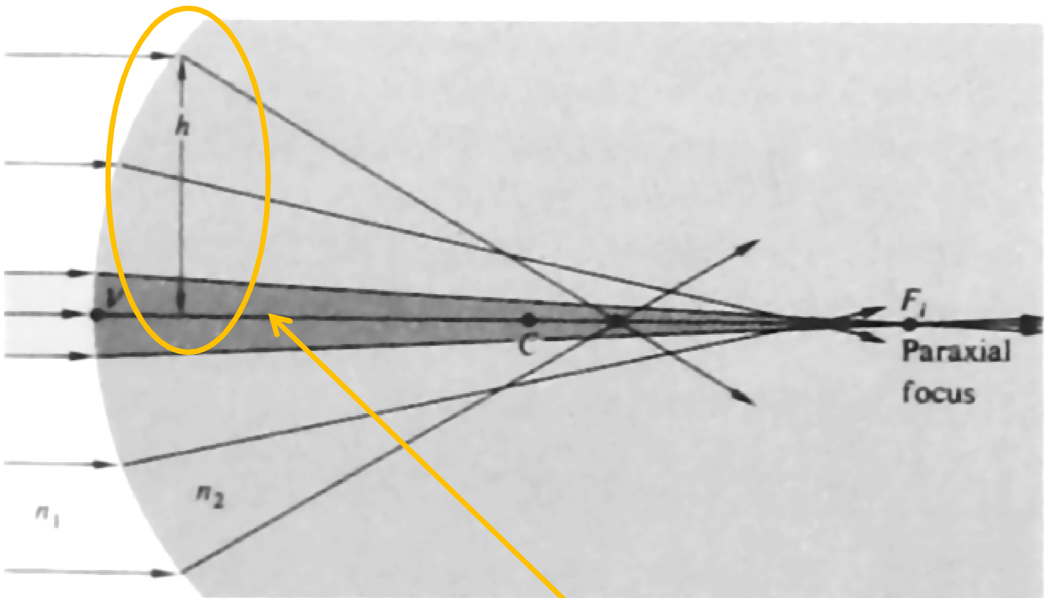
- Departure from the linear theory at third-order were classified into five types of ***primary aberrations*** by Phillip Ludwig Seidel (1821-1896):
  - Spherical aberration
  - Coma
  - Astigmatism
  - Field curvature
  - Distortion



Spherical Aberration

- We first derived the shape of a surface that changes spherical waves into plane waves
  - It was either a parabola, ellipse or hyperbola
- But this only worked for light sources that were on the optical axis
- To form an image, we need to bring rays into focus from points that lie off the optical axis
- A sphere looks the same from all directions so there are no "off-axis" points
- It is still not perfect – there are aberrations

Spherical Aberration



Paraxial approximation:

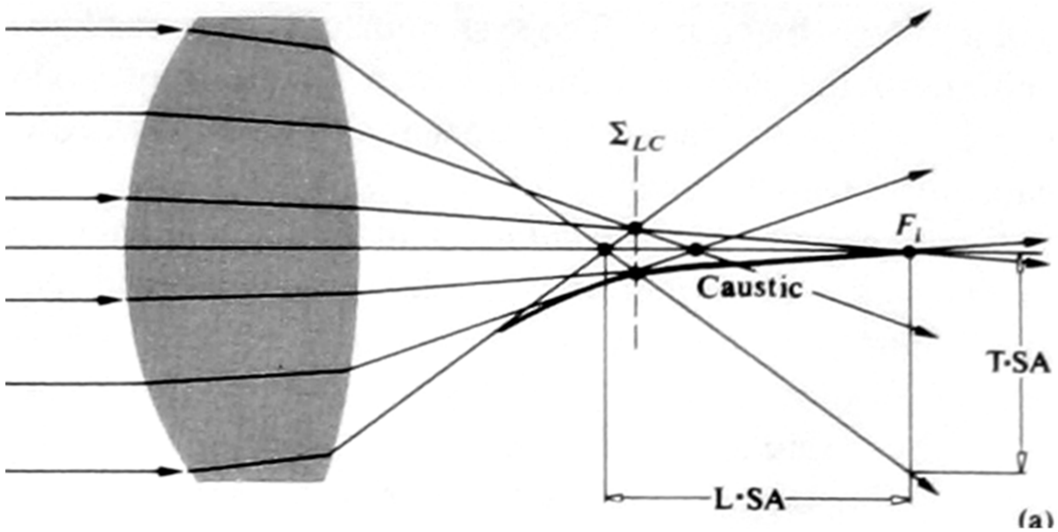
$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

Third order approximation:

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} + h^2\left[\frac{n_1}{2s_o}\left(\frac{1}{s_o} + \frac{1}{R}\right)^2 + \frac{n_2}{2s_i}\left(\frac{1}{R} - \frac{1}{s_i}\right)^2\right]$$

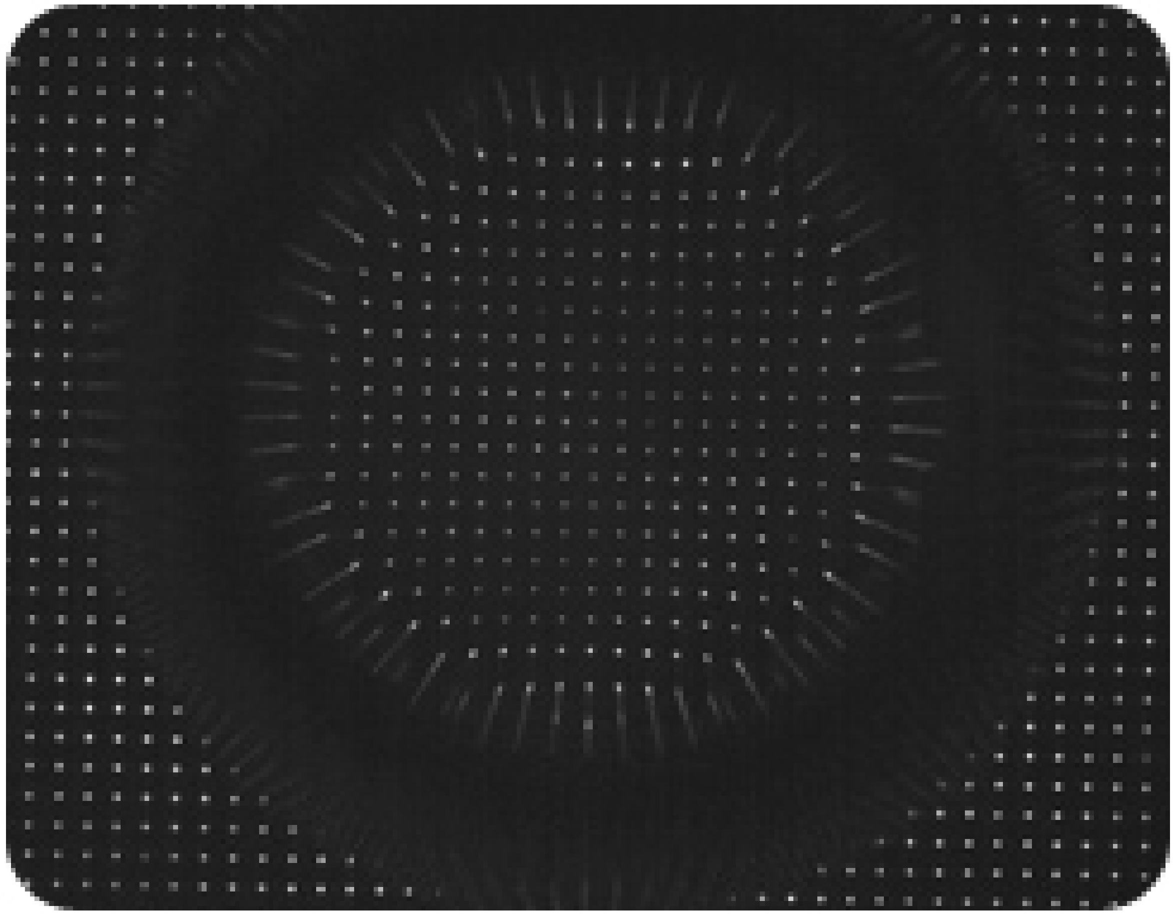
Deviation from first-order theory

Spherical Aberrations



- Longitudinal Spherical Aberration: $\mathrm{L} \cdot SA$
  - Image of an on-axis object is longitudinally stretched
  - Positive $\mathrm{L} \cdot SA$ means that marginal rays intersect the optical axis in front of F_i (paraxial focal point).
- Transverse Spherical Aberration: $\mathrm{T} \cdot SA$
  - Image of an on-axis object is blurred in the image plane
- Circle of least confusion: Σ_{LC}
  - Smallest image blur

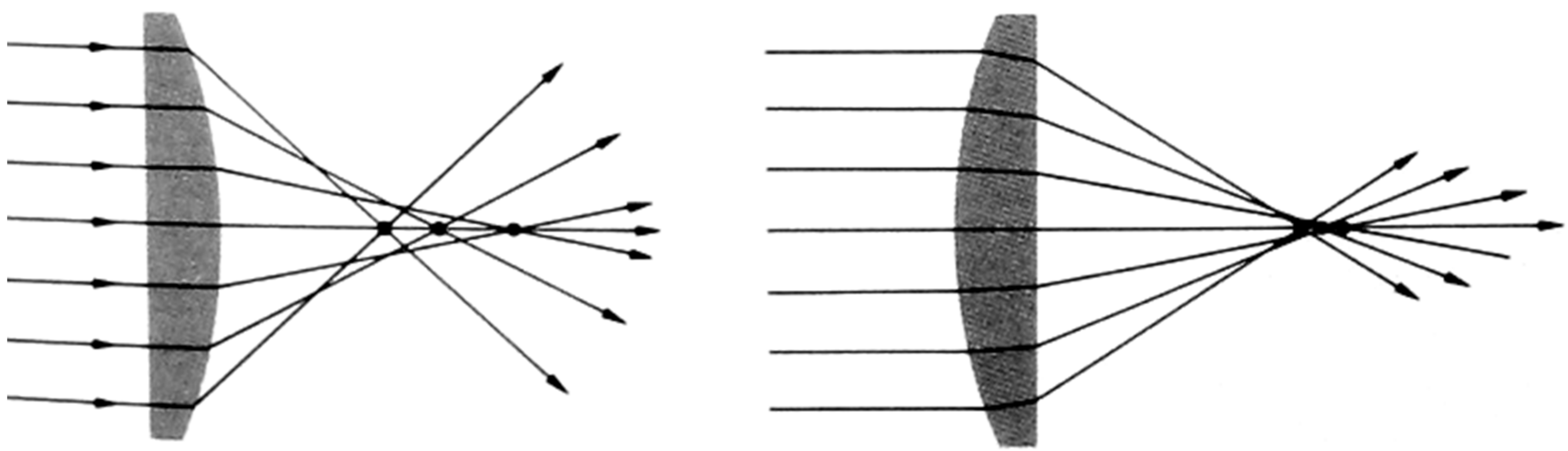
Spherical Aberration



Example from http://www.spot-optics.com/index.htm

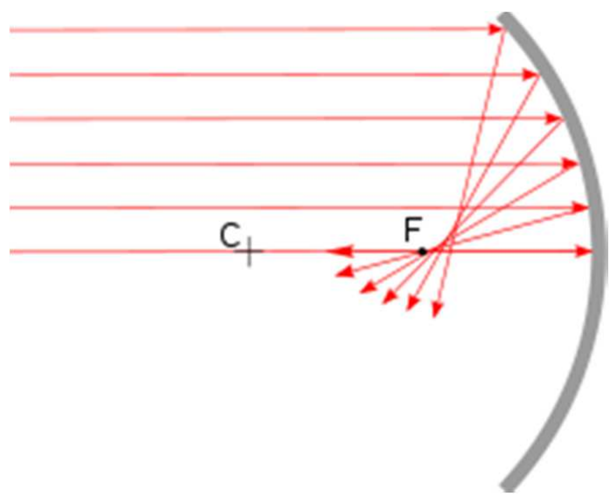
Spherical Aberration

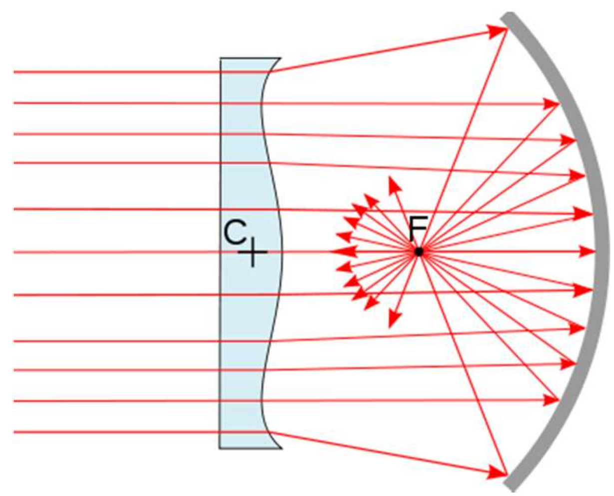
- In third-order optics, the orientation of the lenses does matter
- Spherical aberration depends on the lens arrangement:



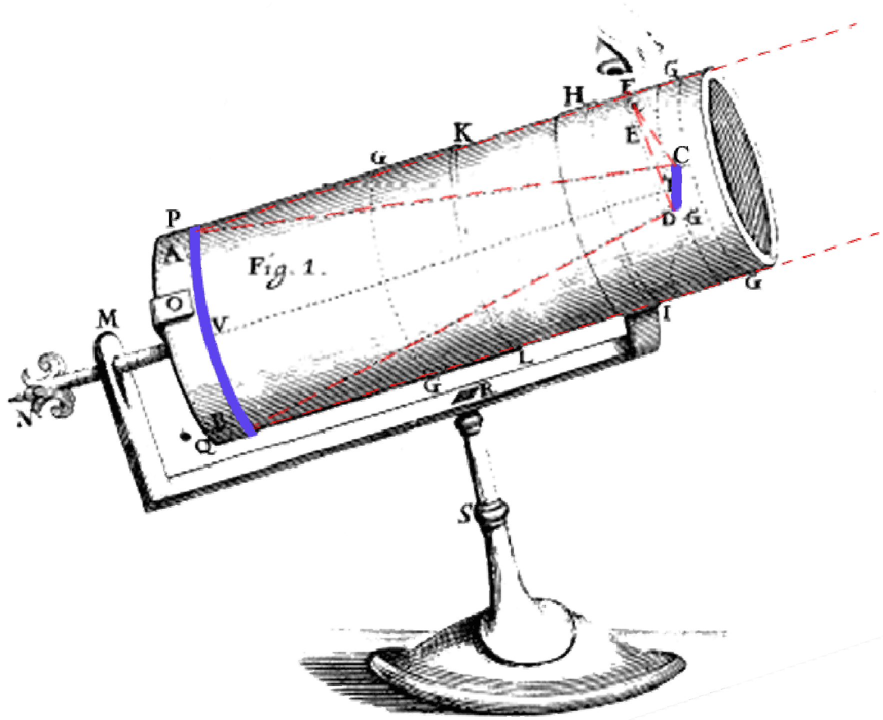
Spherical Aberration of Mirrors

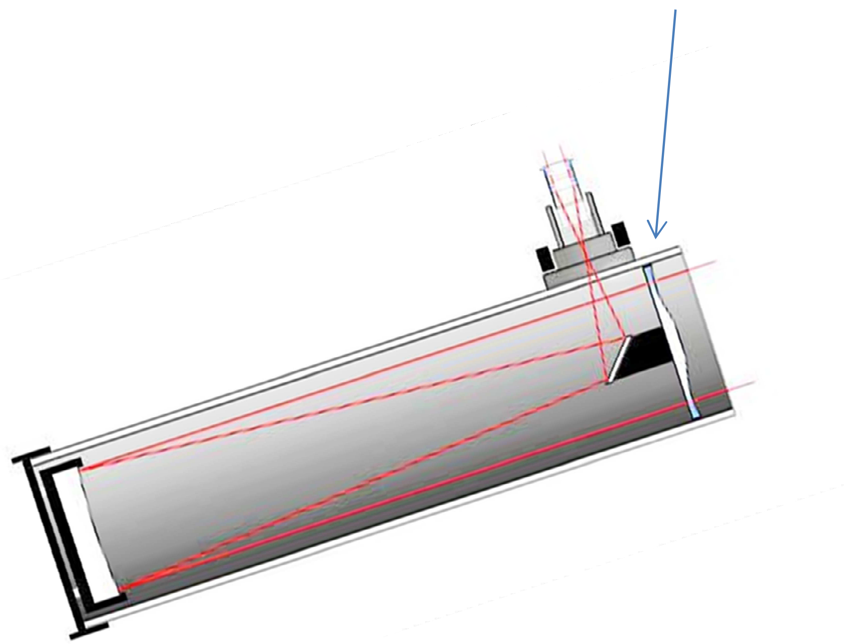
- Spherical mirrors also suffer from spherical aberration
  - Parabolic mirrors do not suffer from spherical aberration, but they distort images from points that do not lie on the optical axis
- ***Schmidt corrector plate*** removes spherical aberration without introducing other optical defects.



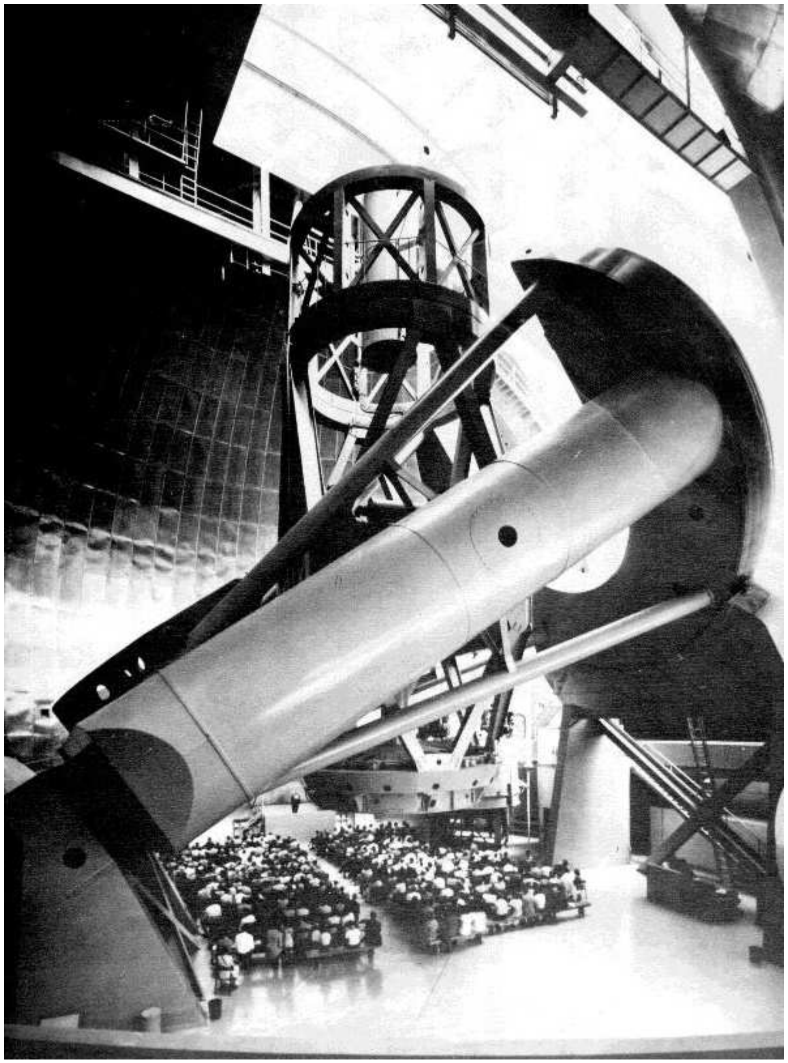


Newtonian Telescope

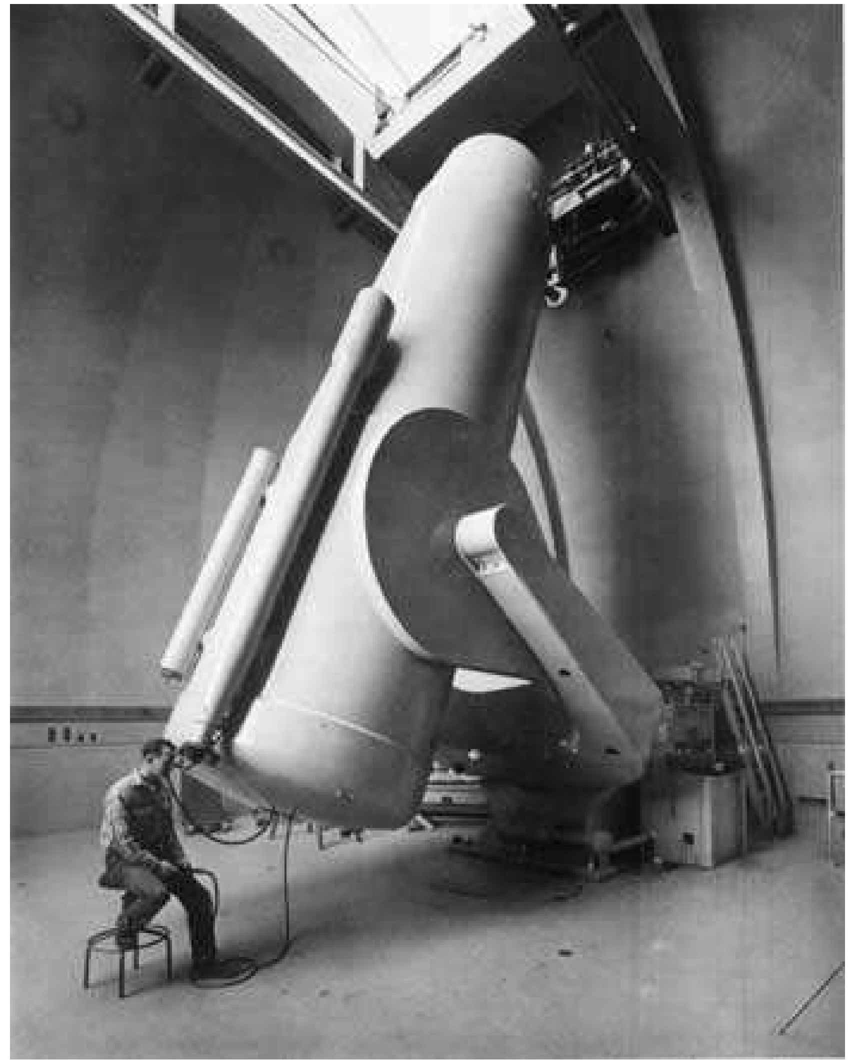




Schmidt 48-inch Telescope



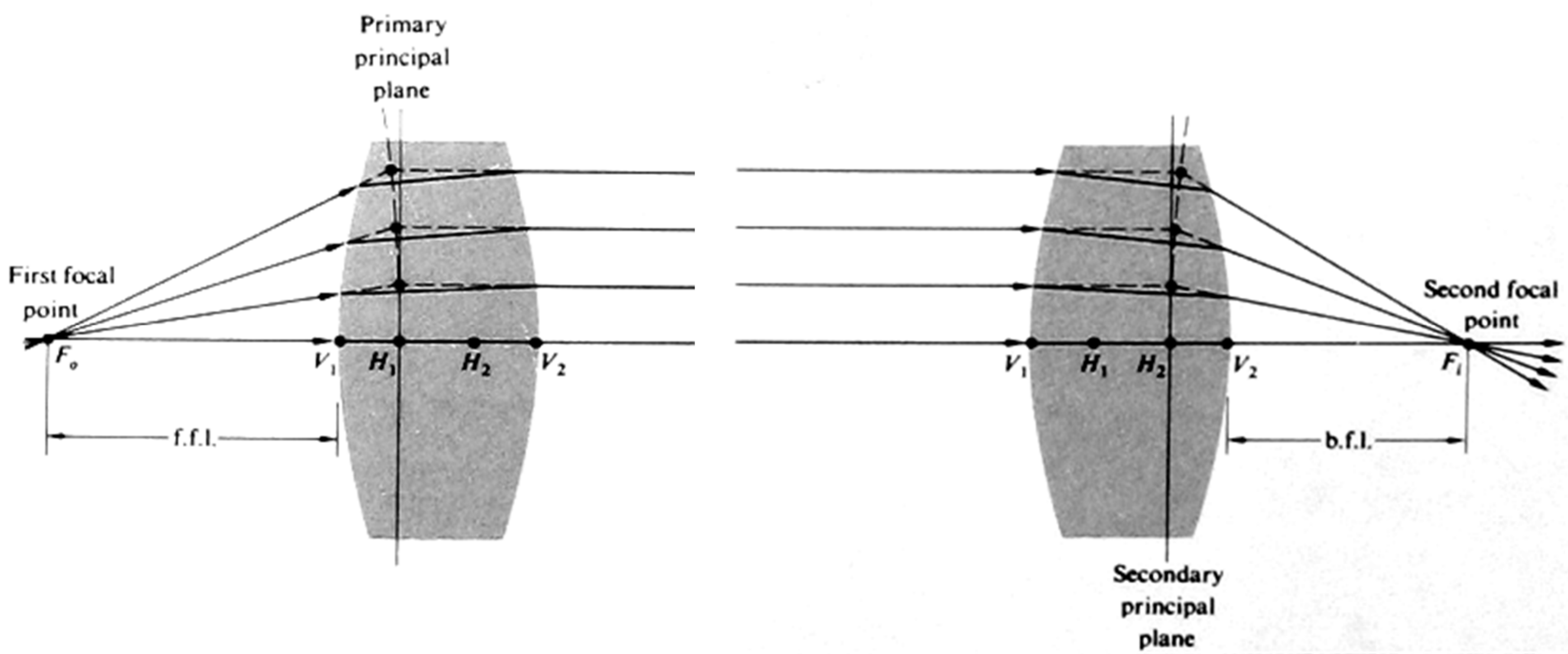
200 inch Hale telescope



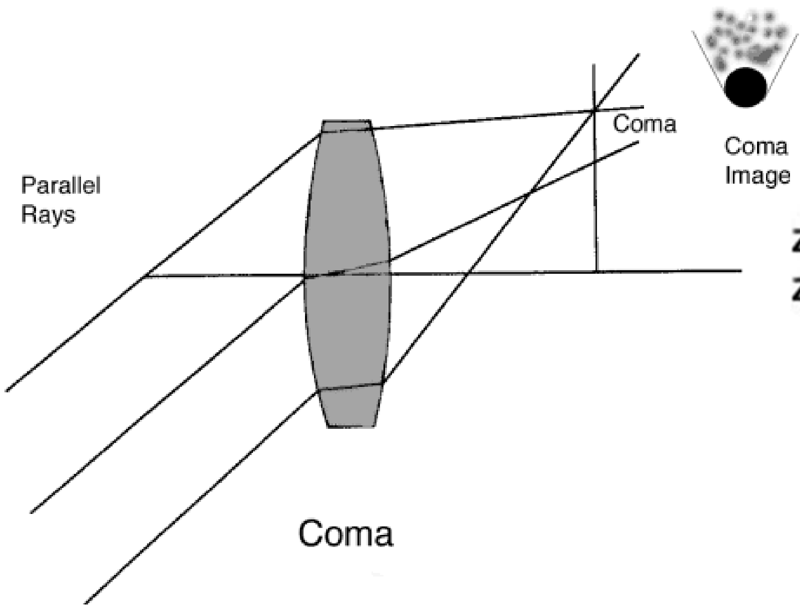
48-inch Schmidt telescope

Coma (comatic aberration)

- Principle planes are not flat – they are actually curved surfaces.
- Focal length is different for off-axis rays



Coma



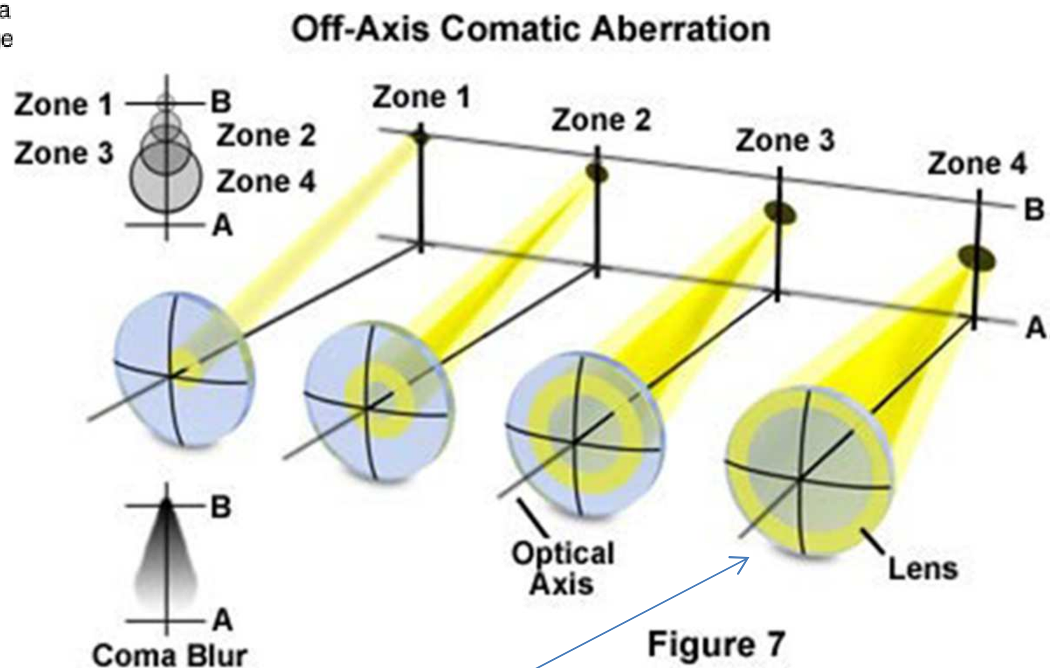
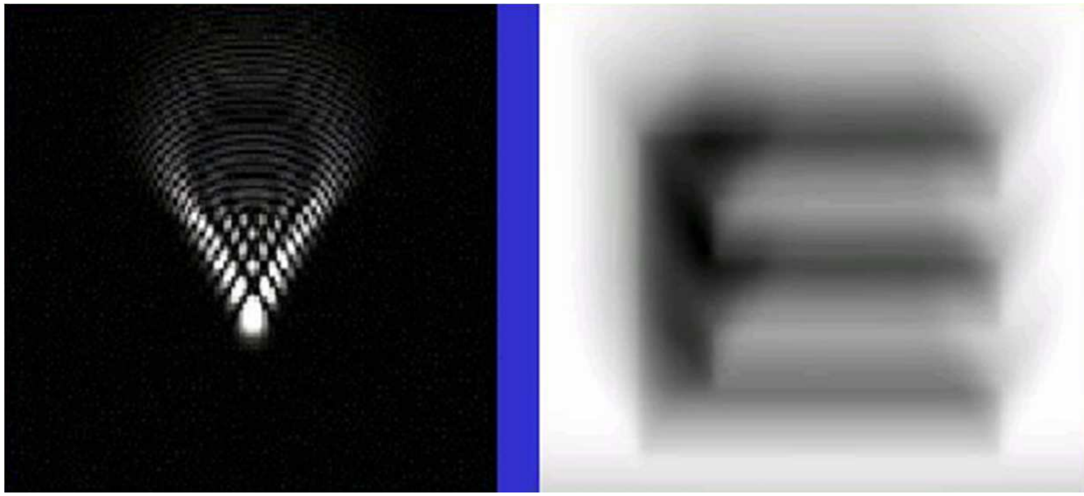


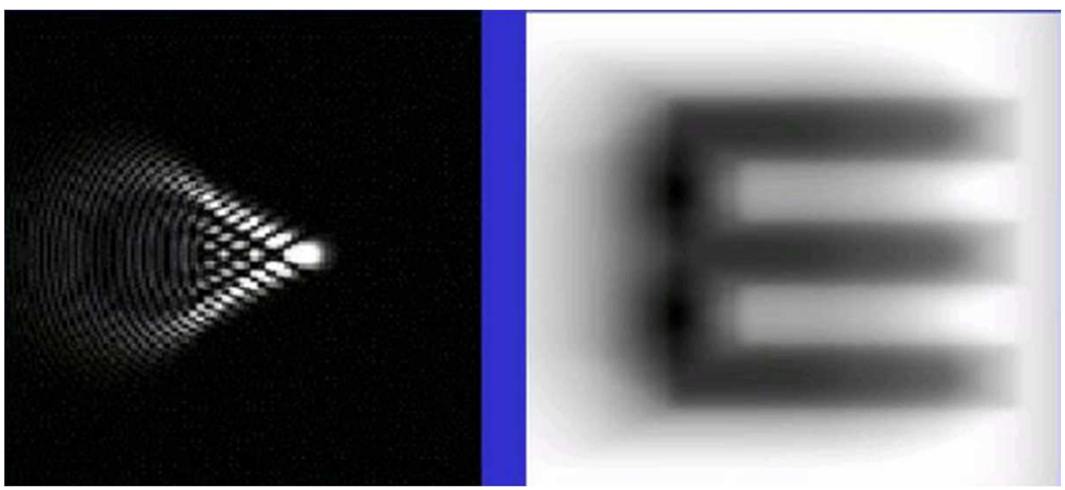
Figure 7

- Negative coma: meridional rays focus closer to the principal axis

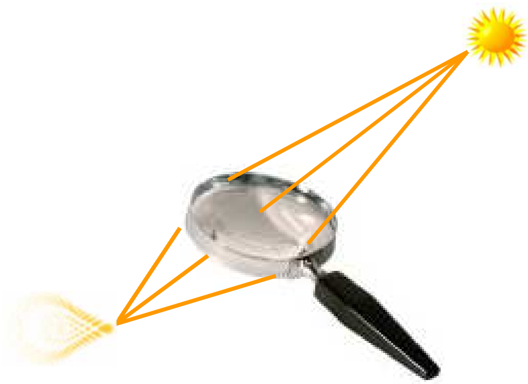
Coma



Vertical coma



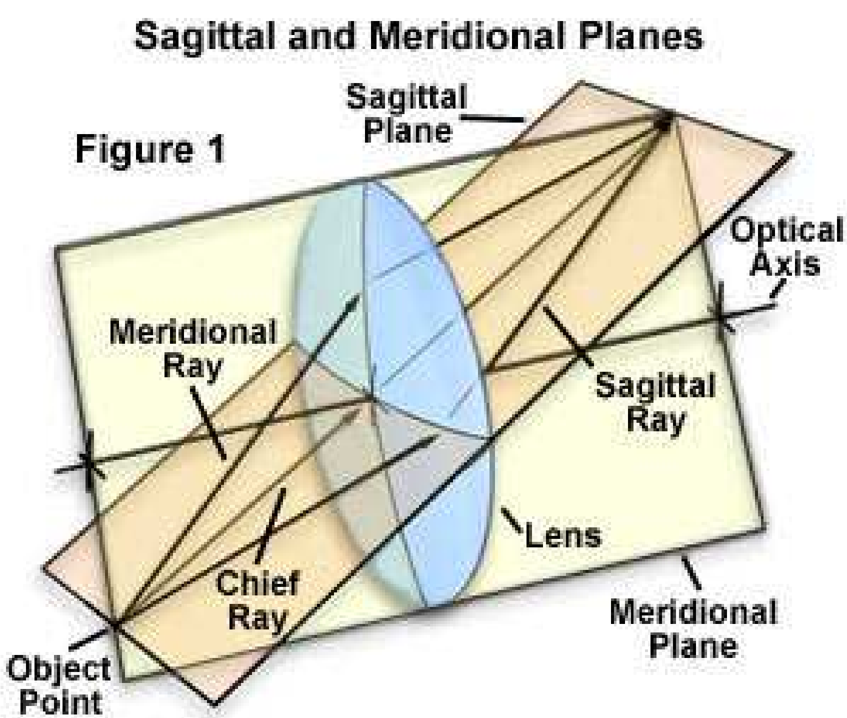
Horizontal coma

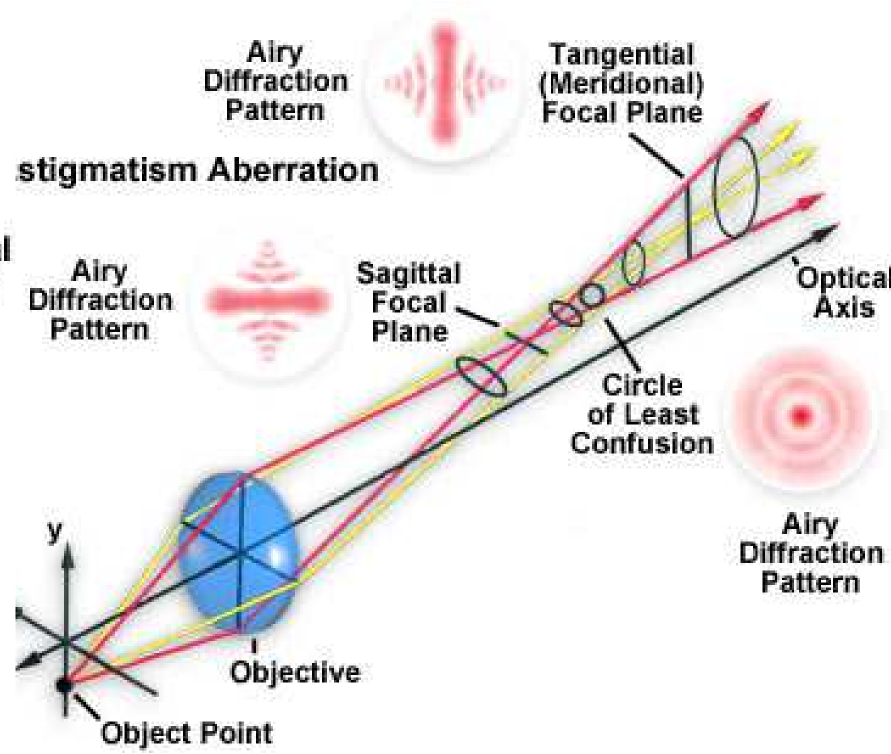


Coma can be reduced by introducing a stop positioned at an appropriate point along the optical axis, so as to remove the appropriate off-axis rays.

Astigmatism

- Parallel rays from an off-axis object arrive in the plane of the lens in one direction, but not in a perpendicular direction:

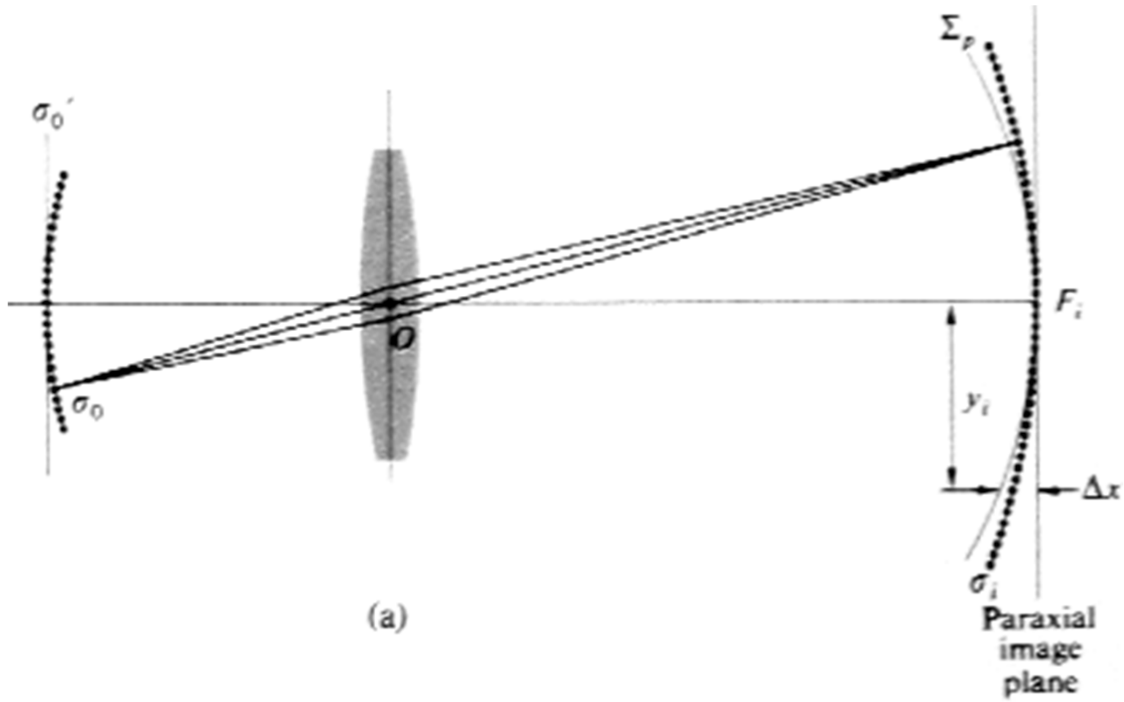




Astigmatism

- This formal definition is different from the one used in ophthalmology which is caused by non-spherical curvature of the surface and lens of the eye.

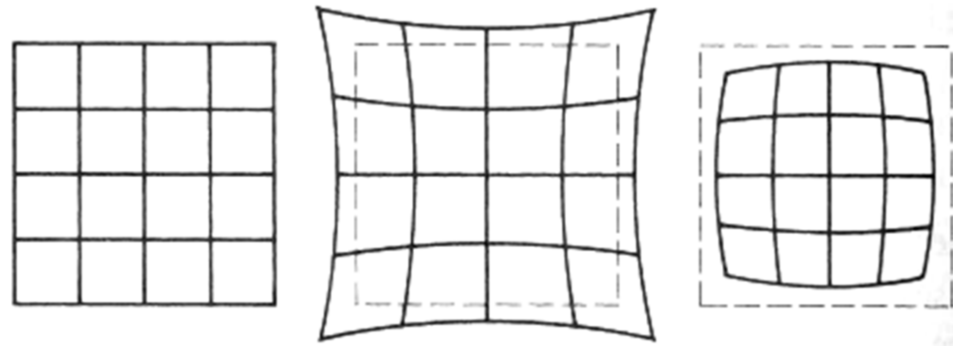
Field Curvature



- The focal plane is actually a curved surface
- A negative lens has a field plane that curves away from the image plane
- A combination of positive and negative lenses can cancel the effect

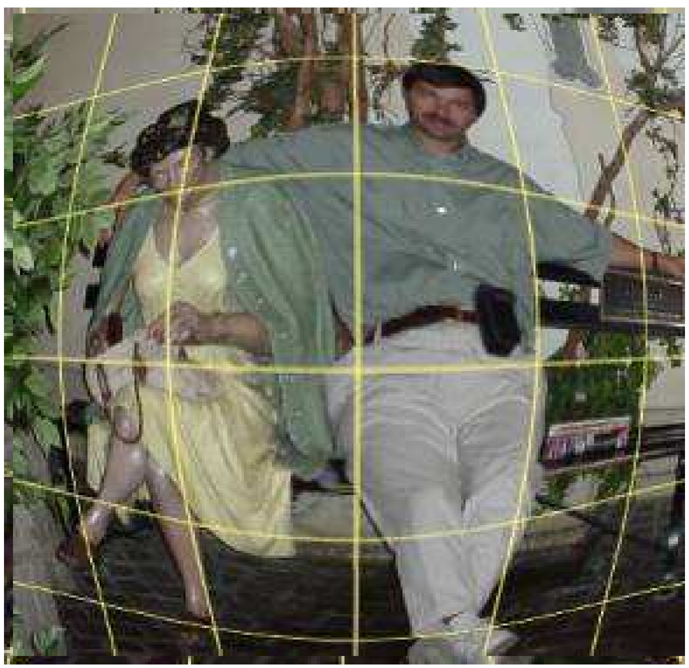
Field Curvature

- Transverse magnification, m_T, can be a function of the off-axis distance:



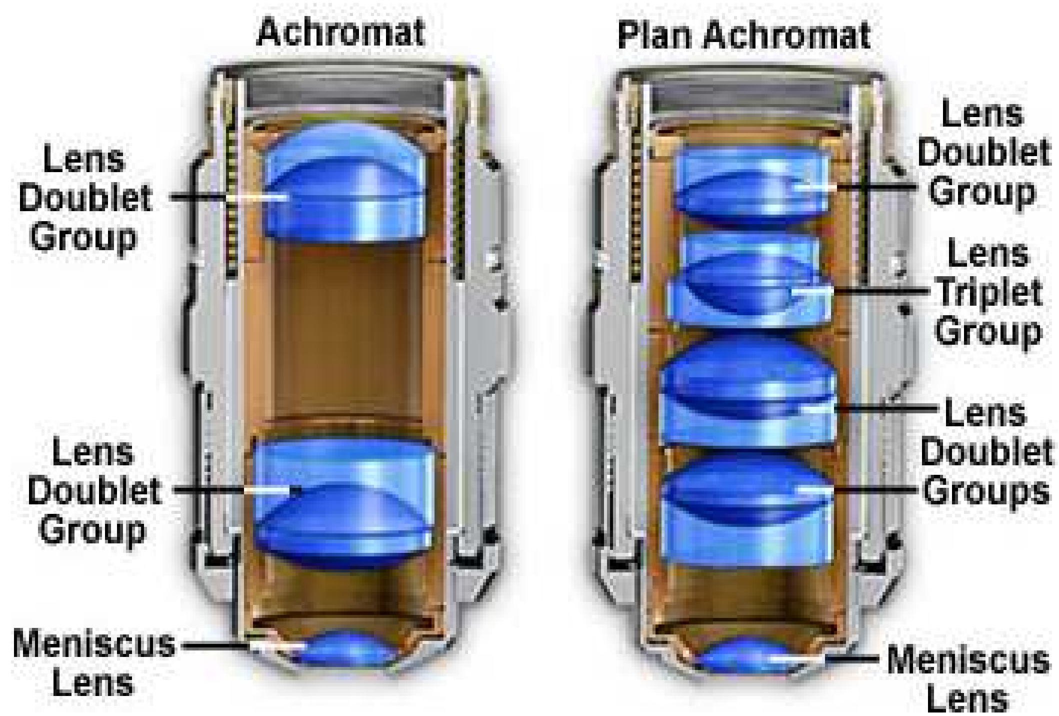
Positive (pincushion) distortion

Negative (barrel) distortion



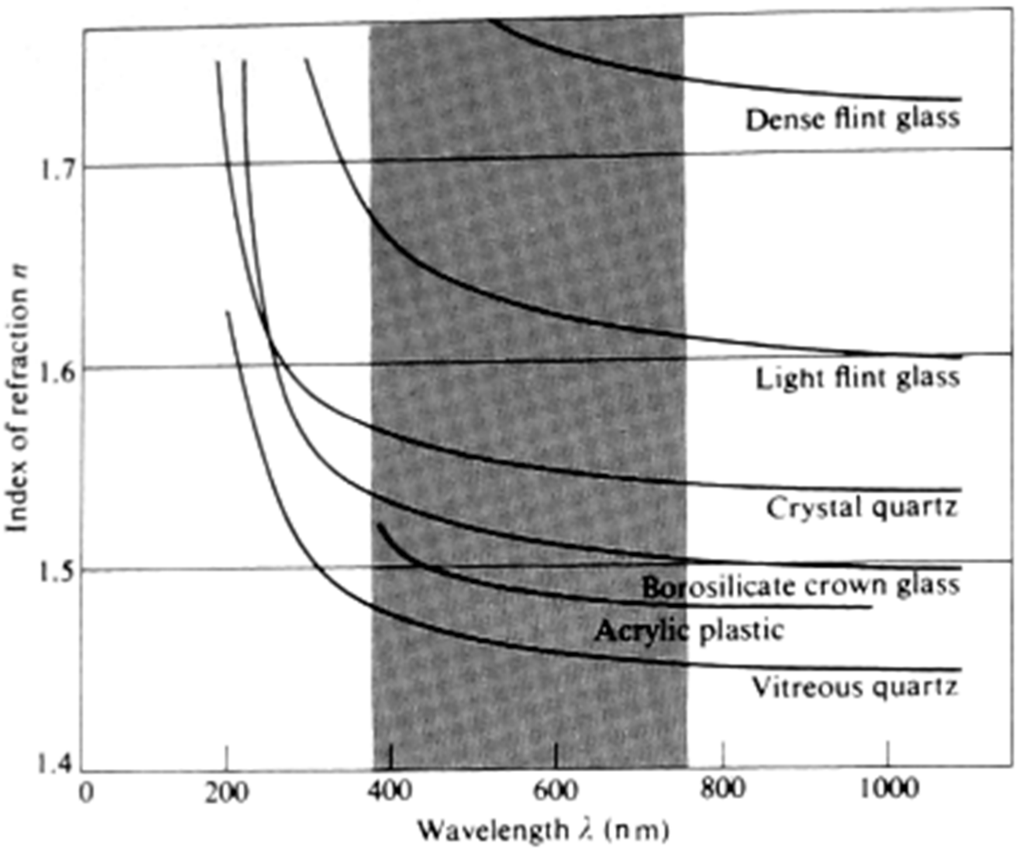
Correcting Monochromatic Aberrations

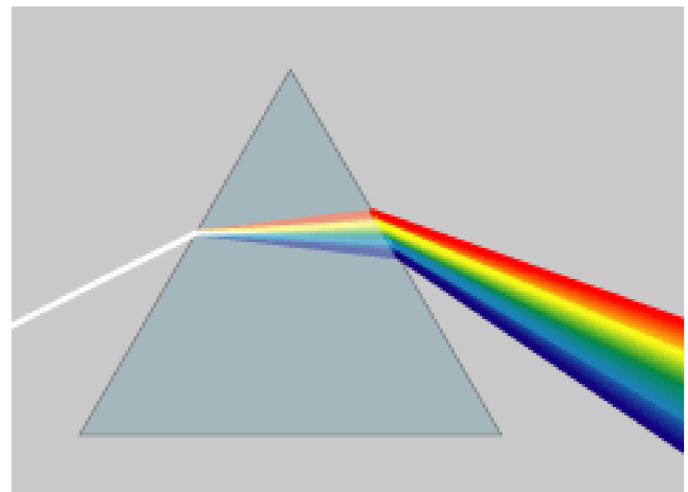
- Combinations of lenses with mutually cancelling aberration effects
- Apertures
- Aspherical correction elements.



Chromatic Aberrations

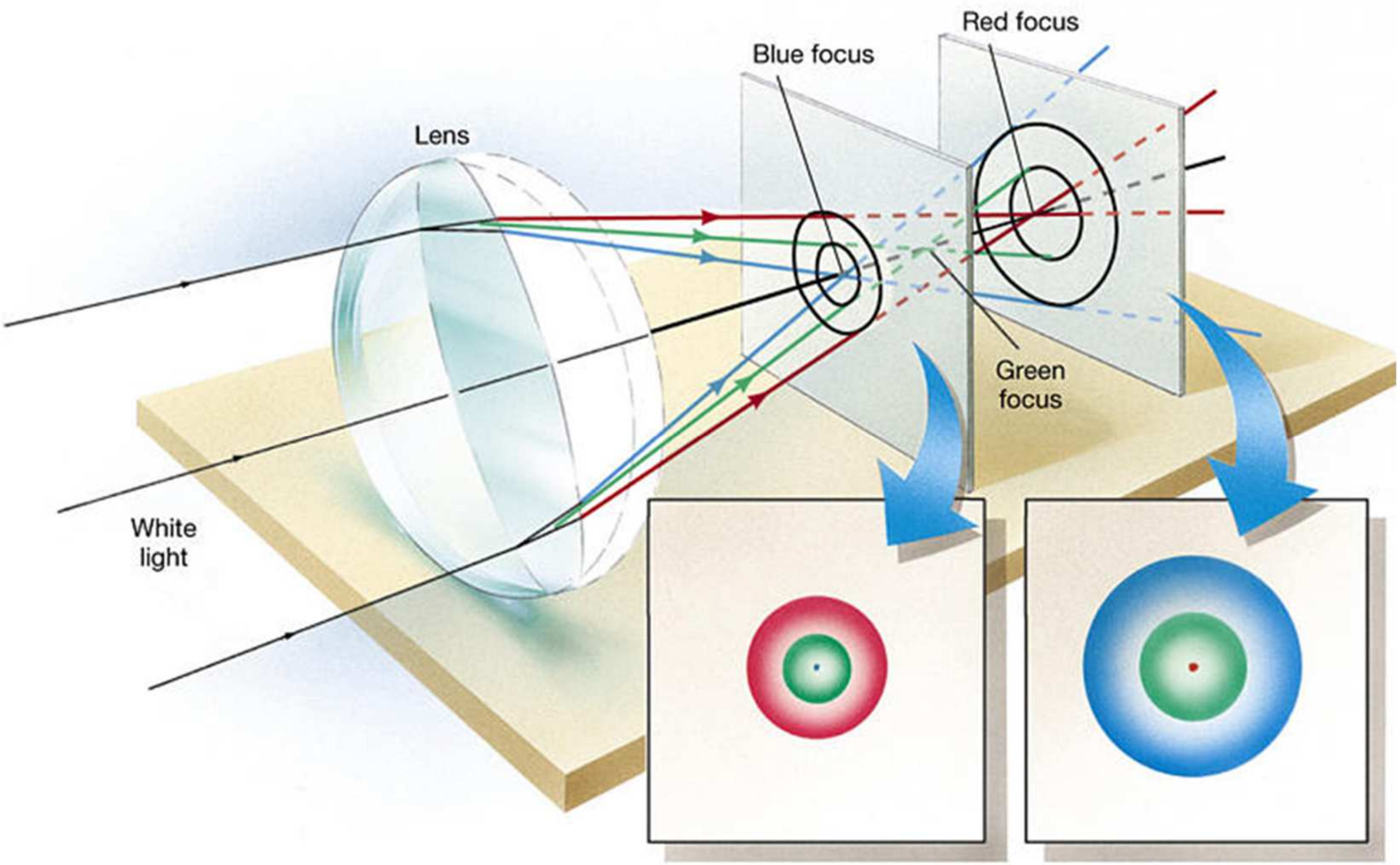
- Index of refraction depends on wavelength





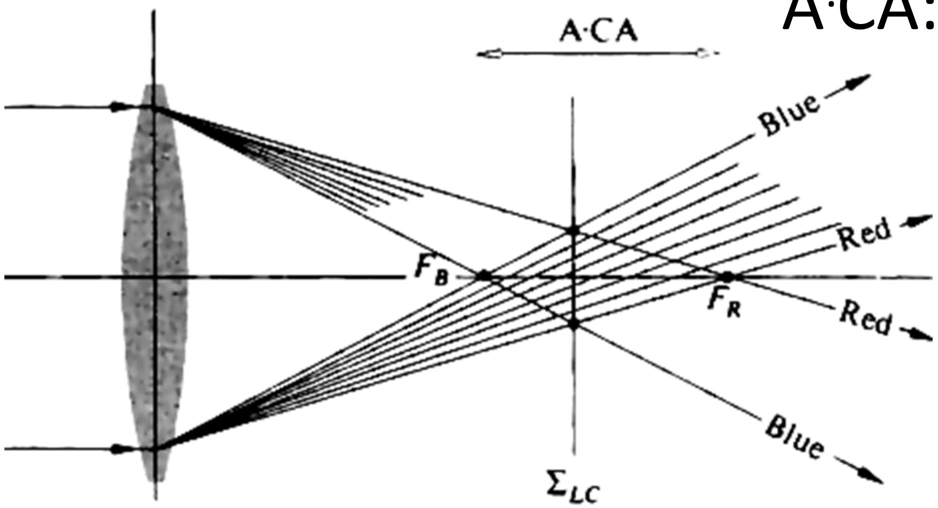
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Chromatic Aberrations

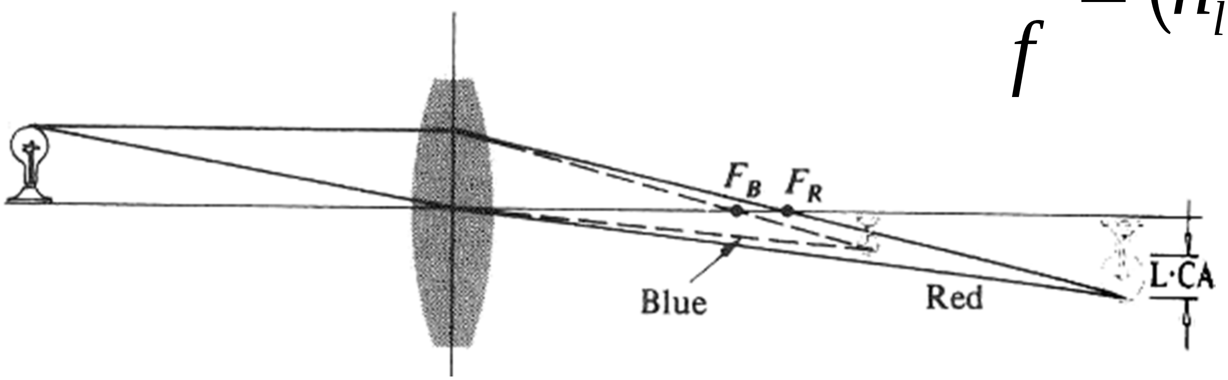


Chromatic Aberrations

A·CA: axial chromatic aberration



$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

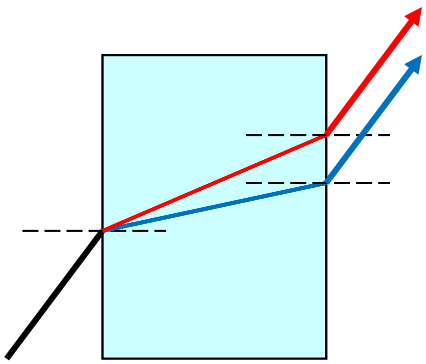


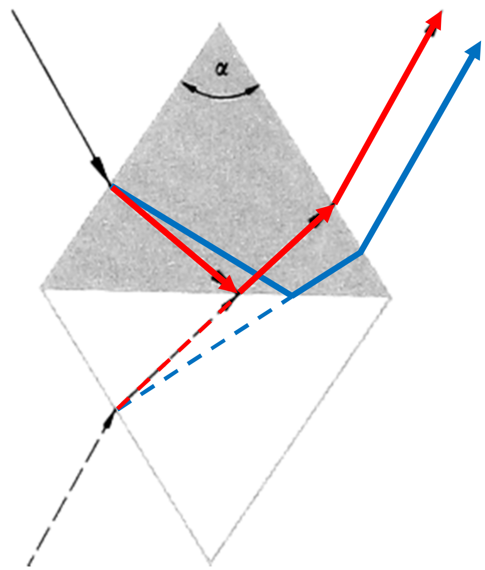
L·CA: lateral chromatic aberration

Chromatic Aberration

Correcting for Chromatic Aberration

- It is possible to have refraction without chromatic aberration even when n is a function of λ:
  - Rays emerge displaced but *parallel*
  - If the thickness is small, then there is no distortion of an image
  - Possible even for non-parallel surfaces:
  - Aberration at one interface is compensated by an opposite aberration at the other surface.





Chromatic Aberration

- Focal length:

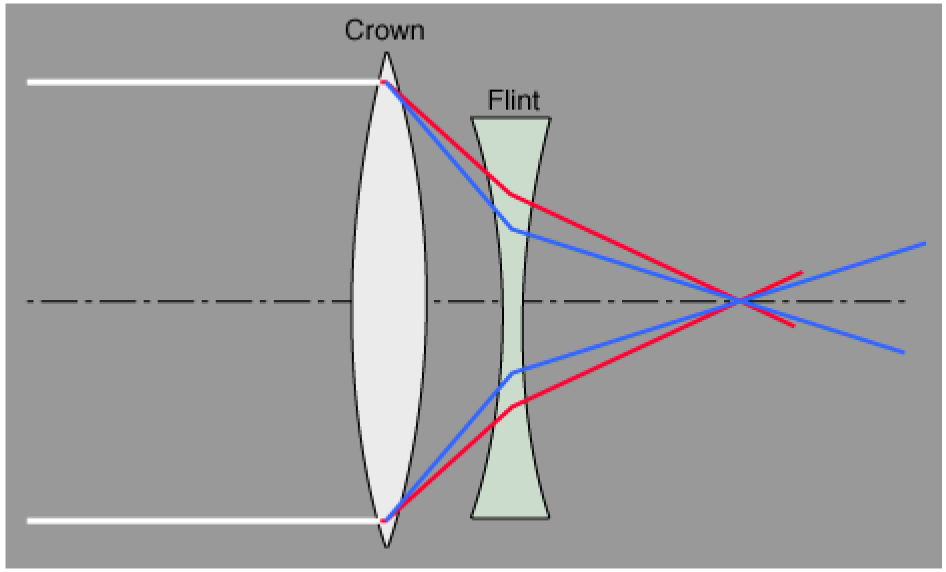
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

- Thin lens equation:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

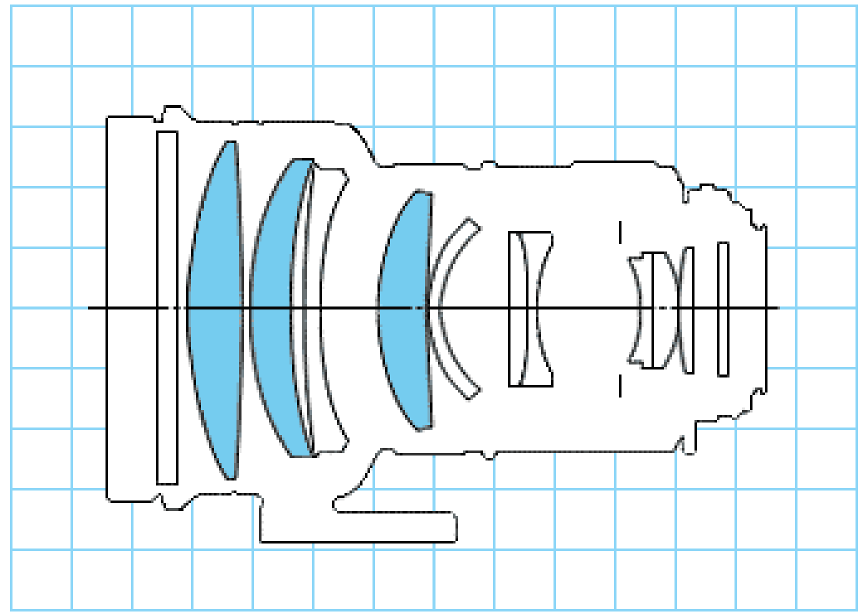
- Cancel chromatic aberration using a combination of concave and convex lenses with different index of refraction

Chromatic Aberration



- This design does not eliminate chromatic aberration completely – only two wavelengths are compensated.

Commercial Lens Assemblies

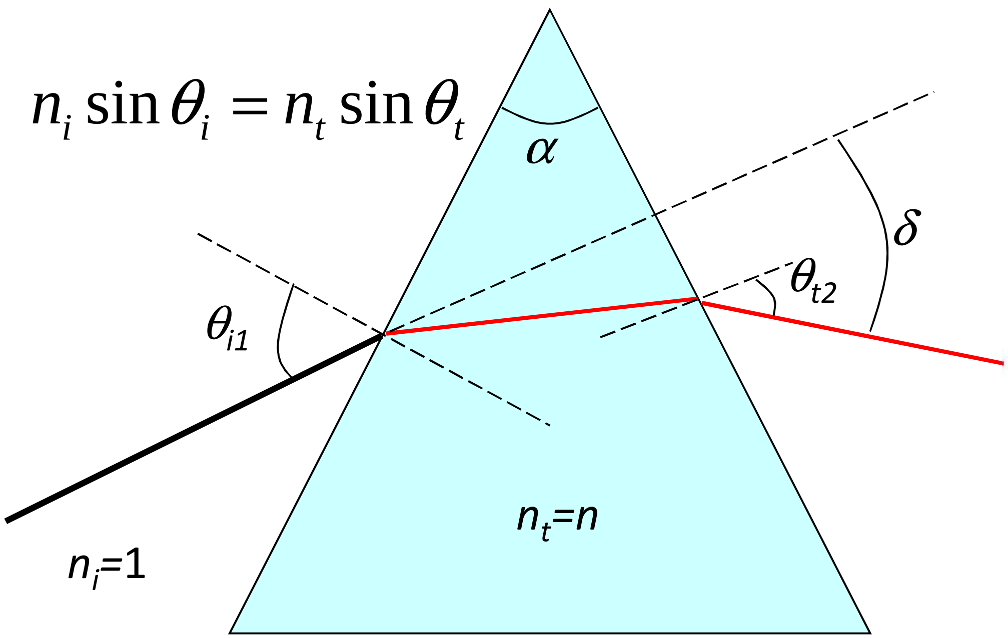


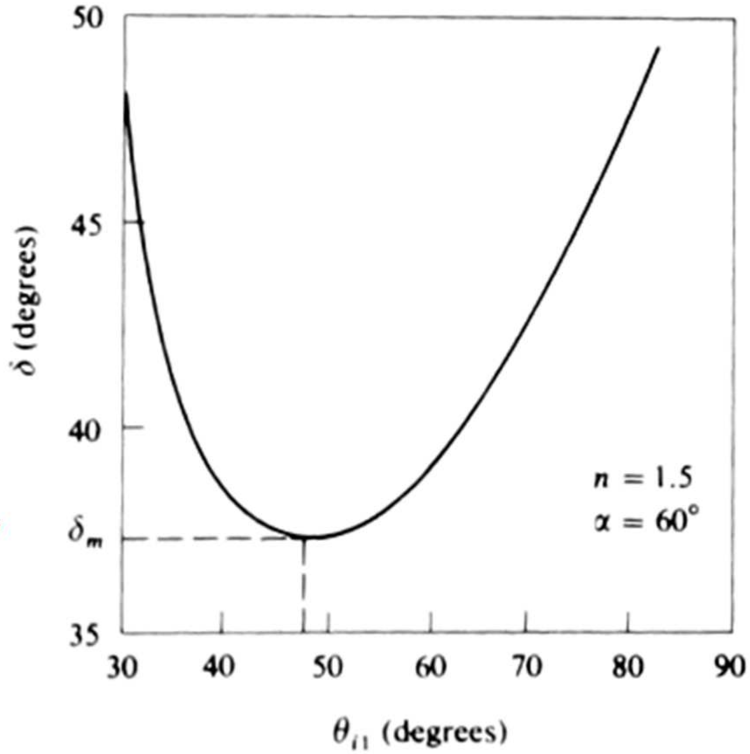
- Some lens components are made with ultra-low dispersion glass, eg. calcium fluoride

Prisms

- Dispersing prism:

$$n_i \sin\theta_i = n_t \sin\theta_t$$



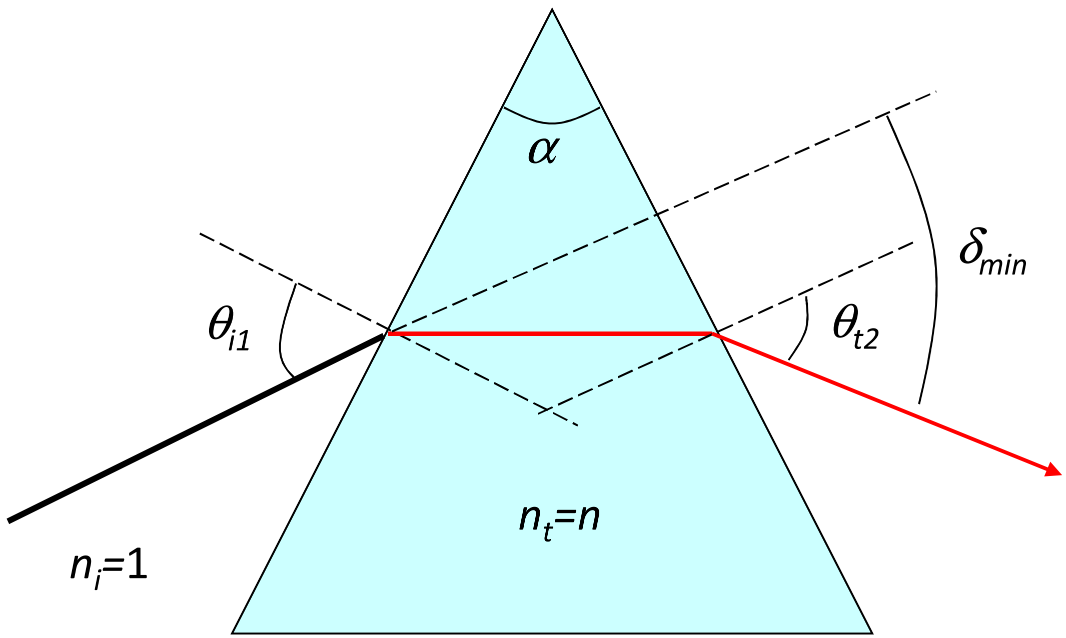


- Total deviation:

$$\delta = \theta_{i1} + \sin^{-1}\left[(\sin\alpha)\sqrt{n^2 - \sin^2\theta_{i1}} - \sin\theta_{i1}\cos\alpha\right] - \alpha$$

Prisms

- The minimum deflection, δ_{min}, occurs when $\theta_{i1} = \theta_{t2}$:

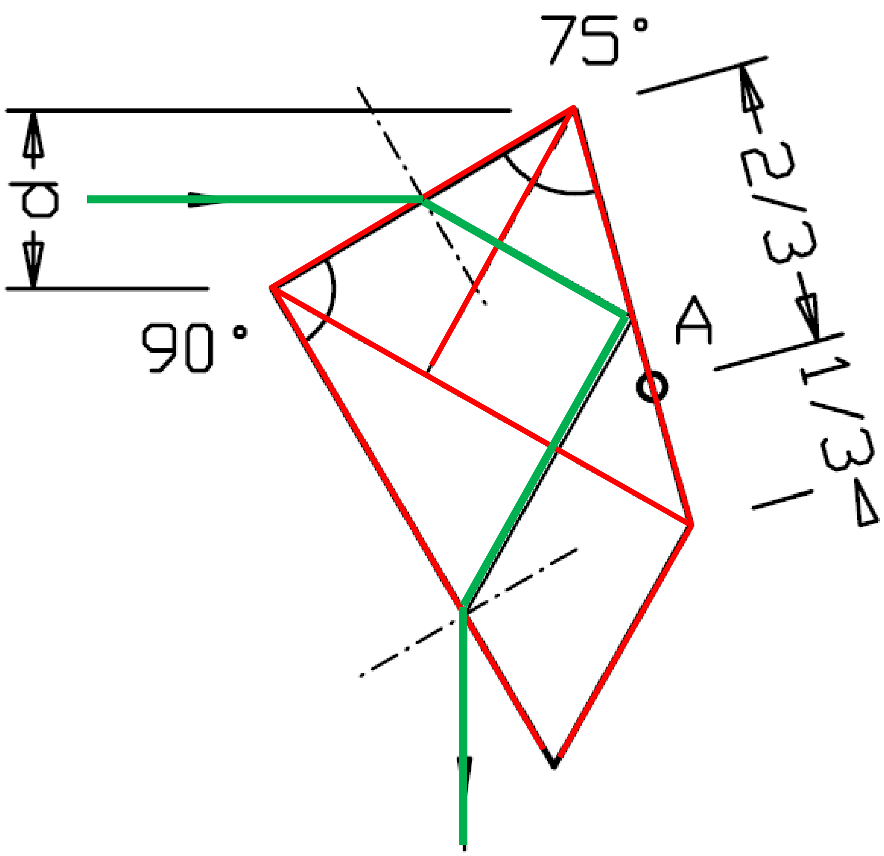


$$n = \frac{\sin[(\delta_{min} + \alpha)/2]}{\sin(\alpha/2)}$$

This can be used as an effective method to measure n

A disadvantage for analyzing colors is the variation of δ_{min} with θ_{i1} - the angle of incidence must be known precisely to

Pellin-Broca Prism



One color is refracted through exactly 90°.

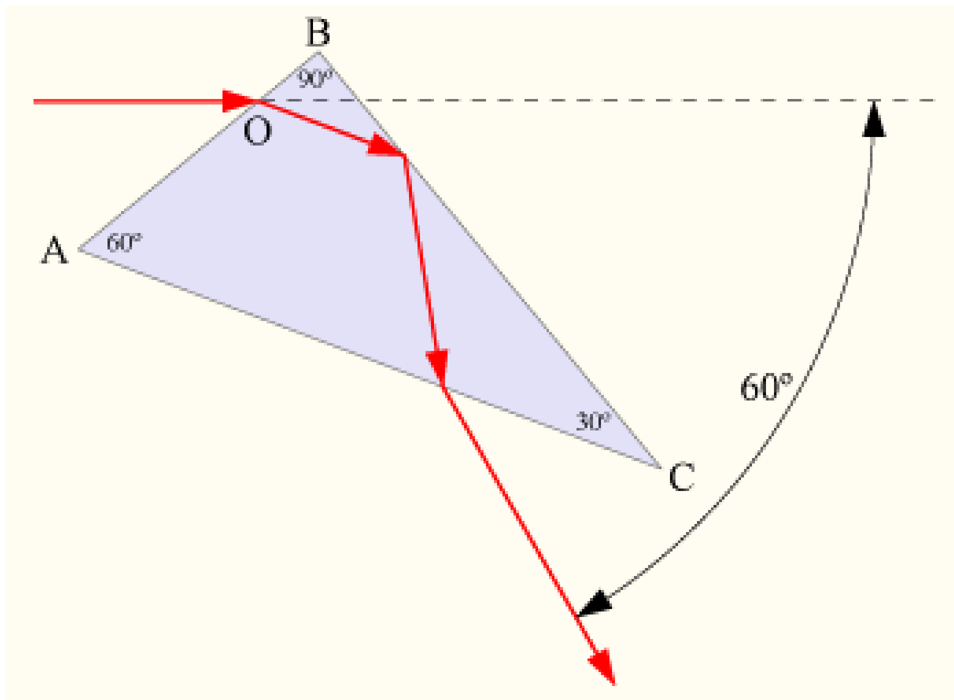
Rotating the prism about point A selects different colors.

Ideal for selecting a particular wavelength with minimal change to an optical system.

Abbe Prism

- A particular wavelength is refracted through 60°
- Rotating the prism about point O selects different colors.

Ernst Abbe
1840-1905

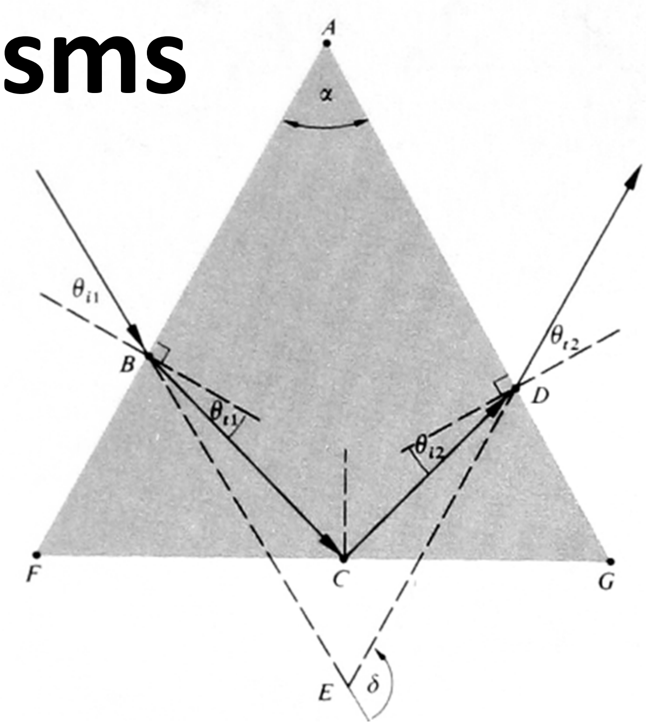


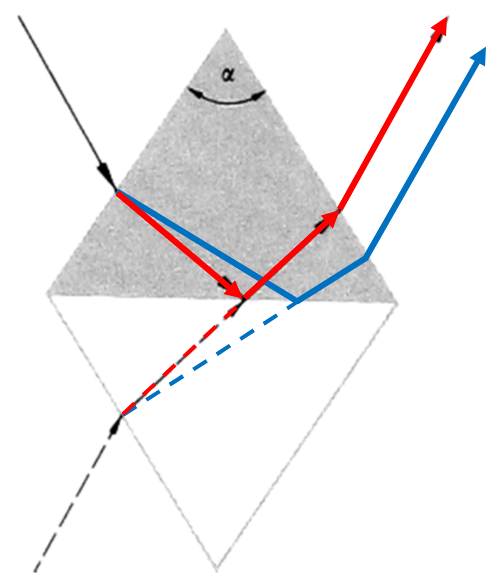
Reflective Prisms

- Total internal reflection on one surface
- Equal and opposite refraction at the other surfaces
- Deflection angle:

$$\delta = 2\theta_{i1} + \alpha$$

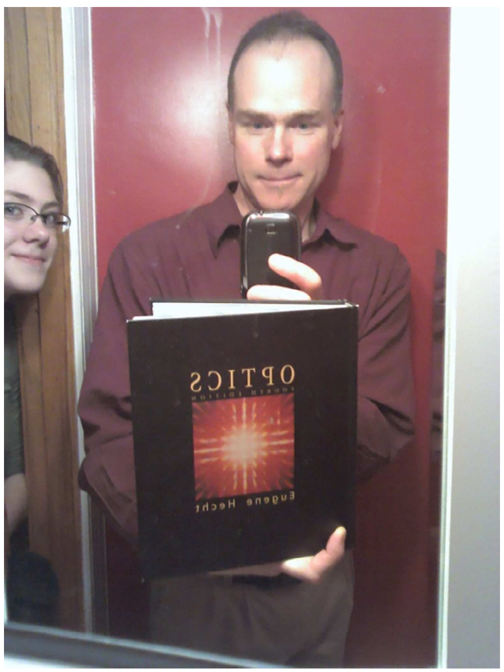
- Independent of wavelength (non-dispersive or achromatic prism)

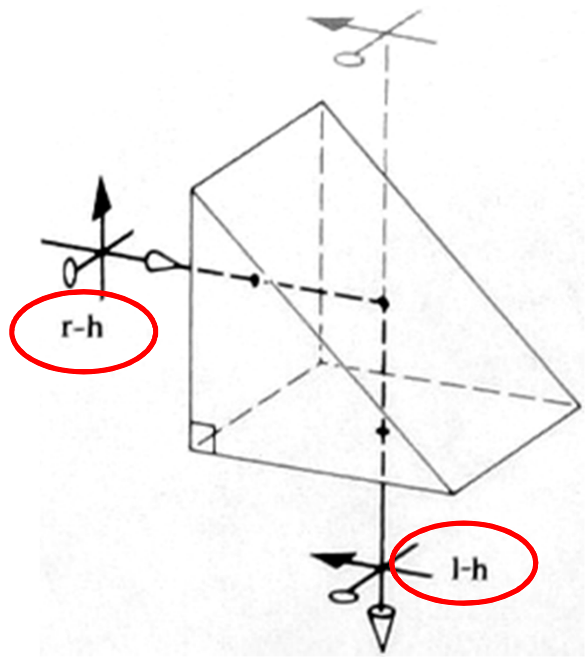




Reflecting Prisms

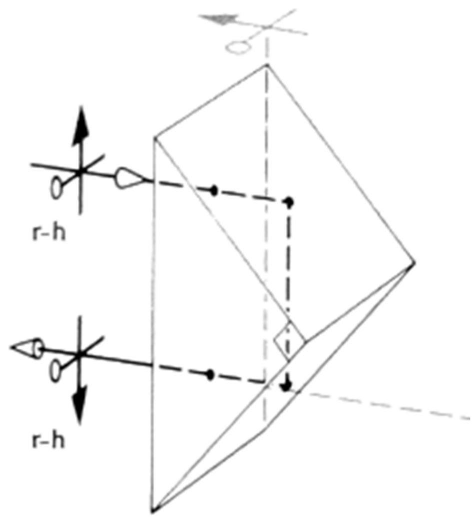
- Why not just use a mirror?
  - Mirrors produce a reflected image
- Prisms can provide ways to change the direction of light while simultaneously transforming the orientation of an image.



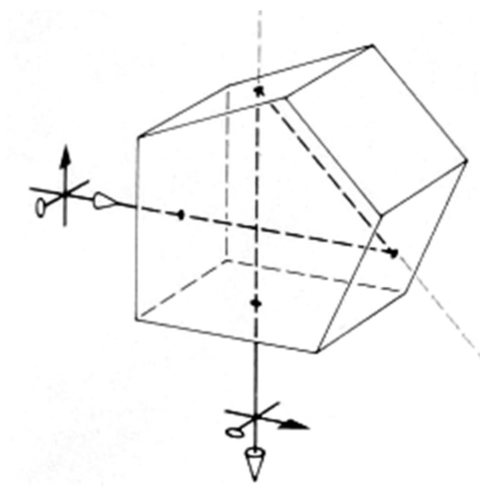


Reflecting Prisms

- Two internal reflections restores the orientation of the original image.

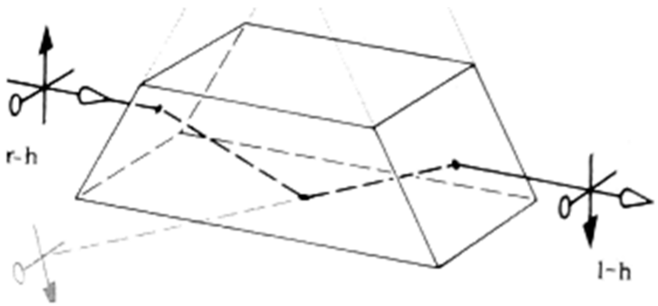


The Porro prism



The penta prism

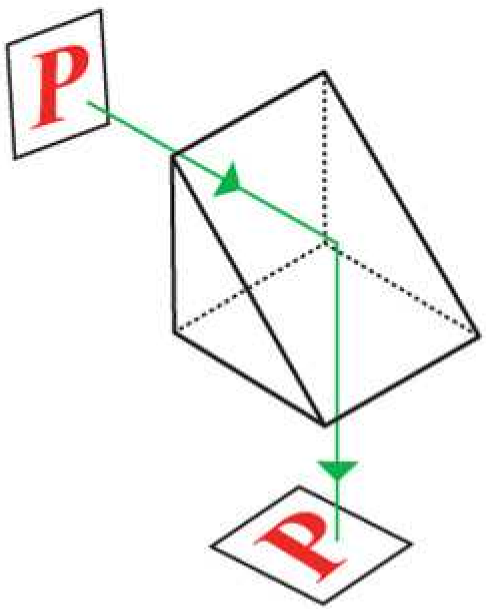
Dove Prism/Image Rotator

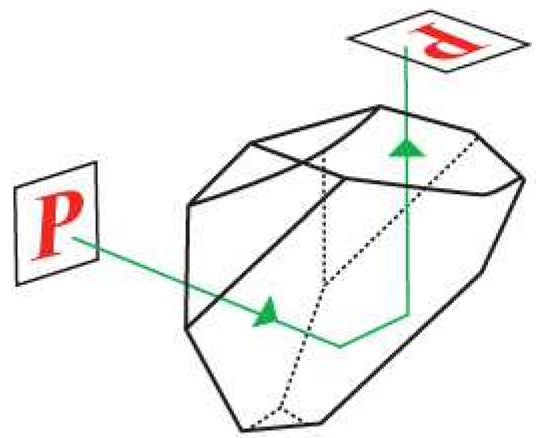


The Dove prism

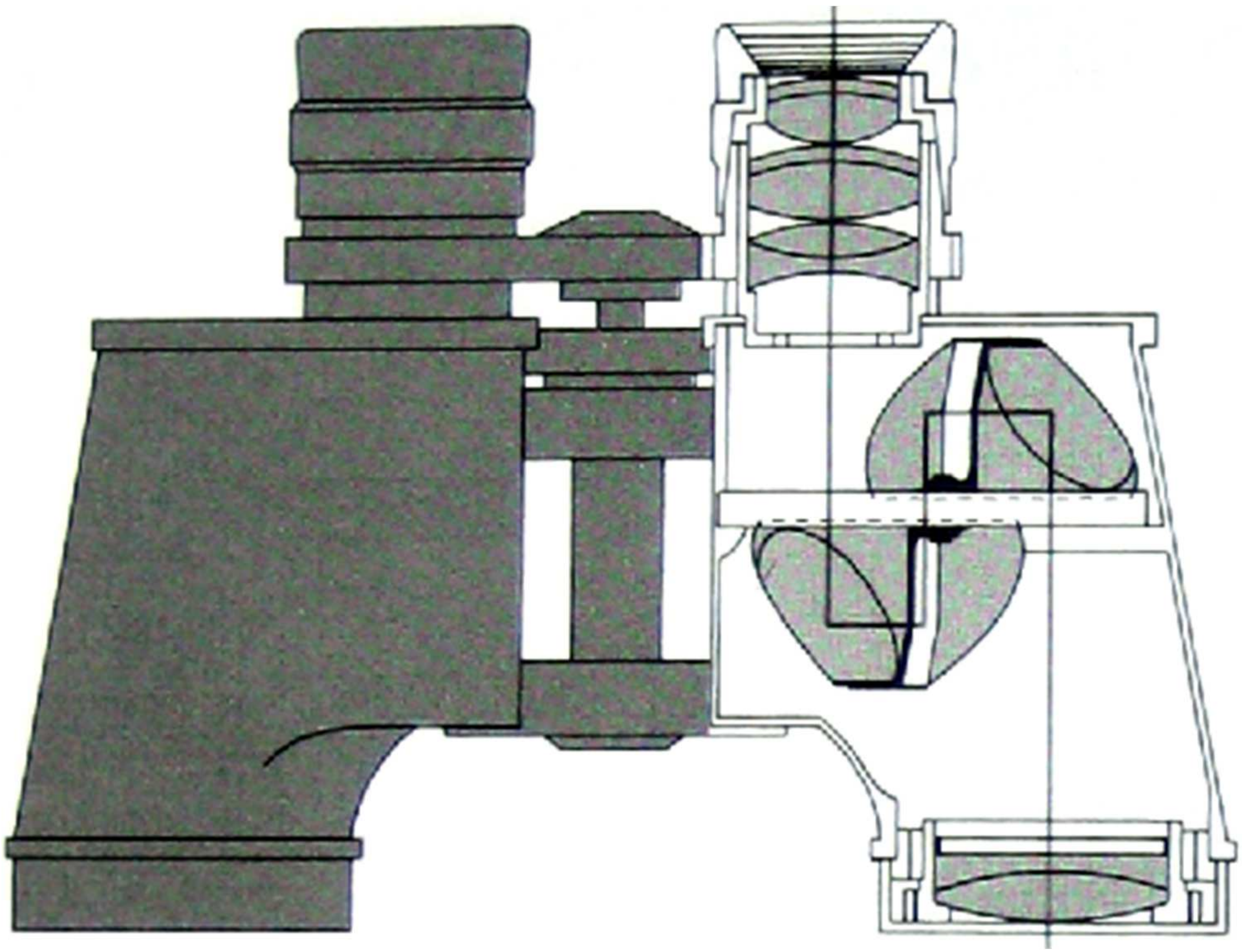
Roof Prism

- Right-angle reflection without image reversal (image rotation)

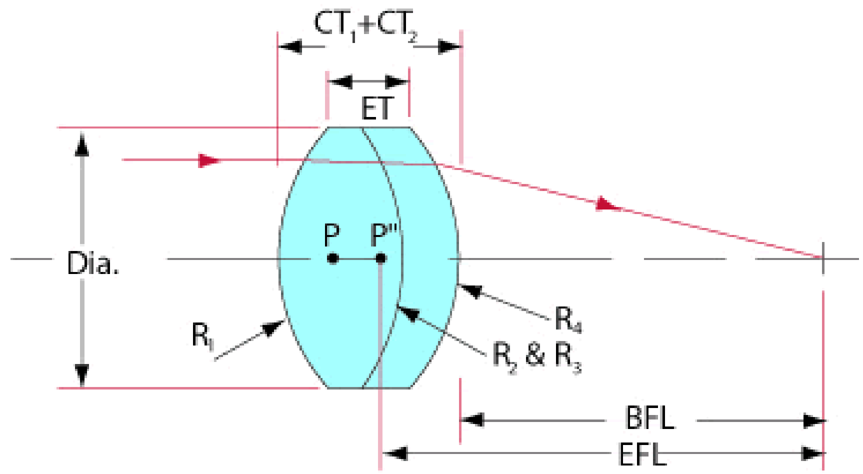


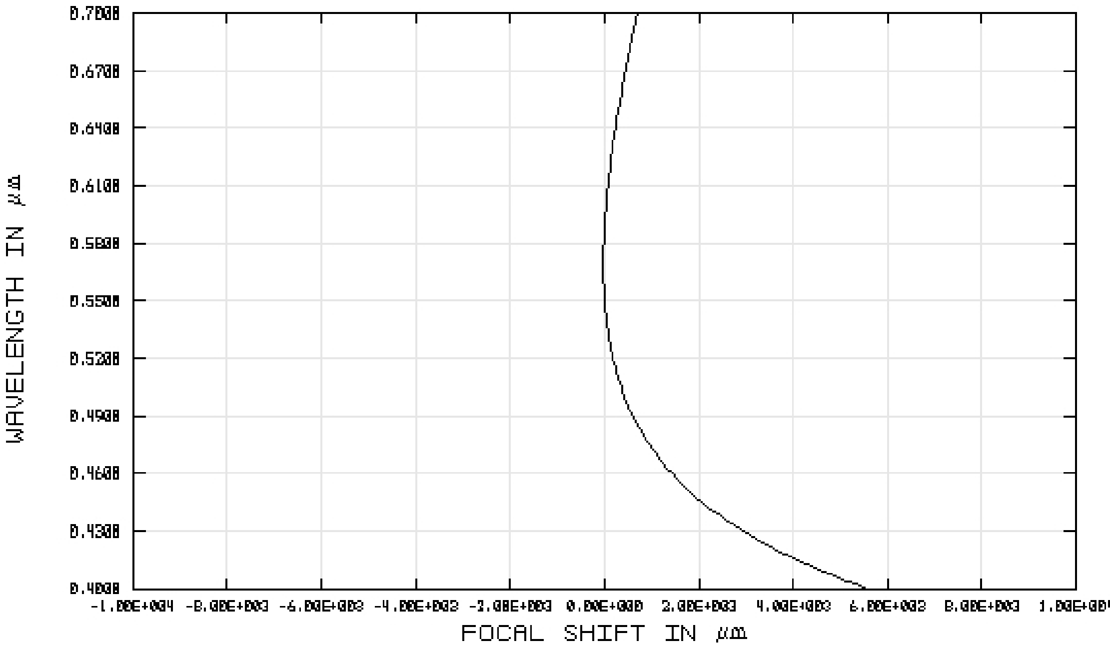


Binoculars



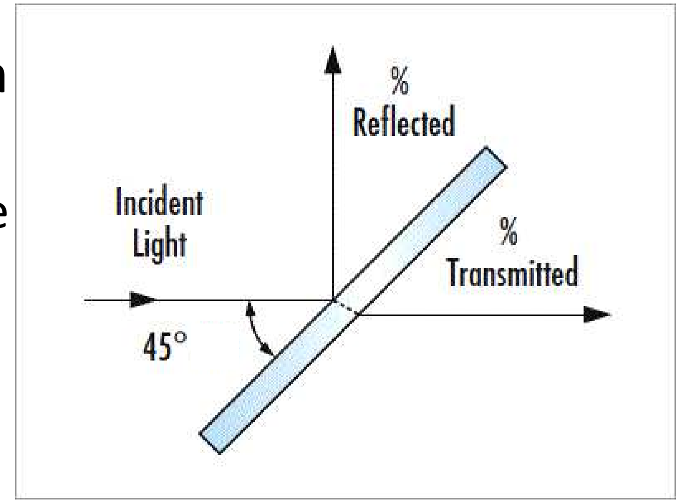
Achromatic Doublets





Beam Splitters

- Reflect half the light in a different direction
- Important application: interferometry
  - Transmitted and reflected beams are phase coherent.
- Beam splitter plate
  - Partially reflective surfaces



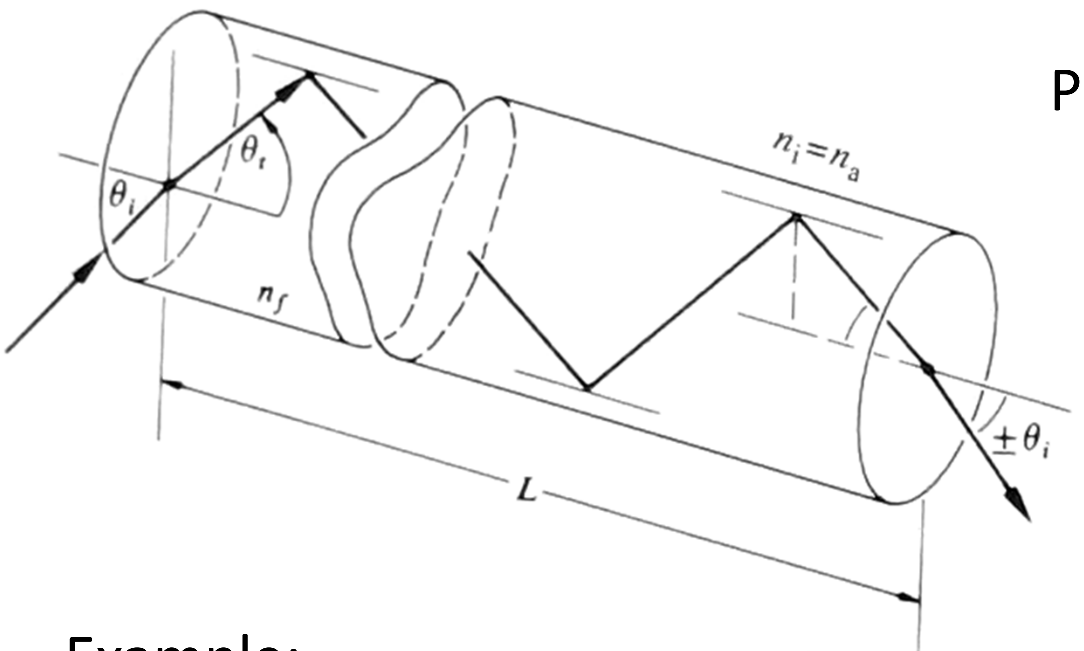
- Beam splitter cube:
  - Right angle prisms cemented together
  - Match transmission of both polarization components

Fiber Optics

- Development of fiber optics:
  - 1854: John Tyndall demonstrated that light could be bent by a curved stream of water
  - 1888: Roth and Reuss used bent glass rods to illuminate body cavities for surgical procedures
  - 1920's: Baird and Hansell patented an array of transparent rods to transmit images
- Significant obstacles:
  - Light loss through the sides of the fibers
  - Cross-talk (transfer of light between fibers)
  - 1954: Van Heel studied fibers clad with a material that had a lower index of refraction than the core

Fiber Optics: Losses

Consider large fiber: diameter $D >> \lambda \rightarrow$ can use geometric optics



Path length traveled by ray:

$$l = L / \cos\theta_t$$

Number of reflections:

$$N = \frac{l}{D / \sin\theta_t} \pm 1$$

Using Snell's Law for θ_t:

$$N = \frac{L\sin\theta_i}{D\sqrt{n_f^2 - \sin^2\theta_i}} \pm 1$$

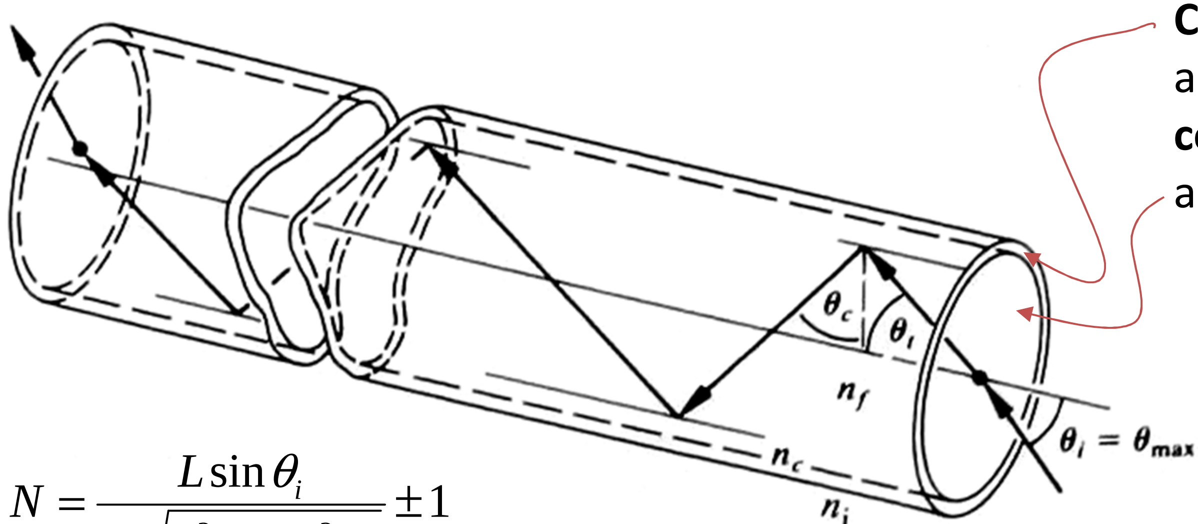
Example:

L = 1 km, D = 50 μm, n_f=1.6, θ_i = 30°

N = 6,580,000

Note: frustrated internal reflection, irregularities $\rightarrow$ losses!

‘Step-index’ Fiber



Cladding - transparent layer around the **core** of a fiber (reduces losses and *f/#*)

For total internal reflection need $n_c<n_f$

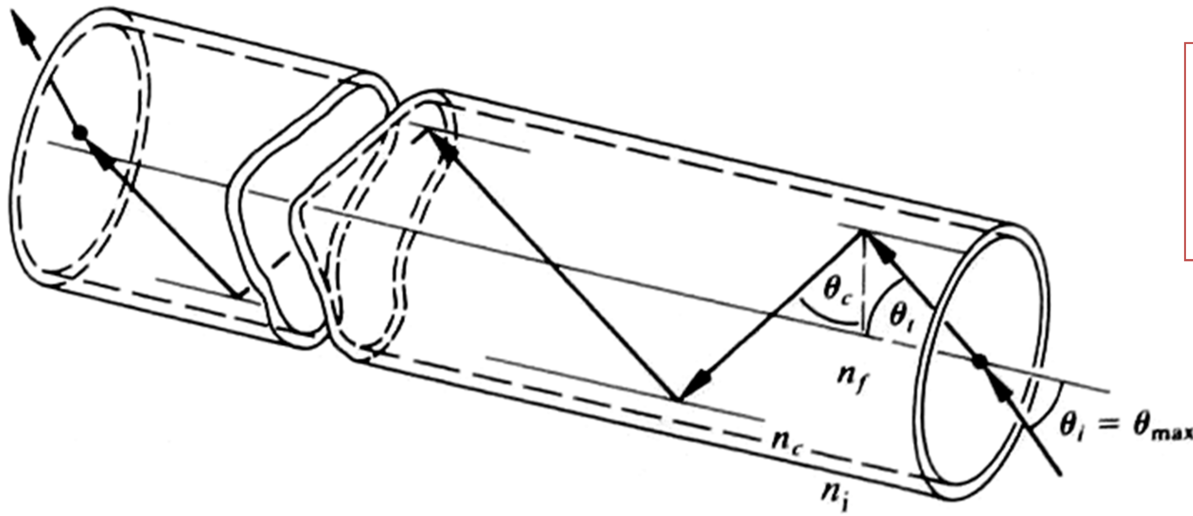
$$N = \frac{L\sin\theta_i}{D\sqrt{n_f^2 - \sin^2\theta_i}} \pm 1$$

For lower losses need to reduce *N*, or maximal θ_i, the latter is defined by critical angle for total internal reflection:

$$\sin\theta_c = \frac{n_c}{n_f} = \sin(90^\circ - \theta_t) = \cos(\theta_t) \Rightarrow \boxed{\sin\theta_{max} = \frac{\sqrt{n_f^2 - n_c^2}}{n_i}}$$

n_i=1 for air

Fiber and *f*/#



$$\sin\theta_{\max} = \frac{\sqrt{n_f^2 - n_c^2}}{n_i}$$

Angle θ_{max} defines the light gathering efficiency of the fiber, or numerical aperture NA:

$$NA \equiv n_i \sin\theta_{\max} = \sqrt{n_f^2 - n_c^2}$$

And *f*/# is:

$$f/\# \equiv \frac{1}{2(NA)}$$

Largest NA=1
Typical NA = 0.2 … 1

Data Transfer Limitations

1. Distance is limited by losses in a fiber. Losses α are measured in decibels (dB) per km of fiber (dB/km), i.e. in logarithmic scale:

$$\alpha \equiv -\frac{10}{L}\log\left(\frac{P_o}{P_i}\right) \quad \Rightarrow \quad \frac{P_o}{P_i} = 10^{-\alpha L/10}$$

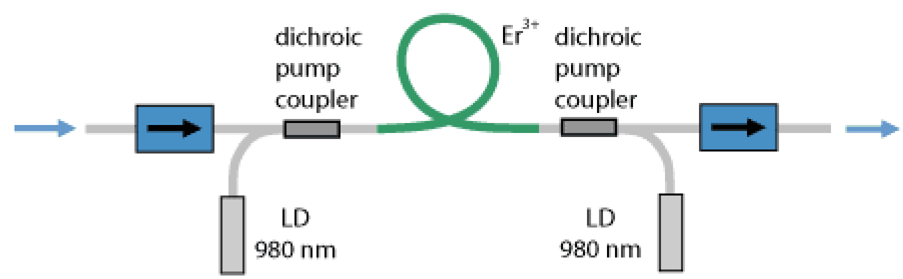
P_o - output power
P_i - input power
L - fiber length

Example:

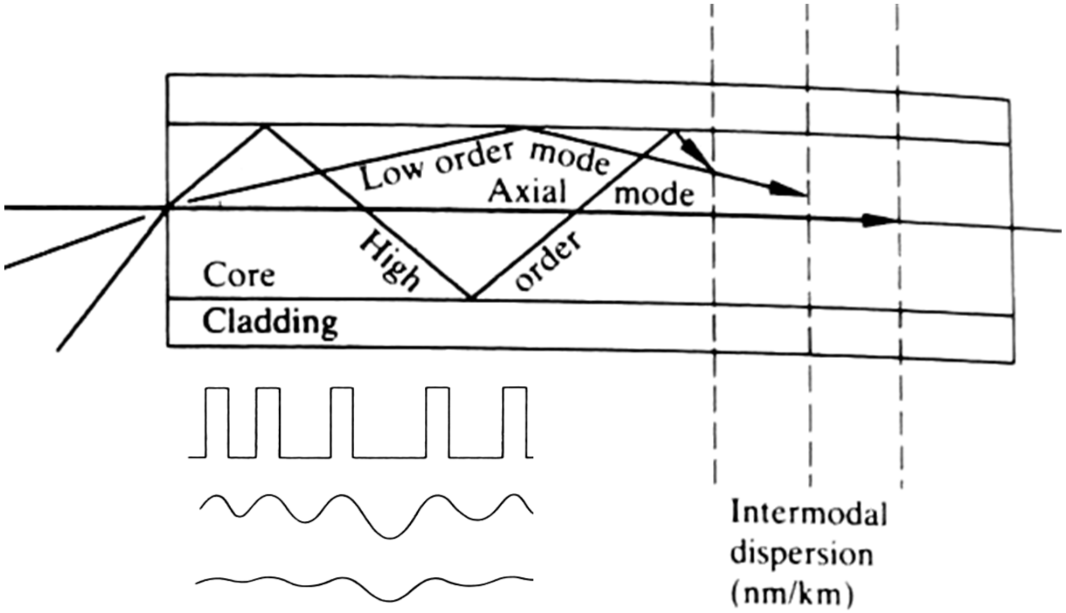
α	P_o/P_i over 1 km
10 dB	1:10
20 dB	1:100
30 dB	1:1000

Workaround: use light amplifiers to boost and relay the signal →

2. Bandwidth is limited by pulse *broadening* in fiber and processing electronics



Pulse Broadening



Multimode fiber: there are many rays (modes) with different OPLs and initially short pulses will be broadened (*intermodal dispersion*)

For ray along axis: $$t_{\min} = L/v_f = Ln_f/c$$

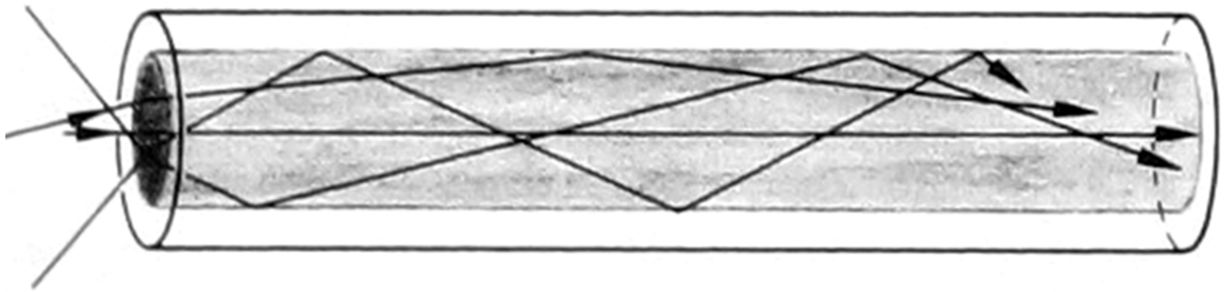
For ray entering at θ_{max}: $$t_{\max} = l/v_f = Ln_f^2/(cn_c)$$

The initially short pulse will be broadened by:

Making n_c close to n_f reduces the effect! →

$$\Delta t = t_{\max} - t_{\min} = \frac{Ln_f}{c}\left(\frac{n_f}{n_c} - 1\right)$$

Pulse Broadening: Example



n_f = 1.5
n_c=1.489
Estimate the bandwidth limit for 1000 km transmission.

Solution:

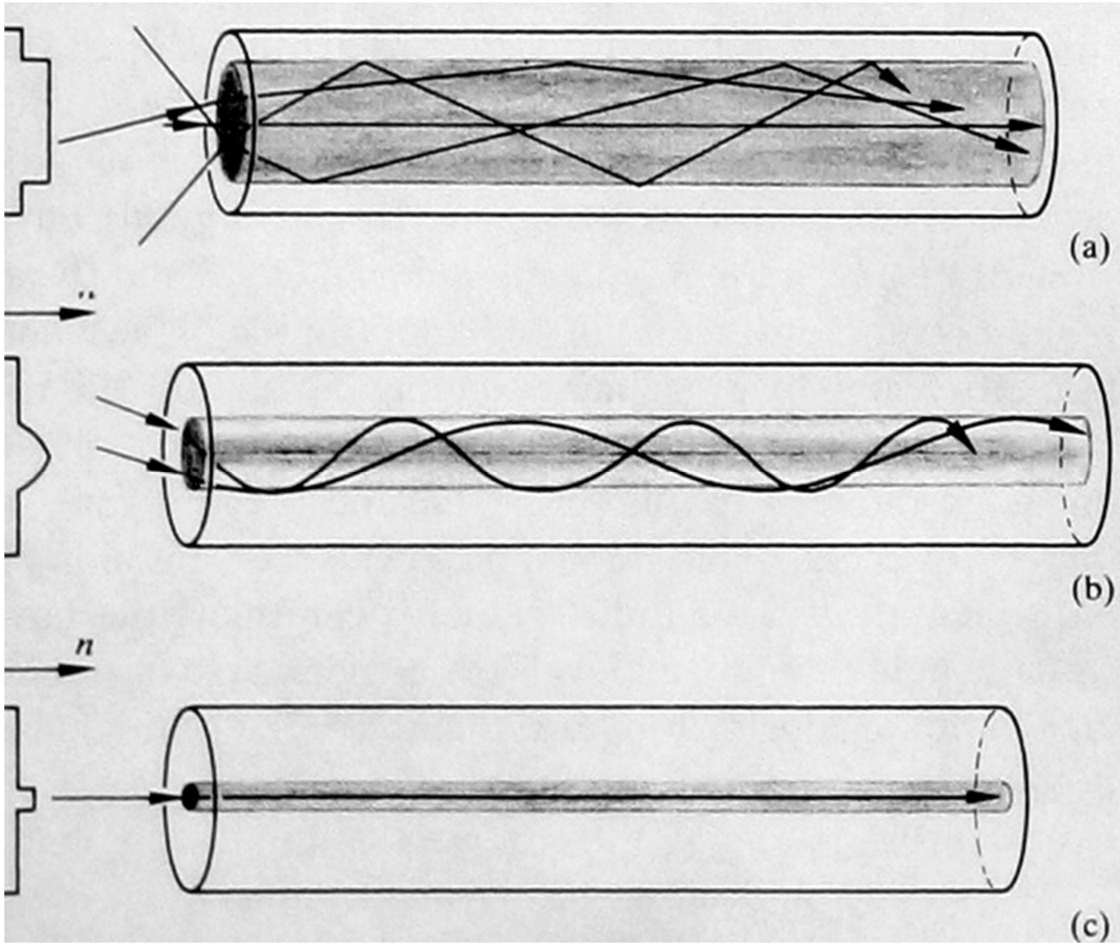
$$\Delta t = \frac{Ln_f}{c}\left(\frac{n_f}{n_c}-1\right) = \frac{10^6 \cdot 1.5}{3\times10^8}\left(\frac{1.5}{1.489}-1\right)s = 3.7\times10^{-5}s = 37\mu s$$

Even the shortest pulse will become ~37 μs long

Bandwidth ~ $\dfrac{1}{3.7\times10^{-5}s} = 27\ \text{kbps}$ ← kilobits per second = ONLY 3.3 kbytes/s

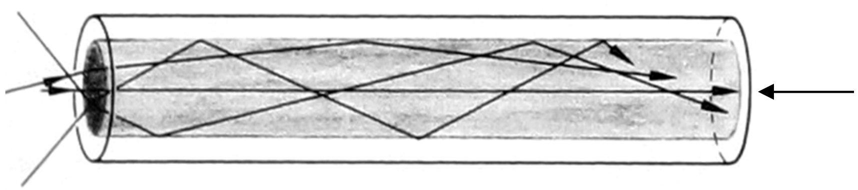
Multimode fibers are not used for communication!

Graded and Step Index Fibers



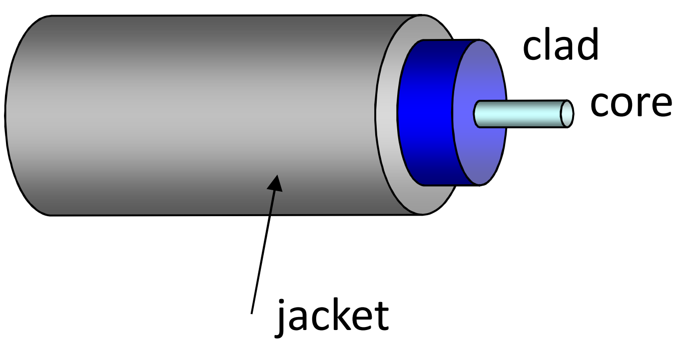
Step index: the change in n is abrupt between cladding and core
Graded index: n changes smoothly from n_c to n_f

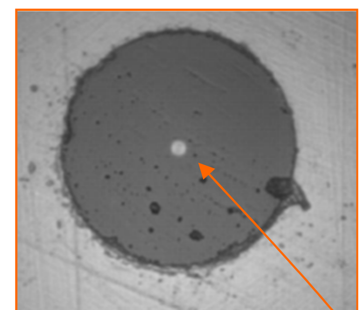
Single Mode Fiber



To avoid broadening need to have only one path, or mode

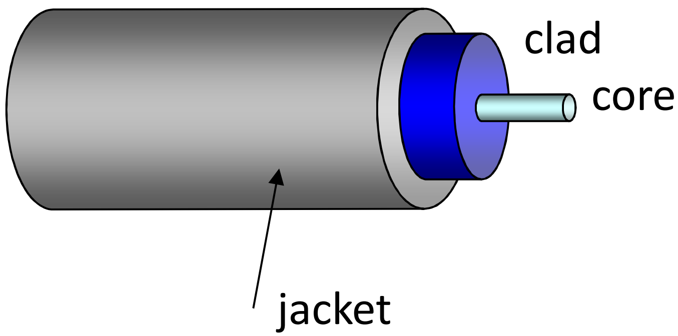
Single mode fiber: there is only one path, all other rays escape from the fiber





Geometric optics does not work anymore: need wave optics.
Single mode fiber core is usually only 2-7 micron in diameter

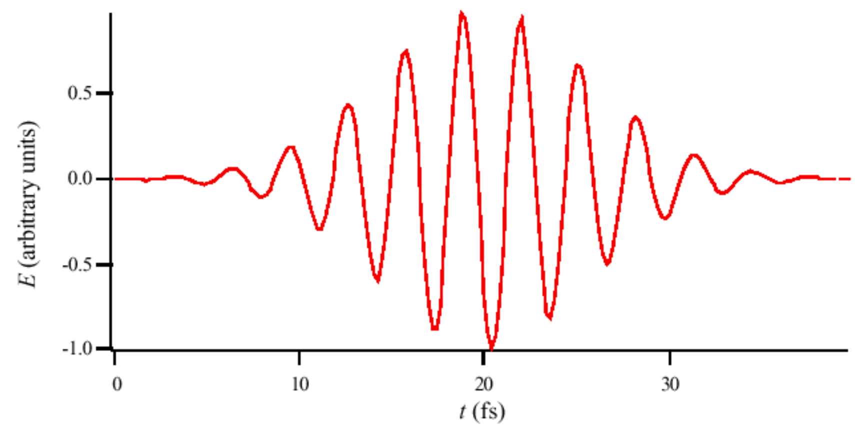
Single Mode Fiber: Broadening



Problem: shorter the pulse, broader the spectrum.
refraction index depends on wavelength

'Transform' limited pulse
product of spectral full width at half maximum (fwhm) by time duration fwhm:

$$\Delta f \Delta t \approx 0.2$$



A 10 fs pulse at 800 nm is ~40 nm wide spectrally
If second derivative of *n* is not zero this pulse will broaden in fiber rapidly

Solitons: special pulse shapes that do not change while propagating